Variational Quantum Compressed Sensing for Joint User and Channel State Acquisition in Grant-Free Device Access Systems

Bryan Liu
Toshiaki Koike-Akino
Ye Wang
Kieran Parsons

May 16, 2022

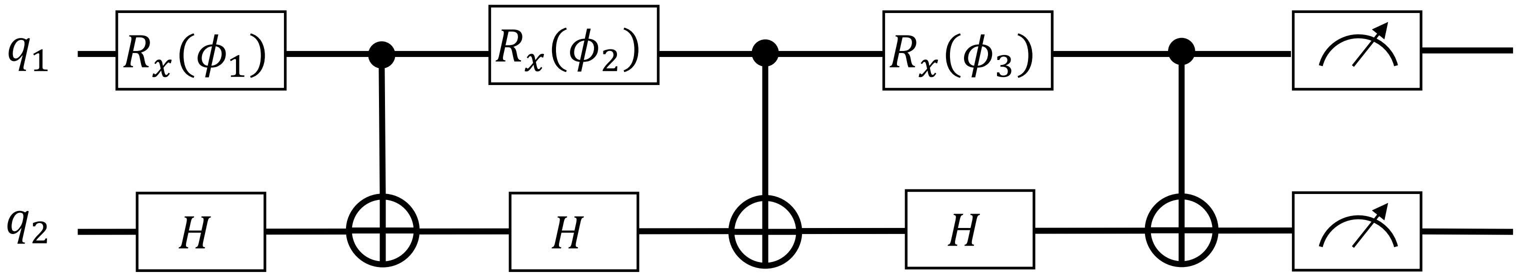
MITSUBISHI ELECTRIC RESEARCH LABORATORIES (MERL)
Cambridge, Massachusetts, USA
http://www.merl.com

Agenda

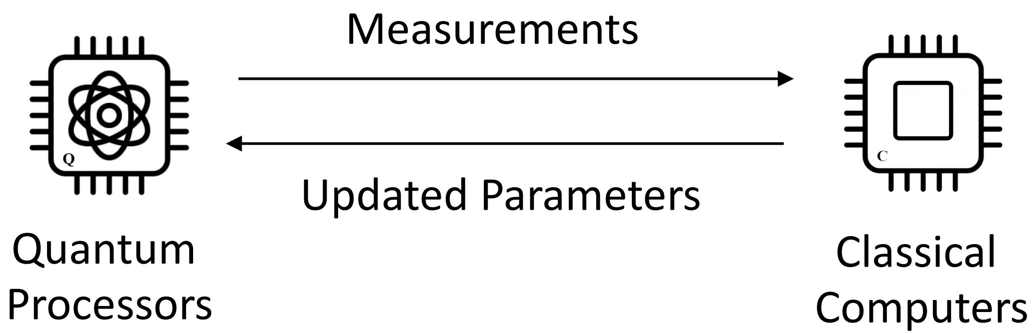
- Quantum Machine Learning
- System Model
- Compressed Sensing
- Proposed Variational Quantum Compressed Sensing
  - Embedding
  - Denoising
  - State Preparation
  - Variational Quantum Circuit
- Numerical Results
- Conclusion

Quantum Machine Learning

An Example of a Quantum Circuit:



In a quantum machine learning system, these rotation gates parameters, ϕ_1, ϕ_2 and ϕ_3 can be trainable to achieve tasks such as classification and regression.



