Variational Quantum Demodulation for Coherent Optical Multi-Dimensional QAM

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Outline

• High-dimensional modulation (HDM)
• Quantum technology trend
  – Quantum approximate optimization algorithm (QAOA)
• QAOA hybrid quantum-classical demodulation
• Simulation and real quantum processor results
High-Dimensional Modulation (HDM)

Regular lattice in 1-D:
Regular pulse-amplitude modulation (PAM)

Lattice in 2-D:
Regular quadrature-amplitude modulation (QAM)

Optimally packed lattice in 2-D:
Hexagonal lattice: max 0.8dB gain
• Start with lattice in N-dimensions
• Select $2^p$ neighboring points
• Optimize selection
• Remove mean
• Labeling?
Block-Coded HDM

- Selecting a good subset of non-adjacent points from a Gray-coded hypercube

- Codewords define the set of points that are selected; e.g., [3,2,2] single-parity-check (SPC) code

- Labeling optimization is not required
Increasing dimensionality provides higher power efficiency: \( \gamma = \frac{d_{\text{min}}^2}{4\varepsilon_b} \)

- Sphere-cut
  - 4-D lattice
  - 6-D lattice
  - 8-D lattice
  - 16-D lattice
  - 24-D lattice

- Block-coded
  - [N,p,h] coded
  - Perfect code

### Codes
- Ext. Golay [24,12,8]
- Golay [23,12,7]
- Sing.-Par. [8,7,2]
- Ext. Hamming [16,11,4]
- Hamming [15,11,3]
- Ext. Golay [24,12,8]
- Ext. Hamming [16,5,8]
- Ext. Hamming [16,11,4]
- Sing.-Par. [4,3,2]
- (PS-QPSK)
- Sing.-Par. [7,4,3]
- Uncoded [p,p,1]
- (DP-QPSK)
- Reed-Muller [16,5,8]
- Repetition [4,1,4]
- DP-BPSK
- Perfect code

**6dB with 24D**
Nonlinearity-Mitigating HDM

- Some HDM can further mitigate nonlinear interference (NLI)
  - 4-dimensional constant-modulus modulation (4D-2A8PSK) [Kojima et al., ECOC14]
  - 8-dimensional Grassmann modulation [Koike-Akino et al., SPPCom15]
  - 8-dimensional X-constellation (8D-X) [Shiner et al., OpEx14]
HDM Demodulation

• In exchange of higher packing gain, HDM can increase the demodulation complexity for higher dimensions

• Log-likelihood ratio (LLR) calculation is not straightforward in general to feed into soft-decision decoders

• Simplified LLR calculation based on min-sum belief propagation over algebraic constrains

• We propose quantum processing unit (QPU)-based demodulator
Quantum Computing

- Morgan Stanley: Quantum tech can drive 4th industrial revolution
- Escalating government funds: National Quantum Initiative $1.2B
- Quantum chip providers: IBM, Google, Microsoft, Honeywell, Intel, Nokia, AirBus, IONQ, rigetti

Free libraries to evaluate quantum computing on realistic simulators or real devices
Post-2014 Trend: Variational Quantum Principle

- Hybrid use of quantum measurement and classical optimization
  - VQE: Variational Quantum Eigensolver (2014)
  - QAOA: Quantum Approximate Optimization Algorithm (2014)
  - VQF: Variational Quantum Factoring (2018)
Variational Quantum Algorithms for NISQ

- Current quantum processors are noisy and coherence-limited: quantum gates are imperfect.

- For noisy intermediate-scale quantum (NISQ) devices, variational hybrid quantum-classical algorithms may be a viable driver for quantum supremacy due to shallow gates and noise resilience.

V.Q.E. QUANTUM-CLASSICAL HYBRID ALGORITHM
QAOA: Quantum Approximate Optimization Alg.

- Alternating cost Hamiltonian and mixer Hamiltonian like annealing
- Convergence theorem to eigenstate for infinite-level QAOA
  - Infinite Suzuki-Trotter decomposition with adiabatic annealing
- Classical optimization of variational angle parameters given quantum measurement
  \[
  \lim_{p \to \infty} F_p^* = \max_{z} C(z)
  \]
- Theoretical analysis showed better accuracy than classical counterparts; e.g. MaxCut, MaxSat, MaxClique

A Quantum Approximate Optimization Algorithm

Edward Farhi and Jeffrey Goldstone
Center for Theoretical Physics
Massachusetts Institute of Technology
Quantum Application to Optical DSP

• This talk is not “Quantum Error Correction Codes (QECC)” nor “Quantum Communications”

• We want to offload classical demodulation computation on CPU towards QPU
Classical Channel Decoding

- Hamming codes, Reed-Muller codes, Golay codes, convolutional codes, turbo codes, low-density parity-check (LDPC) codes, polar codes, ...

