





# SAC-GNC: SAmple Consensus for adaptive Graduated Non-Convexity

Valter Piedade, Chitturi Sidhartha, José Gaspar, Venu Madhav Govindu, and Pedro Miraldo





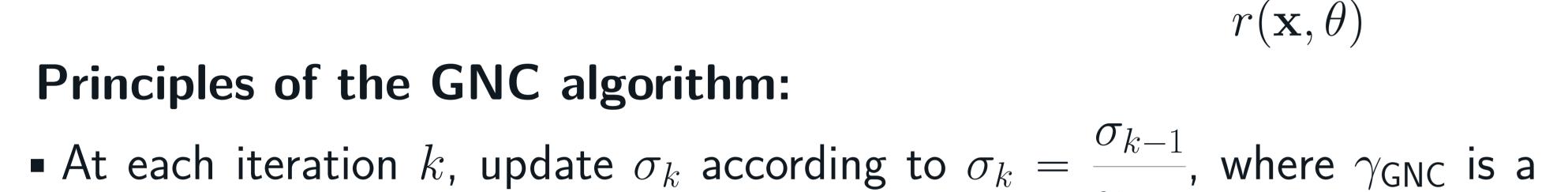
## Graduated Non-Convexity (GNC)

> GNC is a robust estimation algorithm that removes the dependency of the shape of the robust loss function by solving: Geman-McClure loss

$$heta^* = \mathop{\mathsf{argmin}}_{ heta} \sum_{i=1}^N 
ho_\sigma \left( r\left( \mathbf{x}_i, heta 
ight) 
ight),$$

where  $\rho_{\sigma}(\cdot)$  is, e.g., the Geman-McClure loss;

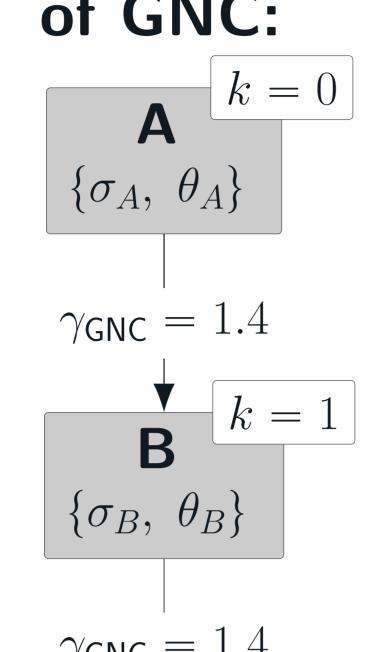
➤ GNC is highly effective in computer vision tasks like 3D registration and pose graph optimization, where robustness is essential.

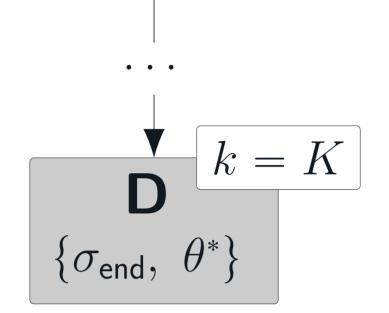


• Iterate until a predefined  $\sigma_{\rm end}$  is reached.

fixed annealing factor;

