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DUAL-REGULARIZED ITERATIVE ADAPTIVE APPROACH FOR DOA SPECTRUM RECONSTRUCTION IN LIMITED ANGLE SECTOR

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ABSTRACT

The Regularized Iterative Adaptive Approach (IAA-R) is a powerful method for high-resolution Direction-of-Arrival (DOA) spectrum reconstruction. However, its performance degrades severely when the angular search is restricted to a narrow sector. This paper reveals that this degradation stems from a structural failure of IAA-R's adaptive regularization in such ill-posed scenarios. To overcome this limitation, we propose the Dual Regularized IAA (DR-IAA), which augments the adaptive regularization of IAA-R with a novel, complementary term. This second regularizer is theoretically derived from the geometric incompleteness of the limited-scan array manifold, which we quantify as an "observation capability loss." This dual-regularization framework determines the baseline regularization level directly from the array manifold's geometric properties and the received signal power, ensuring algorithmic stability. Numerical simulations demonstrate that DR-IAA maintains high angular accuracy and provides a clean, stable DOA spectrum, significantly outperforming IAA-R in severely constrained scan-angle scenarios, including those with out-of-sector interference.

Index Terms— IAA, IAAR, DR-IAA, DOA estimation, signal reconstruction, linear inverse problem

1. INTRODUCTION

Sensor array signal processing is essential to applications such as radar, sonar, and passive surveillance. Signal reconstruction in these fields hinges on a linear signal model that represents the received data vector as the product of an array manifold matrix and an unknown signal vector, which represents the angular power spectrum of impinging signals. Historically, beamforming—multiplying the received data by the Hermitian transpose of the array manifold—has been the prevalent technique. While computationally efficient, beamforming is fundamentally limited by poor angular resolution and the generation of high sidelobes.

More recently, methods rooted in linear inverse problem theory have emerged to address these shortcomings [1]. These methods directly estimate the signal vector, yielding a high-resolution angular response that is inherently sidelobe-free. The signal model formulation typically results in an underdetermined linear inverse problem, as the dimensionality of the signal vector exceeds that of the received data vector. Consequently, regularization is essential for obtaining a physically meaningful solution, typically by incorporating prior information such as signal sparsity [1][2][3] or its probability distribution [4][5]. The reconstruction performance is critically dependent on the selection of hyperparameters, yet tuning these parameters for optimal performance is often a formidable challenge in practical operational systems.

To address this challenge, the Iterative Adaptive Approach (IAA) was developed [6]. It is a high-resolution method that circumvents the need for manual hyperparameter tuning by iteratively estimating the signal vector using a weighted least-squares algorithm. Given its significant advantages, including strong performance even with a single snapshot, IAA has gained considerable attention. Much of the subsequent research has focused on enhancing its computational efficiency, for instance, by reducing the cost of its iterative matrix inversions or reducing the dimensionality of the received data vector. Further details can be found in [7][8] and the references therein.

This paper, however, addresses a different challenge that arises when adapting IAA to physically constrained operational scenarios. In many practical applications, such as Direction-of-Arrival (DOA) estimation, the angular search can be limited to a specific sector of interest. For instance, a radar may have a limited search coverage, or the locations of potential emitters in passive surveillance may be approximately known. Consider DOA estimation with a linear array. Here, it is often sufficient to perform a limited-angle scan (e.g., over ± 30 deg) rather than a full-range angle scan (over ± 90 deg). This scenario naturally leads to a simplified signal model where the number of angular grid points—and thus the number of columns in the array manifold—is reduced. While this reduction offers the benefit of a lower computational load, it introduces a critical issue: from an inverse problem standpoint, this model reduction can render the problem ill-posed, thereby degrading the reconstruction performance of IAA.

Regularized IAA (IAA-R) was introduced to mitigate this problem [9]. This method incorporates an adaptive diagonal loading step within each iteration to regularize the estimated structured covariance matrix. While its effectiveness has been demonstrated, we show in this paper that its performance deteriorates sharply when the scan range is severely narrowed to the order of an array beamwidth. This degradation stems directly from the reduced efficacy of the diagonal loading as a regularizer in such constrained scenarios.