- Software toolkits:
  - Pennylane
  - Qiskit
- Hardware supply:
  - IBM
  - Xanadu
  - Microsoft Azure Quantum

Quantum Machine Learning

- Literatures on Quantum Machine Learning and Wireless Communications
  - Quantum Approximate Optimization Algorithm (QAOA) and channel decoding (approximate ML):
    - T. Matsumine, T. Koike-Akino, and Y. Wang, "Channel decoding with quantum approximate optimization algorithm," *arXiv: 1903.02537*, 2019.
  - QAOA detection:
    - J. Cui, Y. Xiong, S. X. Ng, and L. Hanzo, "Quantum approximate optimization algorithm based maximum likelihood detection," *arXiv: 2107.05020,* 2021.
  - Belief-propagation with quantum messages:
    - Rengaswamy, N., Seshadreesan, K.P., Guha, S. *et al.* Belief propagation with quantum messages for quantum-enhanced classical communications. *NPJ Quantum Inf* 7**,** 97 (2021).
  - Quantum machine learning for communication networks
    - S. J. Nawaz, S. K. Sharma, S. Wyne, M. N. Patwary and M. Asaduzzaman, "Quantum Machine Learning for 6G Communication Networks: State-of-the-Art and Vision for the Future," in *IEEE Access*, vol. 7, pp. 46317-46350, 2019.

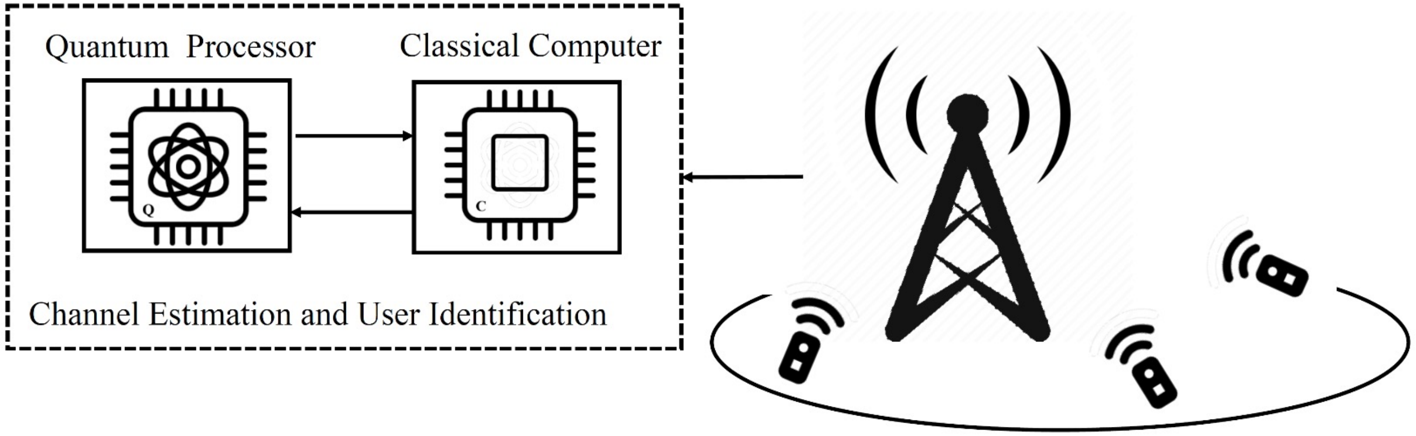
System Model

A massive machine-type communication system with a grant-free access scheme is assumed:

Unknown sequence

$$\boldsymbol{y} = \boldsymbol{A}\boldsymbol{x} + \boldsymbol{z},$$

- where $\boldsymbol{y} \in \mathbb{C}^{M}$, $\boldsymbol{A} \in \mathbb{C}^{M\times N}$, $\boldsymbol{x} = \boldsymbol{a}\boldsymbol{h}$, $\boldsymbol{h} \in \mathbb{C}^{N}$ and $\boldsymbol{z} \in \mathbb{C}^{M}$. $\boldsymbol{a}$ is the activity vector of each user that has $a_n \in \{0,1\}$ and $P(a_n = 1) = \rho$, $0 \le \rho \le 1$ for $n \in \{1, \ldots, N\}$. For a massive machine-type communication system, we assume that $M < N$.
- $h_i \sim \mathcal{N}(0, \rho^{-1})$. $\boldsymbol{z} \sim \mathcal{CN}(\boldsymbol{0}, \sigma^2 \boldsymbol{I})$. corr($a_i$, a_j)= $\gamma^{|i-j|}$ follows an autoregressive model, where corr(·) indicates the correlation coefficient between two variables.