### Diagram of GNC:

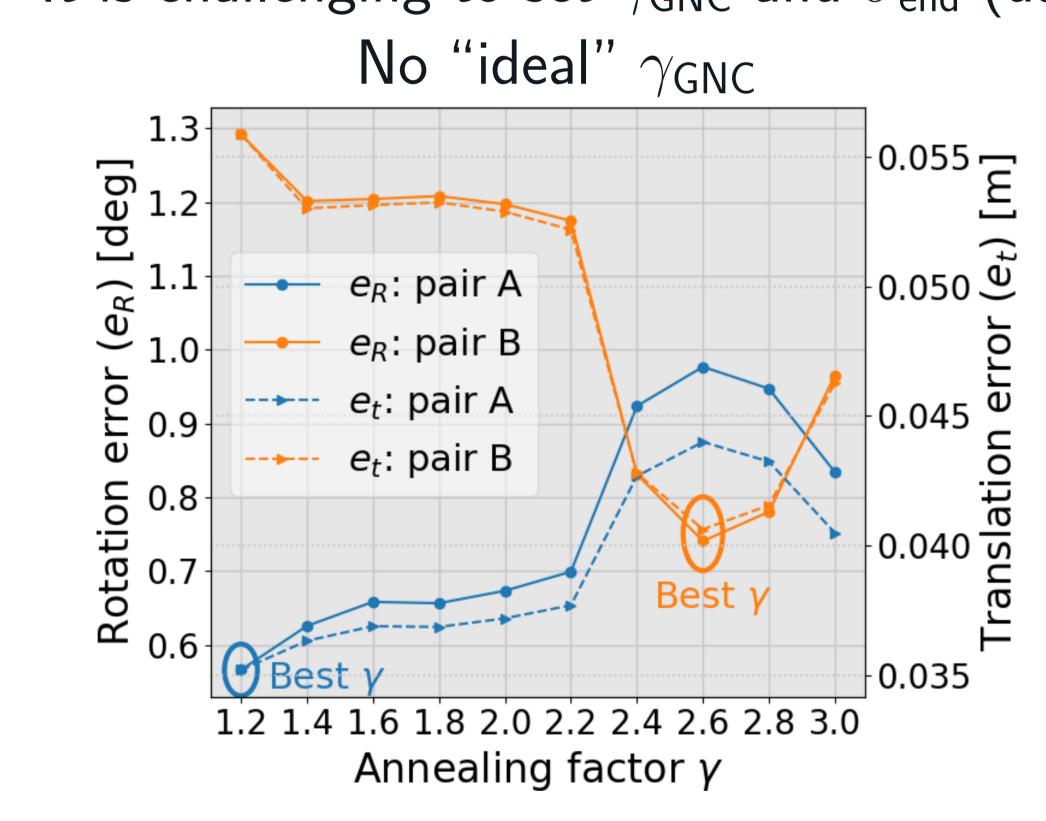


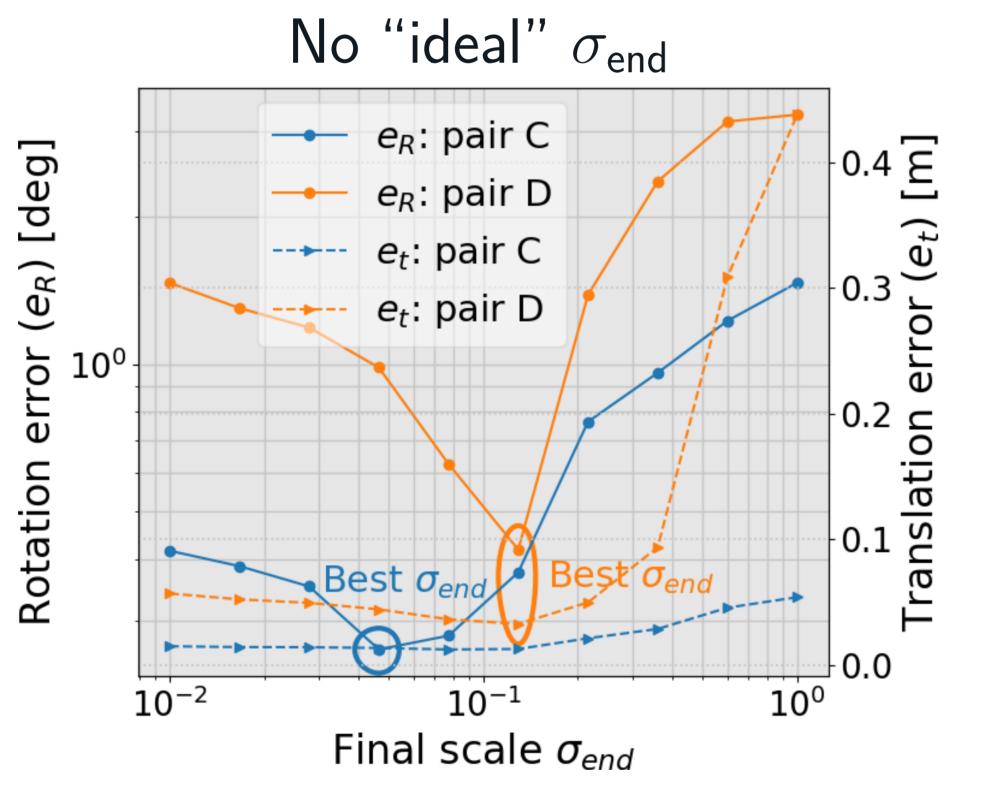


 $\rightarrow$  Previous GNC-based approaches use a fixed annealing factor  $\gamma_{\rm GNC}$  and a predefined  $\sigma_{\rm end}$ ;

GNC's Limitations