Other approaches, such as the General Regularized IAA [10], have been proposed to address the ill-posed problem by adaptively controlling the condition number of the covariance matrix. However, this method is not only computationally expensive due to iterative eigenvalue decompositions but also requires prior knowledge of the signal-to-noise ratio (SNR). Since a significant advantage of the IAA framework is its applicability to single-snapshot scenarios where SNR estimation is itself a challenge, IAA-R remains the most relevant benchmark for a hyperparameter-free approach. Our work focuses on overcoming the fundamental limitations of this IAA-R framework.

In this paper, we propose the Dual Regularized IAA (DR-IAA) to overcome this performance degradation, especially in highly constrained scan-angle scenarios. DR-IAA employs a dual-regularization framework, augmenting the existing regularization term in IAA-R with a novel, complementary term. Our approach is founded on the perspective that a limited-scan model suffers from an "observation capability deficiency" relative to its full-range counterpart. We quantify this deficiency as an "observation capability loss," which captures the geometric incompleteness of the limited-scan array manifold. This loss term, pre-calculated before the iterations start, is introduced as a second regularizer into the covariance matrix estimation at each step. This dual-regularization mechanism robustly enhances signal reconstruction performance compared to IAA-R when the scan range is narrow.

The remainder of this paper is organized as follows. In Section 2, we formulate the problem. Section 3 presents our proposed DR-IAA method. In Section 4, we demonstrate its effectiveness through numerical simulations. Finally, in Section 5, we conclude the paper and suggest directions for future work.

2. PROBLEM FORMULATION

2.1. Signal Model

The signal model for the received data vector $\mathbf{y} \in \mathbb{C}^M$ across M channels is defined as follows:

$$\mathbf{y} = \mathbf{A}\mathbf{x} + \mathbf{n} = \mathbf{A}_\text{s}\mathbf{x}_\text{s} + \mathbf{A}_\text{i}\mathbf{x}_\text{i} + \mathbf{n} \tag{1}$$

where $\mathbf{A} \in \mathbb{C}^{M\times N}$ and $\mathbf{x} \in \mathbb{C}^N$ are the array manifold matrix and the signal vector, respectively. The matrix $\mathbf{A}$ is the array manifold composed of N steering vectors that span the full angular scan range. The matrix $\mathbf{A}_\text{s} \in \mathbb{C}^{M\times N_\text{s}}$ is the array manifold composed of N_s steering vectors that span the desired (limited) scan range, and $\mathbf{x}_\text{s} \in \mathbb{C}^{N_\text{s}}$ is the corresponding desired signal vector. Similarly, $\mathbf{A}_\text{i} \in \mathbb{C}^{M\times N_\text{i}}$ is the array manifold composed of N_i steering vectors that span the out-of-sector scan range, and $\mathbf{x}_\text{i} \in \mathbb{C}^{N_\text{i}}$ is the corresponding the out-of-sector signal vector. The following relationships hold:

$$\mathbf{A} = [\mathbf{A}_\text{s}, \mathbf{A}_\text{i}] \tag{2}$$

$$\mathbf{x} = [{\mathbf{x}_\text{s}}^\text{T}, {\mathbf{x}_\text{i}}^\text{T}]^\text{T} \tag{3}$$

$$N = N_\text{s} + N_\text{i} \tag{4}$$

The vector $\mathbf{n} \in \mathbb{C}^M$ represents the receiver noise, which is assumed to be uncorrelated across the array channels. The covariance matrix $\mathbf{R}_\text{y}$ of the received data vector in (1) is given by

$$\mathbf{R}_\text{y} = E[\mathbf{y}\mathbf{y}^\text{H}] = \mathbf{A}_\text{s}\mathbf{P}_\text{s}\mathbf{A}_\text{s}^\text{H} + \mathbf{A}_\text{i}\mathbf{P}_\text{i}\mathbf{A}_\text{i}^\text{H} + \sigma_\text{n}^2\mathbf{I} \tag{5}$$