Grant-Free Access Scheme

Iterative Compressed Sensing Algorithm

- Iterative shrinkage-thresholding algorithm (ISTA) [1]
  - Two-step iterative algorithm which follows the updates of:

$$\text{(Linear Estimation): } \boldsymbol{l}^t = \boldsymbol{q}^t + \boldsymbol{A}^H(\boldsymbol{y} - \boldsymbol{A}\boldsymbol{q}^t)$$

$$\text{(Non-linear Estimation): } \boldsymbol{q}^{t+1} = \eta_t(\boldsymbol{l}^t)$$

  ISTA has a low computational complexity but correspondingly, the convergence speed of ISTA might be slow, especially in extreme underdetermined problems

- Approximate message passing (AMP) algorithm [2] employs an "Onsager" term to manipulate and maintain the residual errors of the linear estimation to be Gaussian distributed. However, AMP replies on the assumption that the entries in the measurement matrix are samples from a Gaussian distribution. Starting from $\hat{\boldsymbol{x}}^0 = \boldsymbol{0}$ and $\hat{\boldsymbol{v}}^0 = \boldsymbol{y}$, the AMP algorithm has a two-step update of:

$$\hat{\boldsymbol{x}}^{t+1} = \eta_t(\boldsymbol{A}^H\hat{\boldsymbol{v}}^t + \hat{\boldsymbol{x}}^t)$$

$$\hat{\boldsymbol{v}}^{t+1} = \boldsymbol{y} - \boldsymbol{A}\hat{\boldsymbol{x}}^{t+1} + \frac{N}{M}\langle\eta'(\boldsymbol{A}^H\hat{\boldsymbol{v}}^t + \hat{\boldsymbol{x}}^t)\rangle$$

References:
1. I. Daubechies, M. Defrise, and C. De Mol, "An iterative thresholding algorithm for linear inverse problems with a sparsity constraint," Commun. Pure Appl. Math: J. Issued by the Courant Institute of Math Sciences., vol. 57, no. 11, pp. 1413-1457, Nov. 2004.
2. D. L. Donoho, A. Maleki, and A. Montanari, "Message-passing algorithms for compressed sensing," Proc. Natl. Acad. Sci., vol. 106, no. 45, pp. 18914-18919, Nov 2009.

Orthogonal Approximate Message Passing

- Starting from $\boldsymbol{q}^0 = \boldsymbol{0}$, conventional orthogonal approximate message passing algorithm [3] contain two-step iteration, linear estimation (LE) and non-linear estimation (NLE):

$$\text{(Linear Estimation): } \boldsymbol{l}^t = \boldsymbol{q}^t + \boldsymbol{D}_t(\boldsymbol{y} - \boldsymbol{A}\boldsymbol{q}^t)$$