 $\rightarrow$  It is challenging to set  $\gamma_{\rm GNC}$  and  $\sigma_{\rm end}$  (data-dependent):





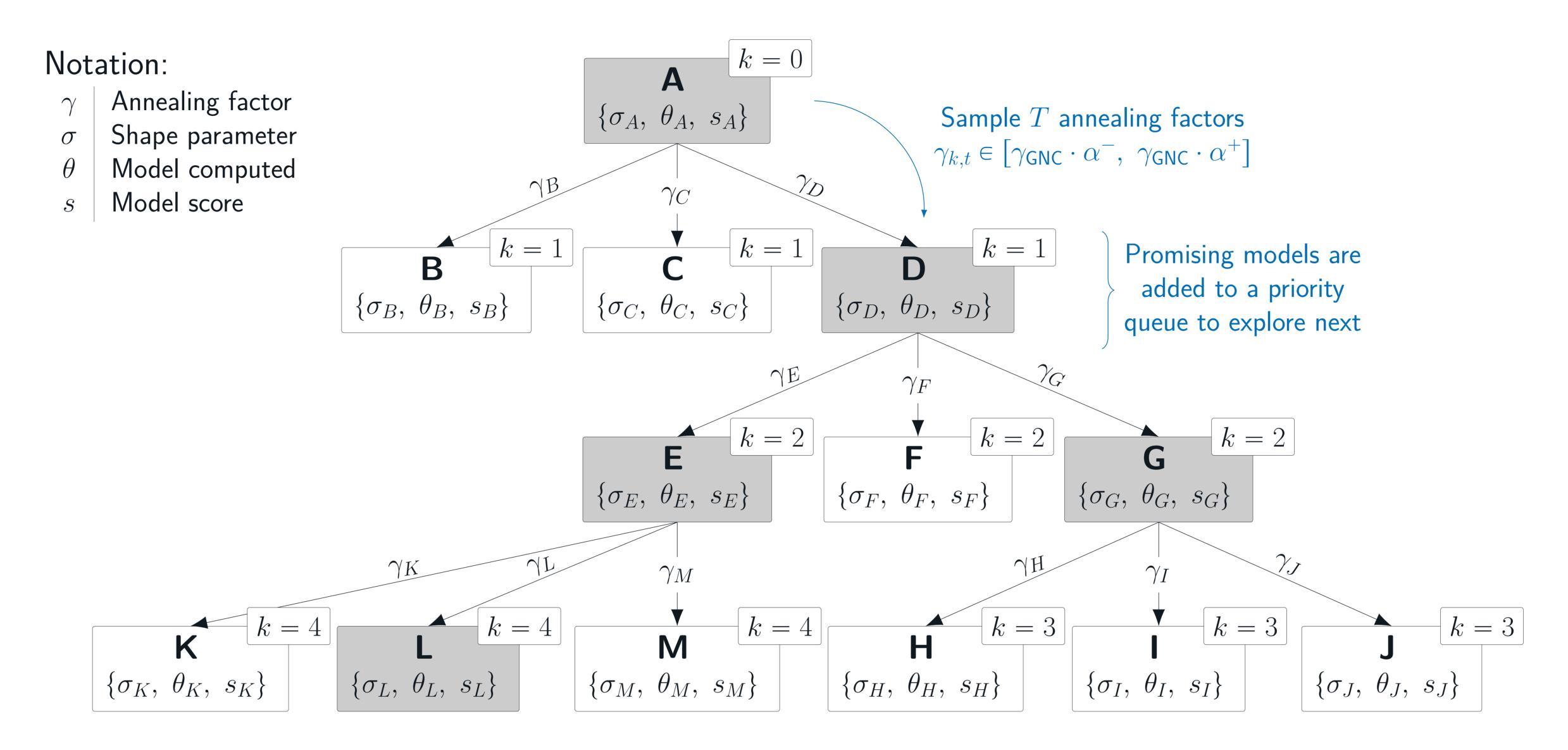
 $\rightarrow$  Previous methods assume a continuous decrease of  $\sigma$  leads to the best solution, *i.e.*, the model estimate of the next iteration will be better than the current model.