where $(\cdot)^\text{H}$ denotes the Hermitian transpose and $E[\cdot]$ is the statistical expectation operator. Here, the incoming signals comprising the desired signal vector $\mathbf{x}_\text{s}$ and the out-of-sector signal vector $\mathbf{x}_\text{i}$ are assumed to be mutually uncorrelated. Consequently, their respective covariance matrices, $\mathbf{P}_\text{s}$ and $\mathbf{P}_\text{i}$, are diagonal and are modeled as follows:

$$\mathbf{P}_\text{s} = E[\text{diag}\left(\left[\,|\,(\mathbf{x}_\text{s})_1\,|^2, |\,(\mathbf{x}_\text{s})_2\,|^2, \ldots, |\,(\mathbf{x}_\text{s})_{N_\text{s}}\,|^2\right]\right)] \tag{6}$$

$$\mathbf{P}_\text{i} = E[\text{diag}\left(\left[\,|\,(\mathbf{x}_\text{i})_1\,|^2, |\,(\mathbf{x}_\text{i})_2\,|^2, \ldots, |\,(\mathbf{x}_\text{i})_{N_\text{i}}\,|^2\right]\right)] \tag{7}$$

where $|\,(\mathbf{x}_\text{s})_{n_\text{s}}\,|^2$ and $|\,(\mathbf{x}_\text{i})_{n_\text{i}}\,|^2$ represent the power of the n_s-th and n_i-th signal component in the desired and out-of-sector vectors, respectively. Furthermore, σ_n^2 is the average receiver noise power, defined by the relationship:

$$\sigma_\text{n}^2\mathbf{I} = E[\mathbf{n}\mathbf{n}^\text{H}] \tag{8}$$

In practice, the covariance matrix $\mathbf{R}_\text{y}$ is generally unknown and must be estimated. A widely used method for this purpose is the Sample Covariance Matrix (SCM) estimation which requires many snapshots [11]. In contrast, the IAA family of algorithms can obtain a highly accurate structured covariance matrix estimate from a single snapshot.

2.2. Limitation of IAA-R

In the IAA-R, the first term on the right-hand side of (5), $\mathbf{A}_\text{s}\mathbf{P}_\text{s}\mathbf{A}_\text{s}^\text{H}$, is estimated at each iteration, following the original IAA algorithm[6]. The remaining terms of (5) are not estimated by directly finding $\mathbf{P}_\text{i}$ and σ_n^2. Instead, they are modeled as a single diagonal matrix $\mathbf{\Sigma}_\text{R}$, whose diagonal components σ_m^2 are estimated at each iteration of the IAA-R algorithm:

$$\mathbf{A}_\text{i}\mathbf{P}_\text{i}\mathbf{A}_\text{i}^\text{H} + \sigma_n^2\mathbf{I} \equiv \text{diag}(\sigma_1^2, \sigma_2^2, \ldots, \sigma_M^2) = \mathbf{\Sigma}_\text{R} \tag{9}$$

Therefore, the structured covariance matrix for IAA-R at the k-th iteration, $\hat{\mathbf{R}}_\text{IAAR}^{(k)}$, is given by [9]:

$$\hat{\mathbf{R}}_\text{IAAR}^{(k)} = \mathbf{A}_\text{s}\hat{\mathbf{P}}_\text{s}^{(k)}\mathbf{A}_\text{s} + \hat{\mathbf{\Sigma}}_\text{R}^{(k)} \tag{10}$$

From (10), it can be seen that $\hat{\mathbf{\Sigma}}_\text{R}^{(k)}$ functions as a regularization term via adaptive diagonal loading. The signal power matrix $\hat{\mathbf{P}}_\text{s}^{(k+1)}$ is estimated after computing the inverse of $\hat{\mathbf{R}}_\text{IAAR}^{(k)}$, and the diagonal loading matrix $\hat{\mathbf{\Sigma}}_\text{R}^{(k)}$ is updated concurrently. In essence, given only the received data vector $\mathbf{y}$ and the limited-scan array manifold $\mathbf{A}_\text{s}$, the algorithm selectively estimates the signal power matrix $\hat{\mathbf{P}}_\text{s}^{(k)}$ and the diagonal loading matrix $\hat{\mathbf{\Sigma}}_\text{R}^{(k)}$.

