**Phase Unwrapping in Correlated Noise for FMCW Lidar Depth Estimation**

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**Motivation**

- Object distance \( d \) causes a delay \( \tau \) in received light RX
- Optically mixing RX and local oscillator LO creates an interference pattern with beat frequency \( f_{\text{beat}} \) proportional to \( \tau \)
- Goal: estimate distance from \( f_{\text{beat}} \)
- Problem: phase noise causes deviation from \( f_{\text{beat}} \) that degrades performance of depth estimators

**Measurement Model**

- Transmitted signal: \( E(t) = \cos(\pi r^2 + \omega t + \phi(t)) \)
- Received signal: \( E(t) = \sqrt{R} \cos((\pi r^2 + \omega t + \phi(t)) + \phi(\tau - \tau)) + w(t) \)
- Interference signal intensity:
  \[
  i(t) = \sqrt{R} \cos((\pi r^2 + \omega t + \phi(t)) + \phi(\tau - \tau)) + w(t)
  \]
- Phase noise variance and correlation increases with delay \( \tau \)

**Periodogram maximization (Rife and Boorstyn 1974)**
- ML estimator of frequency in AWGN
- Doesn’t account for phase noise

**Lorentzian fitting (Kim et al. 2018)**
- PSD of interference signal approaches Lorentzian distribution
- Robust to phase noise
- Approximate noise model lacks precision at high SNR

**Phase unwrapping + linear regression (Tretter 1985)**
- Unwrapped phase is affine function in additive Gaussian noise
- Requires accurate phase unwrapping

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**Proposed Depth Estimation Algorithm**

- Our method estimates the beat frequency by alternating between phase unwrapping and linear regression
- Both steps account for the second-order statistics of the phase noise

**Viterbi Phase Unwrapping**

- Solve for phase unwrapping by predicting the phase noise for each sample
- Select the most likely unwrapping sequence among multiple possible trajectories

**LMMSE prediction of observed phase noise**

\[
\hat{\phi}_{\text{LMMSE}}(t) = \frac{y_k(t)^2}{\sigma^2} + \frac{\hat{\phi}(t)}{\sigma^2} + \hat{\phi}_{\text{wrapped}}(t)
\]

**Approximate log-likelihood of the unwrapped phase**

\[
\log |\mathbb{P}(r_k y_{\text{unwrapped}} - r_k^2 \hat{\phi}_{\text{LMMSE}} + \hat{\phi}_{\text{wrapped}}/2)| + N \cdot \log(2^2)
\]

**Performance vs distance**

\[ \Delta_1 = 1 \text{ MHz} \]

\[ \Delta_2 = 10 \text{ MHz} \]

Proposed approach is the most robust to phase noise (which increases with linewidth \( \Delta_1 \) and distance \( d \))

**Performance vs SNR (additive noise)**

\[ \Delta_1 = 1 \text{ MHz} \]

\[ \Delta_2 = 10 \text{ MHz} \]

Proposed approach is robust to low SNR and achieves best overall performance at high SNR (low additive noise)

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**Conclusion**

1. Introduced a phase unwrapping-based algorithm specifically for accurate depth estimation from FMCW lidar measurements with significant phase noise and over long range
2. Demonstrated that our method can achieve the best accuracy at high SNR
3. Could enable use of cheaper swept-frequency lasers with significant phase noise for use in FMCW lidar, SS-OCT, coherent communications, etc.

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**References**