### Our Contributions

- > A novel GNC-based adaptive annealing strategy for robust and efficient estimation;
- > We show that combining sample & consensus (discrete) into GNC (continuous) has benefits over previous approaches;
- Experiments in real-world data in 3D registration and pose graph optimization show that SAC-GNC outperforms baselines in accuracy and efficiency.

## SAmple Consensus for adaptive GNC

Diagram of the proposed online searching strategy:



### Online search for $\sigma$

- 1. Annealing sampling  $-\gamma_{k,t} \in [\gamma_{\mathsf{GNC}} \cdot \alpha^-, \gamma_{\mathsf{GNC}} \cdot \alpha^+], t \in \{1, \cdots, T\};$
- 2. Model scoring (e.g., MSAC);
- Relaxation ( $\alpha^{\pm}$ ) of the fixed annealing factor  $\gamma_{\text{GNC}}$ ;
- lacktriangle Do not assume decreasing  $\sigma$  always leads to better solutions; 4. Save the best hypothesis.
  - Allows exploring various promising solutions.

### Algorithm 1: SAC-GNC

3. Priority queue search;

**Input** – Data  $\mathcal{D}$ ; annealing parameter  $\gamma_{GNC}$ ; number of trial hypotheses T.

**Output** – Best hypothesis  $\mathcal{H}^* = \{\sigma^*, \theta^*, s^*, d^*\}$ . 1  $k \leftarrow 0$ ,  $Q \leftarrow$  empty queue,  $\mathcal{H}_0 \leftarrow \{\sigma_0, \theta_0, \infty, 0\}$ ; 2  $\theta_0 \leftarrow \text{compute\_initial\_model}(\mathcal{D});$ 

3  $\sigma_0$  ← shape\_initialization  $(\theta_0)$ ; 4 while True do

5  $k \leftarrow k+1$ ; 6  $\{\sigma_{k-1}, \theta_{k-1}, s_{k-1}, d_{k-1}\} \leftarrow \mathcal{H}_{k-1};$ 

**for** t = 1 : T **do**  $\gamma_{k,t} \leftarrow \text{get\_annealing}(\gamma_{\text{GNC}}, \alpha^{\pm});$ 

 $\sigma_{k,t} \leftarrow \mathsf{update\_shape} (\sigma_{k-1}, \ \gamma_{k,t});$  $\theta_{k,t} \leftarrow \text{compute\_model}(\mathcal{D}, \theta_{k-1}, \sigma_{k,t});$ 11  $s_{k,t} \leftarrow \text{compute\_score}(\mathcal{D}, \theta_{k,t});$ 12 |  $d_{k,t} \leftarrow d_{k-1} + 1$ ;

13  $\mathcal{H}_{k,t} \leftarrow \{\sigma_{k,t}, \theta_{k,t}, s_{k,t}, d_{k,t}\};$ 14  $Q \leftarrow add_{to}(\{\mathcal{H}_{k,i}\});$ 15  $\mathcal{H}^* \leftarrow \text{save\_best\_hypothesis}(\mathcal{H}^*, \{\mathcal{H}_{k,i}\});$ 

if  $stopping\_criteria(Q, \mathcal{H}^*)$  then 18  $\mathcal{H}_k \leftarrow \text{get}\_\text{next}\_\text{hypothesis}(\mathcal{Q});$ 

### Stopping criteria

- 1. Check for model or scoring convergence;
- 2. Queue is empty.
  - No need for a predefined  $\sigma_{\text{end}}$ .