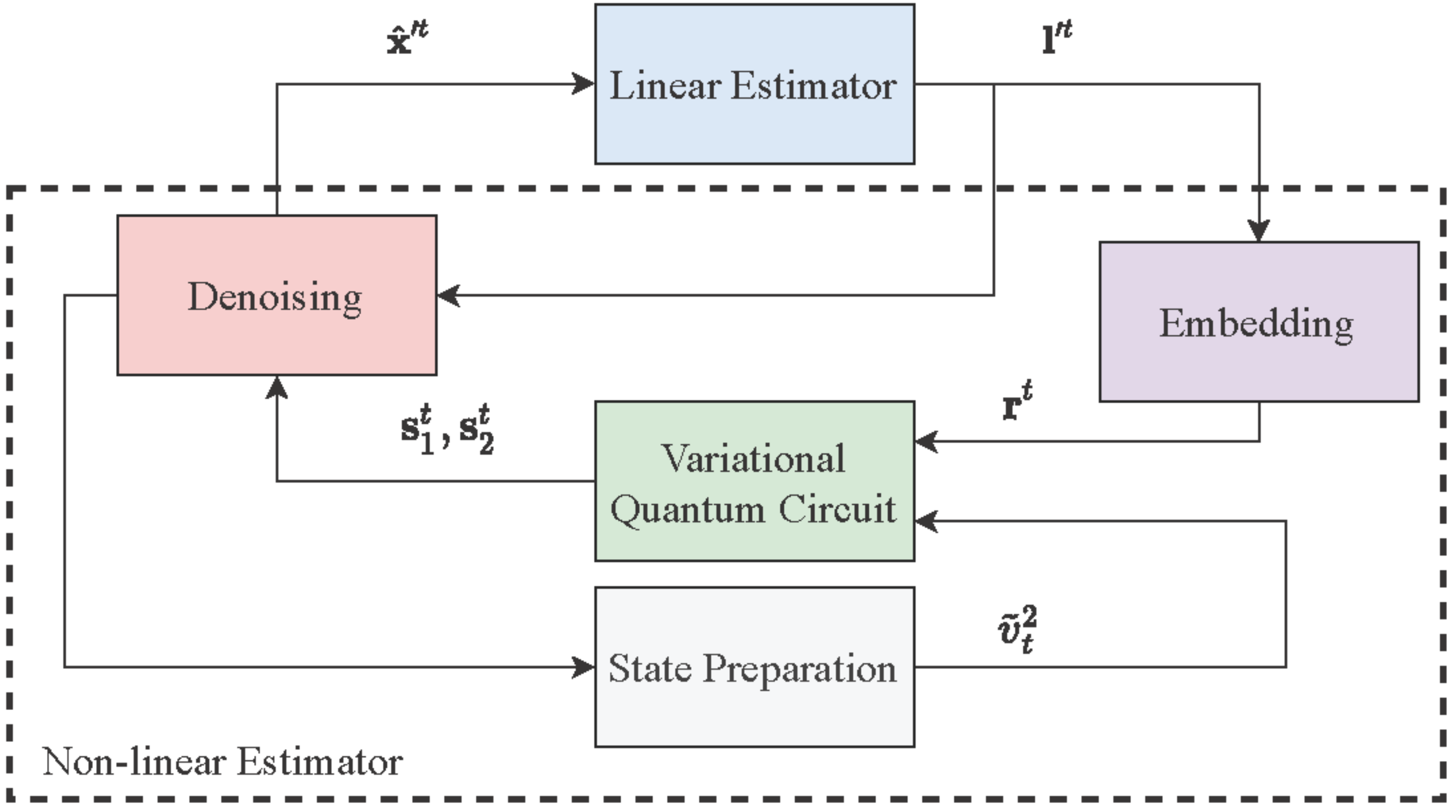
$$\text{(Non-linear Estimation): } \boldsymbol{q}^{t+1} = \eta_t(\boldsymbol{l}^t)$$