The model in (9) treats the interference and noise components as unknown, unwanted signals that are uncorrelated between channels. This effectively reformulates the problem as an ill-posed one, $\mathbf{y} = \mathbf{A}_\text{s}\mathbf{x}_\text{s}$, where only the data vector and the limited-scan manifold are known. Consequently, the estimation performance of IAA-R degrades as the condition number of $\mathbf{A}_\text{s}$ worsens.

3. PROPOSED DR-IAA

To address the performance degradation of IAA-R when the limited-scan range is narrow, we propose the DR-IAA. This approach enhances the IAA-R model by augmenting its existing adaptive regularizer with a novel, complementary regularization term. DR-IAA operates on the principle that the limited-scan signal model ($\mathbf{A}_\text{s}\mathbf{x}_\text{s}$) suffers from a loss of observation capability compared to the full-scan model ($\mathbf{A}\mathbf{x}$). Based on this view, we define this loss as the geometric incompleteness of the limited-scan manifold $\mathbf{A}_\text{s}$ relative to the full-scan manifold $\mathbf{A}$. This loss term, which increases as the limited-scan range narrows, is calculated before the iterative process starts. It is then added as a fixed, baseline regularization term during each iteration. The combination of this fixed baseline regularizer with the original adaptive one yields a robust dual-regularization framework, resulting in a significantly improved signal reconstruction capability compared to IAA-R in scenarios with narrow scan ranges. We begin by considering that the modeling error introduced by replacing $\mathbf{A}$ with $\mathbf{A}_s$ geometrically resides in the column space of a manifold spanning the orthogonal complement of the column space of $\mathbf{A}_s$, which can be expressed as:

$$\mathbf{A}_\text{s}^\perp = \mathbf{P}_\text{s}^\perp\mathbf{A} \tag{11}$$

where $\mathbf{P}_s^{\perp}$ is the projection matrix onto the orthogonal complement of the column space of $\mathbf{A}_s$, given by:

$$\mathbf{P}_s^{\perp} = \mathbf{I} - \mathbf{A}_s \mathbf{A}_s^{\dagger} \tag{12}$$

Here, $\mathbf{A}_s^{\dagger}$ is the pseudo-inverse of $\mathbf{A}_s$.

Next, we define an observation capability loss factor, ρ_F, which represents the portion of signal power that cannot be reconstructed because it lies in the column space of $\mathbf{A}_s^{\perp}$:

$$\rho_F = \frac{||\mathbf{A}_s^{\perp}||_F^2}{||\mathbf{A}||_F^2} \tag{13}$$

where $||\cdot||_F^2$ is the squared Frobenius norm. The term $||\mathbf{A}||_F^2$ represents the total received power from unit-power signals across the full-scan range, while $||\mathbf{A}_s^{\perp}||_F^2$ is the total power of signals that are unobservable by the limited-scan range. Thus, if the scan range is not narrowed, $\rho_F = 0$, and as the range narrows, ρ_F increases asymptotically towards 1.