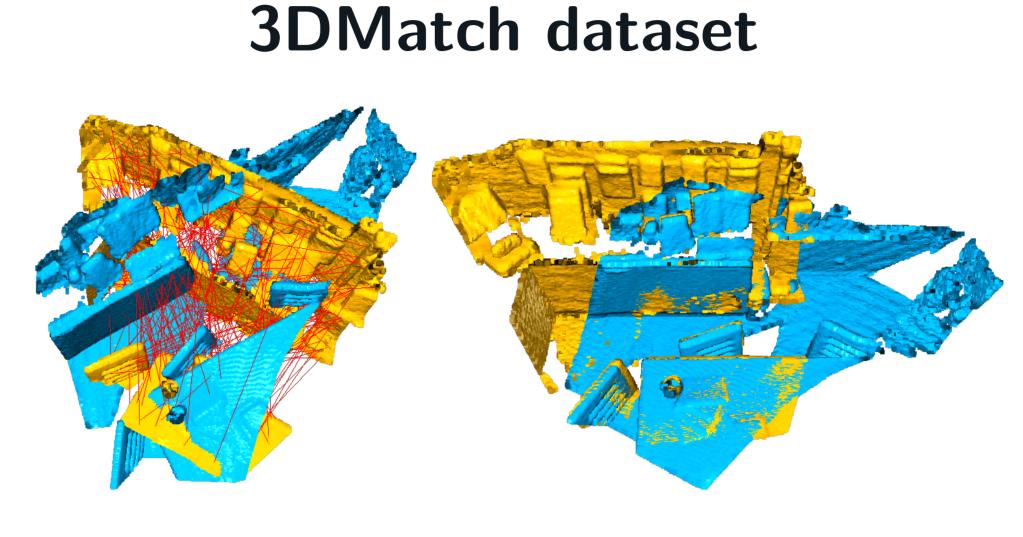
### Initialization

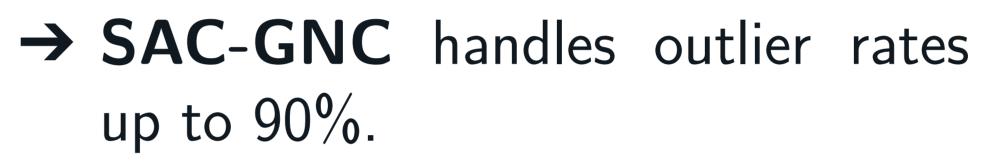
Compute a  $\sigma_0$  that approximates a least-squares solution, by making the data point with maximum residual,  $r_{\rm max}$ , weight  $\approx 1$ :

$$w = \left(\frac{1}{1 + r_{\text{max}}^2/\sigma_0^2}\right)^2 \Leftrightarrow \sigma_0 = \frac{r_{\text{max}}}{\sqrt{\left(w^{-1/2} - 1\right)}}$$