- $\boldsymbol{D}_t$ has three types of form
  - Matched filtering: $\frac{N}{tr(A^T A)} A^T$
  - Pseudo-inverse: $\frac{N}{tr(A^T (AA^T)^{-1} A)} A^T (AA^T)^{-1}$
  - Linear minimum mean-squared error: $\frac{N}{tr\left(\epsilon_t^2 A^T \left(\epsilon_t^2 AA^T + \sigma^2 I\right)^{-1} A\right)} A^T (AA^T)^{-1}$, where ϵ^2 is the mean-squared error of the non-linear estimation $\eta_t(\boldsymbol{l}^t)$
- $\eta_t(l_i^t) = \frac{\tau_t^2}{\tau_t^2 - \mathrm{mmse}(\tau_t^2)}\left(\eta_t^{\mathrm{MMSE}}(l_i^t) - \frac{\mathrm{mmse}(\tau_t^2)}{\tau_t^2} l_i^t\right)$, where $\eta_t^{\mathrm{MMSE}}(l_i^t) = \frac{\frac{\rho^{-1}}{\rho^{-1}+\tau_t^2} l_i^t}{1 + \frac{1-\rho}{\rho}\sqrt{\frac{\rho^{-1}+\tau_t^2}{\tau_t^2}} \exp\left(-\frac{1}{2\tau_t^2}\frac{\rho^{-1}}{(\rho^{-1}+\tau_t^2)}|l_i^t|^2\right)}$.
- $\mathrm{mmse}(\tau_t^2)$ represents the variance of the estimation error between the MMSE denoiser's output and $\boldsymbol{x}$, i.e., $E\{\left(\eta_t^{\mathrm{MMSE}}(l_i^t) - \boldsymbol{x}\right)^2\}$. c^2 is the channel variance.

References:
3. J. Ma and L. Ping, \Orthogonal AMP," IEEE Access, vol. 5, pp. 2020{2033, Jan. 2017.

Variational Quantum Compressed Sensing – Block Diagram

- Motivation: we employ a variational quantum circuit to replace the non-linear function in a conventional compressed sensing algorithm so that the user activity's correlation can be explored.



- Embedding: adjust the input data range before processing by a variational quantum circuit (VQC).
- VQC: the main quantum circuit that explores the characteristics of user activities and find a proper scaling factor for denoising
- Denoising: scale the linear estimate for sparsity recovery
- State Preparation: compute the previous iteration's empirical estimation error serves as an input to the VQC

Variational Quantum Compressed Sensing – Sub-Processors

- Embedding

$$r_i^t = \pi \cdot \tanh\left(\left|l_i^t\right|^2\right)$$

The range of the linear estimate is embedded to $[0, \pi]$, which refers to a one-to-one mapping from the linear estimate to the rotation gate's angle in a VQC.

- Denoising

$$\hat{x}_i^t = \frac{s_{1,i}^t}{1 + s_{2,i}^t} l_i^t$$

Introducing 2 scaling factors to form a function that follows the analogous form of a minimum mean square error denoiser. $s_{1,i}^t$ and $s_{2,i}^t$ are found by 2 VQCs.