- Suppose linear binary codes with generator matrix \( G \in \mathbb{F}_2^{k \times n} \)

Redundancy: Parity

\([01011100101011] \rightarrow [01011100101011 00100101110111011]\)

\[
\begin{align*}
\mathbf{u} &\in \mathbb{F}_2^k \\
\mathbf{x} &= \mathbf{u}G \\
\mathbf{y} &= \mathbf{x} + \mathbf{w} \\
\mathbf{w} &\in \mathbb{R}^n
\end{align*}
\]

- Communication channel exhibits noise

- Maximum-likelihood (ML) decoding for symmetric channels:

\[
\begin{align*}
\text{arg min}_{\mathbf{u}} d_H(\mathbf{y}|\mathbf{x}) &= \text{arg max}_{\mathbf{u}} \sum_{\nu=1}^{n} (1 - 2y_\nu)(1 - 2x_\nu) \\
&\text{NP-hard } 2^k \text{ search for maximum correlation}
\end{align*}
\]
QAOA Channel Decoding

• Convert ML decoding problem into Ising Hamiltonian model [Koike-Akino ISIT19]

\[
\arg \min_{\mathbf{u}} d_H(\mathbf{y} | \mathbf{x}) = \arg \max_{\mathbf{u}} \sum_{\nu=1}^{n} (1 - 2y_{\nu})(1 - 2x_{\nu})
\]

\[\mathbf{x} = \mathbf{uG}\]

\(k\)-bit search: \(2^k\)

\(k\)-qubit parallel operation

\[
C = \sum_{\nu=1}^{n} C_{\nu} = \sum_{\nu=1}^{n} (1 - 2y_{\nu}) \prod_{\kappa \in \mathcal{I}_{\nu}^c} Z_{\kappa}
\]

Pauli-Z

\[Z_1 Z_2 = \text{XOR}\]
Example: Hamming Code Hamiltonian

- [7, 4]-Hamming code with minimum distance 3, corrects 1 bit error
- Generator matrix:

\[
G = \begin{bmatrix}
1 & 0 & 0 & 0 & 1 & 1 & 0 \\
0 & 1 & 0 & 0 & 1 & 0 & 1 \\
0 & 0 & 1 & 0 & 0 & 1 & 1 \\
0 & 0 & 0 & 1 & 1 & 1 & 1
\end{bmatrix}
\]

Degree: [1, 1, 1, 1, 3, 3, 3]

\[
C = r_1 Z_1 + r_2 Z_2 + r_3 Z_3 + r_4 Z_4 \\
+ r_5 Z_1 Z_2 Z_4 + r_6 Z_1 Z_3 Z_4 + r_7 Z_2 Z_3 Z_4
\]

c.f.) MaxCut Hamiltonian is regular degree-2
QAOA Channel Decoding

• CPU:
  – Given generator matrix G and received signal y
  – Construct cost Hamiltonian with variational angles
  – Quantum shots on QPU to obtain quasi-ML decision
  – Re-optimize angles if necessary and re-shot

\[
G = \begin{bmatrix}
1 & 0 & 0 & 0 & 1 & 1 & 0 \\
0 & 1 & 0 & 0 & 1 & 0 & 1 \\
0 & 0 & 1 & 0 & 0 & 1 & 1 \\
0 & 0 & 0 & 1 & 1 & 1 & 1
\end{bmatrix}
\]

\[
C = r_1 Z_1 + r_2 Z_2 + r_3 Z_3 + r_4 Z_4 + r_5 Z_1 Z_2 Z_4 + r_6 Z_1 Z_3 Z_4 + r_7 Z_2 Z_3 Z_4
\]

• QPU: QAOA
  – Initialize quantum state: |+\> with Hadamard gates
  – Apply gamma angle rotation with cost Hamiltonian C
  – Apply beta angle rotation with mixer Hamiltonian B
  – Cascade \( p \)-times for level-\( p \) QAOA
  – Measure

© MERL 3/10/20; T. Koike-Akino Quantum Demodulation
Quantum Circuits for QAOA Decoding

- State preparation: Hadamard $H$
- Mixer Hamiltonian operation: $\exp(i\beta B)$
- Cost Hamiltonian operation: $\exp(i\gamma C)$
Variational Angle Optimization for HDM

- We can obtain optimal variational angles through VQE
- Landscape of cost expectation (quantum eigenvalue)
• Using higher-level QAOA, quantum eigenvalue can approach ideal quantum eigenvalue
Quantum Eigenvalue and Cross Entropy

- Higher cost expectation (quantum eigenvalue) leads to smaller cross entropy loss, i.e., higher generalized mutual information (GMI)
ML decision success probability can be improved by taking multiple measurements of QPU shots.
Real QPU Performance of HDM Demodulation

- Due to quantum errors, higher-level QAOA had no significant gain in real QPUs
- To optimize angles under decoherence scenarios
- Wavefunction amplification helps improving the performance
Conclusions

• We investigated HDM demodulation using quantum processor
• We introduced variational hybrid quantum-classical algorithms for HDM demodulation
  – QAOA demodulation was demonstrated to approach ML performance
  – Higher-level QAOA achieves better performance
• We also showed feasibility on real quantum processor
  – Quantum decoherence degraded performance
  – Parameter finetuning is required on real QPU
• To the best of our knowledge, this is the first proof-of-concept paper investigating variational quantum algorithms for HDM demodulation
  – Further rigorous analysis and improvement to follow
  – Developing quantum-ready DSP algorithms may be important for future quantum era

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