## **Experiments and Results**

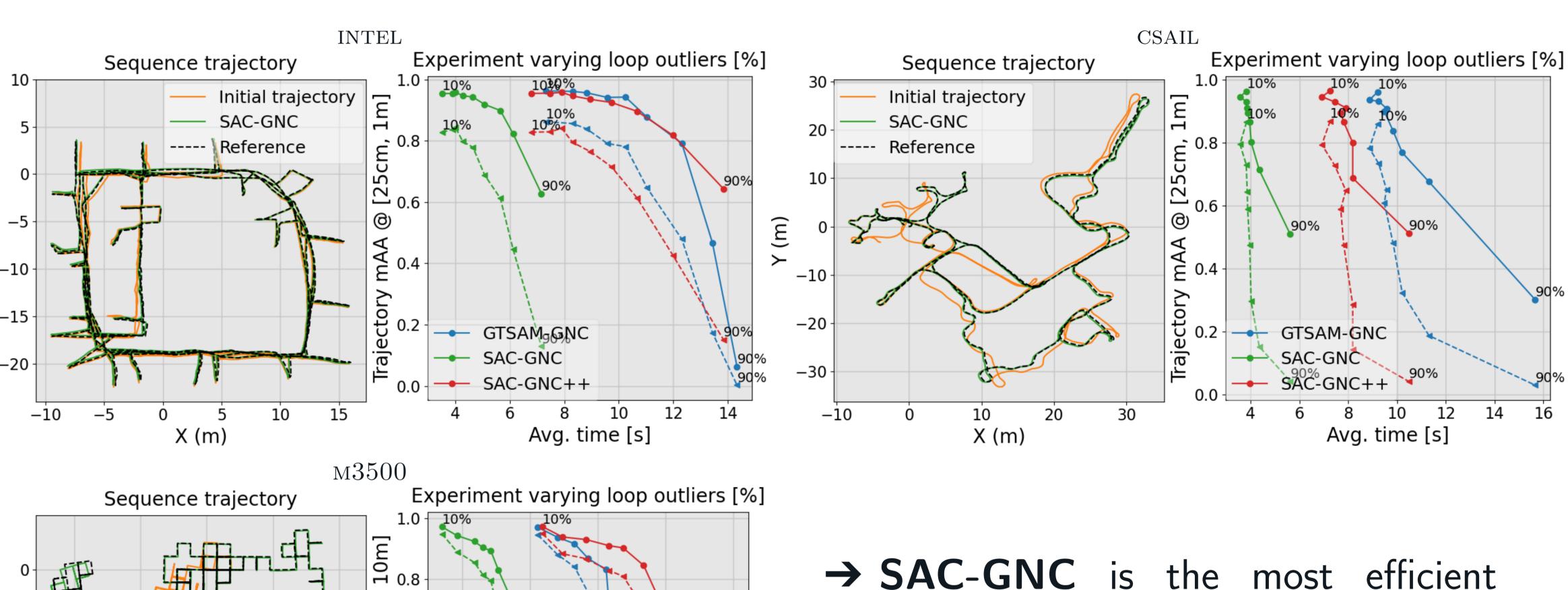
### > 3D registration





Dataset	Method		Time [ms] ↓				
Dataset		$(e_R, 5^\circ)$					
$\uparrow$ inliers $\approx 59.8\%$	RANSAC	0.492	0.707	0.713	0.829	5.13	
	FGR	0.514	0.693	0.706	0.804	16.5	
	TEASER++	Runtime fail ( $>$ 30 minutes per instance)					
	GNCp	0.553	0.726	0.738	0.831	5.61	
	SAC-GNC	0.584	0.750	0.756	0.842	3.98	
· <b>-</b>	SAC-GNC++	0.586	0.753	0.759	0.846	18.4	
$ \downarrow$ inliers $\approx 11.6\%$	RANSAC	0.279	0.446	0.455	0.577	21.6	
	FGR	0.271	0.417	0.443	0.549	12.6	
	TEASER++	0.287	0.441	0.429	0.555	125	
	GNCp	0.348	0.491	0.509	0.603	25.7	
	SAC-GNC	0.421	0.539	0.544	0.616	7.05	
	SAC-GNC++	0.435	0.560	0.565	0.640	56.6	

### ➤ Pose graph optimization on SLAM sequences



→ SAC-GNC outperforms GTSAM in

without losing accuracy;

accuracy for > 50% outliers. Avg. time [s]

### **➤** Ablation studies

Initial trajectory

$\sigma$ Search	Stopping Criteria	Initialization		$(e_{\mathbf{t}}, 0.3m)$	- Iter. ↓ )	Time ↓ [ms]
			0.465	0.612	28	7.45
			0.457	0.605	13.8	5.82
			0.487	0.619	11.9	7.47
			0.487	0.618	10.9	6.80
			0.490	0.622	7.83	6.69
			0.490	0.623	6.72	5.98

- $\rightarrow \sigma$  search improves accuracy the most;
- → Initialization and stopping criteria add efficiency without compromising accuracy.