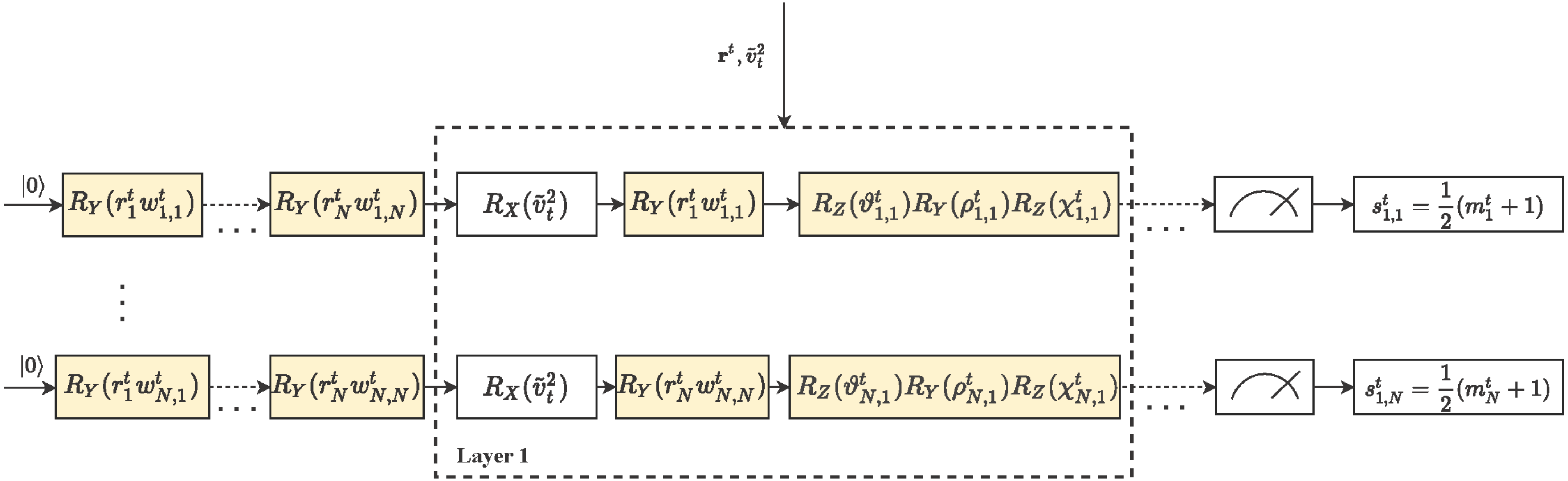
- State Preparation

$$\tilde{v}_t^2 = \pi \cdot \tanh\left(\frac{1}{N}\left|\left|\boldsymbol{y} - \boldsymbol{A}\hat{\boldsymbol{x}}^t\ \right|\right|^2\right)$$

As an input of the VQC, the empirical non-linear recovery error on the received symbols is tracked by the system for every iteration.

Variational Quantum Compressed Sensing – Quantum Circuit

- Variational Quantum Circuit



- Each colored block contains trainable parameters that can be tuned to explore the correlation structure of the user activity.
- By employing data re-uploading method in [1], there could be multiple layers contained for the VQC.