To use ρ_F as a regularization term, it must be scaled to a power value. Approximating the average power per channel as $||\mathbf{y}||^2/M$, we propose the final observation capability loss term, ρ_{DR}, as:

$$\rho_{DR} = \frac{||\mathbf{y}||^2}{M}\rho_F = \frac{||\mathbf{y}||^2}{M}\frac{||\mathbf{A}_s^{\perp}||_F^2}{||\mathbf{A}||_F^2} \tag{14}$$

We introduce this into the covariance matrix as a new regularization term:

$$\hat{\mathbf{R}}_{\mathrm{DRIAA}}^{(k)} = \mathbf{A}_s \hat{\mathbf{P}}_s^{(k)} \mathbf{A}_s^{\mathrm{H}} + \hat{\mathbf{\Sigma}}_{\mathrm{R}}^{(k)} + \rho_{DR}\mathbf{I} \tag{15}$$

As is clear from (15), the proposed term $\rho_{DR}\mathbf{I}$ is computed once before the iterations and remains constant, whereas the original IAA-R term $\hat{\mathbf{\Sigma}}_{\mathrm{R}}^{(k)}$ is adaptively updated at each iteration.

Consequently, the reconstructed signal vector $(\hat{\mathbf{x}}_s)_{n_s}^{(k+1)}$ and the adaptive regularization term $(\sigma_m^2)^{(k+1)}$ in DR-IAA are obtained by the following update rules:

$$(\hat{\mathbf{x}}_s)_{n_s}^{(k+1)} = \frac{\mathbf{a}^{\mathrm{H}}(\theta_{n_s})\left(\hat{\mathbf{R}}_{\mathrm{DRIAA}}^{(k)}\right)^{-1}\mathbf{y}}{\mathbf{a}^{\mathrm{H}}(\theta_{n_s})\left(\hat{\mathbf{R}}_{\mathrm{DRIAA}}^{(k)}\right)^{-1}\mathbf{a}(\theta_{n_s})} \tag{16}$$

$$\left(\hat{\sigma}_m^2\right)^{(k+1)} = \frac{\left(\mathbf{I}^{(m)}\right)^{\mathrm{H}}\left(\hat{\mathbf{R}}_{\mathrm{DRIAA}}^{(k)}\right)^{-1}\mathbf{y}}{(\mathbf{I}^{(m)})^{\mathrm{H}}\left(\hat{\mathbf{R}}_{\mathrm{DRIAA}}^{(k)}\right)^{-1}\mathbf{I}^{(m)}} \tag{17}$$

Here, $\mathbf{a}(\theta_{n_s})$ is the steering vector for the angle θ_{n_s} within the limited-scan range, and $\mathbf{I}^{(m)}$ is the m-th column vector of the M-dimensional identity matrix.

4. NUMERICAL VALIDATION

4.1. Setup

We consider a 32-element uniform linear array with half-wavelength spacing at a frequency of 10 GHz. All simulations use a single snapshot, and the SNR is defined after beamforming. The beamwidth is 3.2 deg at the 3 dB standard, and the Null-to-Null beamwidth is 7.2 deg. Based on the recommendation that the angular grid for the array manifold should be 0.1 to 0.2 times the 3 dB beamwidth [12], we set it to 0.5 deg.

Fig. 1 illustrates the relationship between the limited-scan range and the observability capability loss factor ρ_F for the aforementioned linear array. From the figure, we can observe that when the limited-scan range is wider than $\pm$30 deg, the value of ρ_F becomes close to 0. This indicates that the observable information in this range is not significantly different from that of the full-scan range. Furthermore, when $\rho_F \approx 0$, the double-regularization, which is a feature of DR-IAA, is practically non-functional. Consequently, the signal reconstruction performance is expected to be almost identical to that of IAA-R. On the other hand, as the limited-scan range becomes narrower than $\pm$30 deg, the value of ρ_F increases. This increase is a result of capturing the geometrical incompletion of the limited-scan array manifold, which will be discussed in the next section. Since the proposed DR-IAA performs double regularization by adding a regularization term related to this geometrical incompleteness to IAA-R, it is expected to improve signal reconstruction performance compared to IAA-R.