- We use a Pauli-Z gate to measure the qubits, where the measurement has an output range of $[-1, 1]$. We scale the measurement by $s_{1,i}^t = \frac{1}{2}(m_1^t + 1)$ so that the final scaling factor has a range of $[0, 1]$.

Reference:
1. Perez-Salinas, Adrian and Cervera-Lierta, Alba and Gil-Fuster, Elies and Latorre, Jose I., "Data re-uploading for a universal quantum classifier", *arXiv: 1907.02085,* 2020.

Variational Quantum Compressed Sensing – Loss Function

- Training

By “unfolding” the two-step iterative process of a compressed sensing algorithm, we use a multi-loss function to train the parameters in a variational quantum circuit:

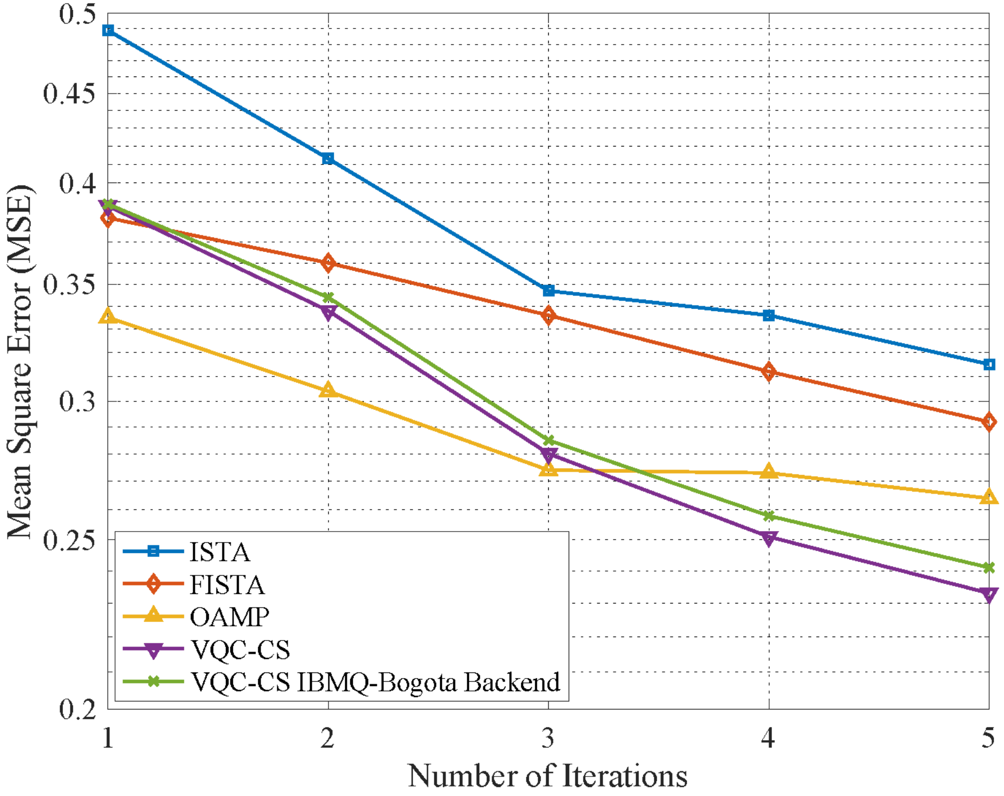
$$\mathcal{C} = \frac{1}{N}\sum_{t=1}^{T} \xi^{T-t} \left|\left|\boldsymbol{x} - \hat{\boldsymbol{x}}^{t}\right|\right|^{2}$$

- System model for the following numerical results:

Parameters	Values
Signal-to-noise ratio	30 dB
Condition number of $\boldsymbol{A}$	1
User activity ratio ρ	0.2
User activity correlation coefficient γ	0.6
Number of data re-uploading layers	3

Numerical Results

- Channel Estimation

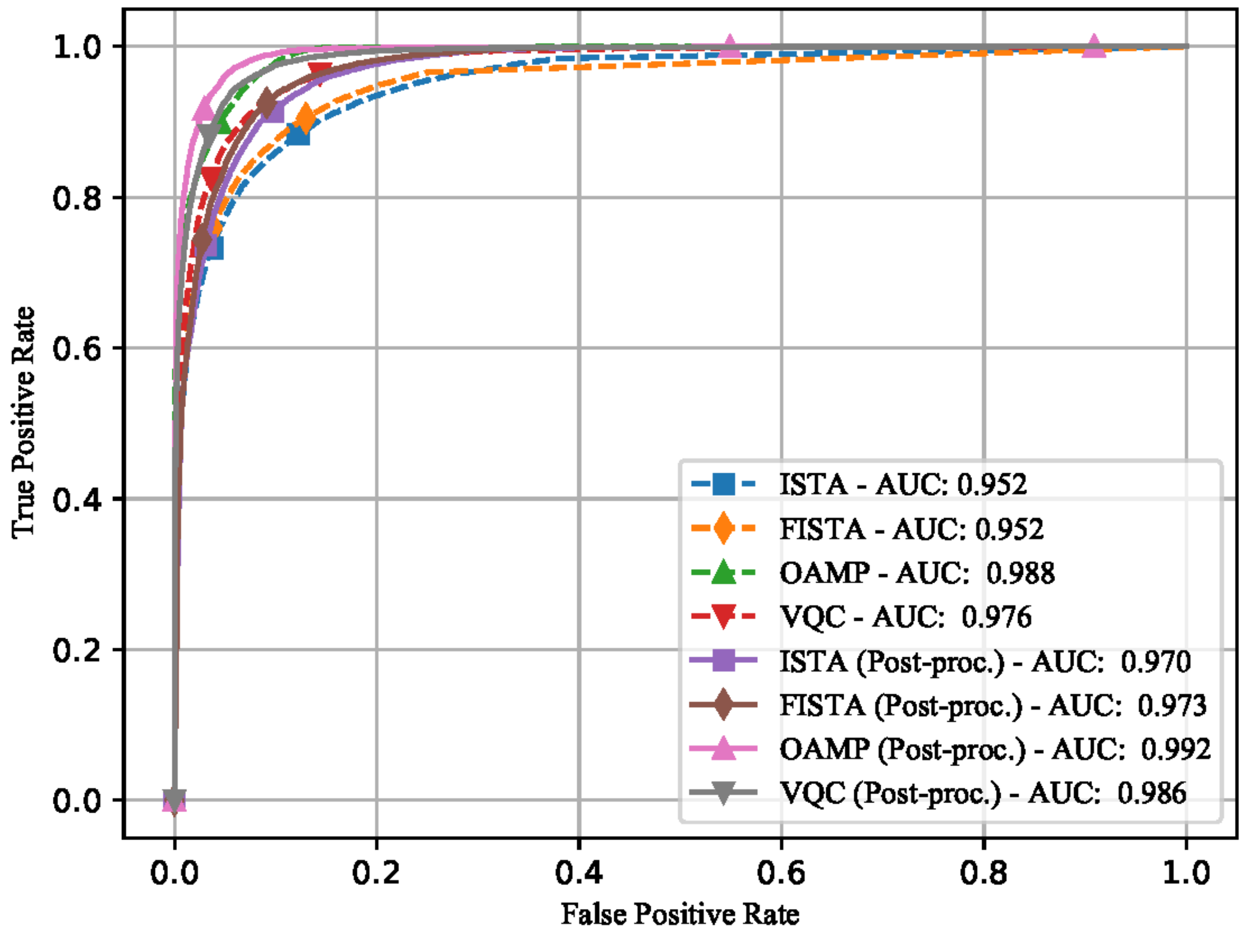


MSE performances in a system that has $N = 10$ devices, $M = 6$ received symbols and correlation factor $\gamma = 0.6$.

- ISTA: Iterative shrinkage-threshold algorithm
  - Matched filtering is used in the linear estimation step
  - Soft-thresholding function is employed in the non-linear estimation step
- FISTA: Fast iterative shrinkage-threshold algorithm
  - Matched filtering is used in the linear estimation step
  - Soft-thresholding function is employed in the non-linear estimation step
- OAMP: Orthogonal approximate message passing algorithm
  - Pseudo-inverse form of a linear matrix is used to decorrelate the elements in linear estimate
  - MMSE denoiser is employed in the non-linear estimation step
- VQC-CS: Variation quantum circuit based compressed sensing
  - Pseudo-inverse form of a linear matrix is used to decorrelate the elements in linear estimate
  - Variational quantum circuit serves as the non-linear estimator
- VQC-CS IBMQ-Bogota Backend: the quantum circuit is simulated with an approximated noisy channel from the IBMQ-Bogota backend.

Numerical Results

- User Identification



ROC charts in a system that has $N = 10$ devices, $M = 6$ received symbols and correlation factor $\gamma = 0.6$.

- For the curves with a label of “Post-proc”, we use a post-processing multi-layer perception (two hidden layers with $4N$ and $2N$ neurons respectively) to optimize the binary cross entropy loss for user activity recognition after the channel estimation.

- We compare the absolute of x_i^t to a threshold g, where the user is detected as active if $|x_i^t| > g$ and inactive otherwise.

Conclusion

- We proposed a new compressed sensing algorithm based on a VQC, that can be applied to joint channel estimation and user identification in grant-free IoT-device access systems.
- The proposed framework is a hybrid classical-quantum computing paradigm, where the NLE step exploits a trainable VQC processor to properly refine the estimate of the LE step as an alternative denoiser.
- As a proof-of-concept study, we showed that VQC-CS can outperform conventional compressed sensing techniques under a challenging system scenario where the device activity is correlated.
- There remain many fascinating challenges for future work, including rigorous performance verification with real quantum processors and quantum ansatz design for large-scale compressed sensing problems.

If you are interested in getting more details on this topic, please feel free to contact us via:

bryanl@merl.com
koike@merl.com
yewang@merl.com
parsons@merl.com

MITSUBISHI ELECTRIC

Changes for the Better