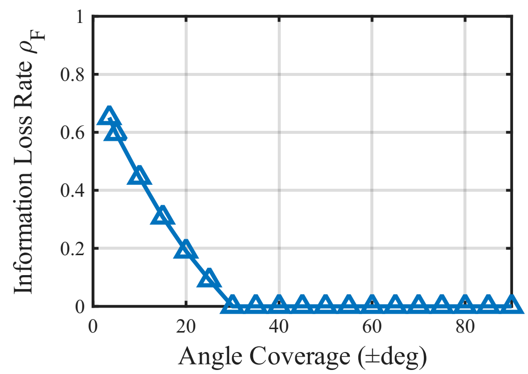
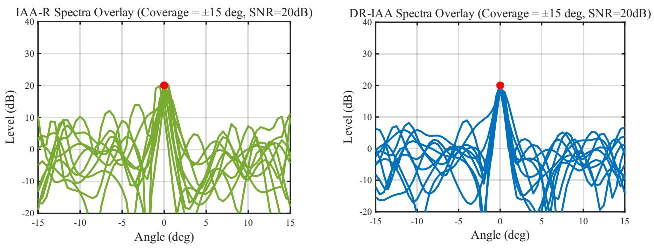


Fig. 1. The Relationship between limited-scan Range and Observation Capability Loss Factor ρ_F

4.2. Single-Source Scenario

As the simplest scenario, we consider the case where a single signal arrives and investigate the DOA estimation accuracy with respect to the limited-scan range to evaluate the superiority of each method. In this scenario, a signal with an SNR of 20 dB arrives from the array normal direction. The reconstructed signal vectors, i.e., the angular spectra, for IAA-R and the proposed DR-IAA with a limited-scan range of $\pm$15 deg are shown in Fig. 2, where 10 trial results are overlaid. The figure shows that DR-IAA exhibits less variation in both peak angle and level than IAA-R with respect to the ground truth marked by the red circle. This is because the observability capability loss factor ρ_F is 0.31 for a limited-scan range of $\pm$15 deg (from Fig. 1), and the effect of the double regularization of the proposed DR-IAA is demonstrated.

Next, we show the results of a Monte Carlo simulation with 1000 trials to compare the DOA estimation accuracy of IAA-R and the proposed DR-IAA for the full-scan range of $\pm$90 deg and the narrowest limited-scan range of $\pm$3.5 deg (equivalent to the Null-to-Null beamwidth of the main beam). The results are shown in Fig. 3. This figure also includes the Cramer-Rao lower bound (CRLB) for comparison purposes. As shown in the figure, for a wider scan range of $\pm$30 deg, the DOA estimation accuracies of both methods are consistent. This is because, as discussed in the previous section, the observability capability loss factor ρ_F is approximately 0, so there is no essential difference between the two methods. As the limited-scan range becomes narrower than $\pm$30 deg, approaching the Null-to-Null beamwidth, the effect of regularization in IAA-R



(a) IAA-R (b) Proposed DR-IAA

Fig. 2. Angular Spectrum with a limited-scan Range of ±15 deg (10 trials overlaid)

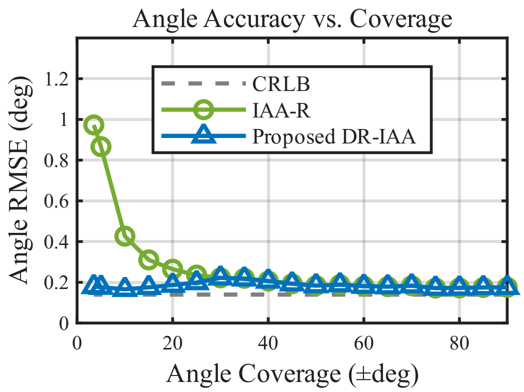


Fig. 3. DOA Estimation Accuracy versus limited-scan Range (SNR=20dB)

diminishes, and its accuracy deteriorates. The DOA estimation accuracy for scan ranges of ±90 deg and ±3.5 deg are approximately 0.2 deg and 1.0 deg, respectively, showing a five-times degradation in estimation accuracy. It is considered that regularization based on modeling components outside the limited-scan range is insufficient, placing the model in a highly ill-posed state.

In contrast, the proposed DR-IAA still maintains the DOA estimation accuracy that asymptotically approaches the CRLB even with a narrower scan range. This is because the effect of the double regularization, which includes regularization based on observability capability loss, is fully demonstrated. Quantitatively, the DOA estimation accuracy for a limited-scan range of ±3.5 deg is approximately 0.2 deg, showing almost no degradation compared to the full-range scan case.

4.3. Multiple-Sources Scenario

Next, we evaluate the DOA estimation accuracy under the same conditions as the previous section, but with the addition of a single interfering signal in the out-of-sector region. For each trial, the interferer's angle is drawn from a uniform random distribution within the out-of-sector region, and its SNR is set to 20 dB. This setup is intended to simulate interference from sources such as multipath, which arrive at a power level comparable to the desired signal, rather than a high-power jammer.

Fig. 4 presents the DOA estimation accuracy, with the CRLB from the interference-free case (Fig.3) included for reference. While the interference understandably degrades the performance, these results dramatically illustrate the effectiveness of the proposed regularization term, ρ_{DR}. First, at a wider scan range of ±65 deg, the accuracy is nearly identical to the interference-free case (Fig.3). This suggests that IAA-R's adaptive regularization term is sufficient to capture the influence of the interferer in this well-conditioned regime. However, as the coverage narrows from ±60 deg to ±45 deg, the effectiveness of this adaptive term gradually diminishes, and the angle RMSE of IAA-R exceeds 0.6 deg between the scan range of ±45 deg and ±30 deg. Below ±30 deg, as the problem becomes more ill-posed, the proposed regularization term of DR-IAA begins to dominate. As a result, its angle accuracy can be seen to improve, reaching an RMSE of 0.3 deg at ±10 deg. For coverages narrower than ±10 deg, the accuracy of DR-IAA degrades sharply. We attribute this to the inverse problem becoming so severely ill-posed that the regularization provided by DR-IAA is no longer sufficient.

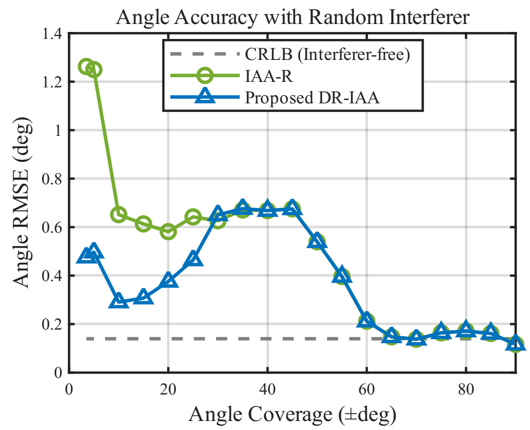


Fig. 4. DOA Estimation Accuracy versus limited-scan Range (SNR=20dB with Single Interference in Out-of-Sector Angle Region)

5. SUMMARY AND FUTURE DIRECTION

In this paper, we have addressed the critical performance degradation of the IAA-R algorithm when applied to DOA spectrum reconstruction in severely limited angular sectors. We have theoretically and experimentally demonstrated that this failure stems from a structural flaw in its adaptive regularization mechanism, which becomes ineffective in severely ill-posed scenarios.

To resolve this fundamental issue, we proposed DR-IAA. The novelty of our approach lies in the introduction of a second, complementary regularization term derived from a new concept we term "observation capability loss." This term quantifies the geometric incompleteness of the limited-scan array manifold and provides a robust baseline for regularization, preventing algorithmic failure.

Numerical simulations have verified that DR-IAA significantly outperforms IAA-R in challenging narrow-scan scenarios, maintaining high angular accuracy and providing a clean, stable spectrum, even in the presence of out-of-sector interference. Our work not only provides a practical and robust solution for constrained high-resolution DOA estimation but also offers a new perspective on regularization, where the problem's geometric structure itself informs the stabilization strategy. Future work will explore the application of this dual-regularization principle to other inverse problems.

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