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Proactive Sequential Phase Swapping Scheduling for Distribution Systems with a Finite Horizon

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Abstract

As renewable energy sources and plug-in electric vehicles increasingly penetrate power distribution systems, phase imbalance becomes more prevalent, posing significant challenges for electric utilities. This paper proactively tackles this issue by implementing phase swapping sequentially at strategic time steps and locations to improve long-term operational cost savings within a given scheduling horizon. An offline data-driven phase-swapping scheduler is developed, mimicking multi-step phase-swapping optimization through an imitation learning framework via supervised learning using random forest regression. This scheduler determines the time steps and associated loads and generations requiring phase swapping. The phase swapping over the finite scheduling horizon is modeled as a multi-step mixed-integer nonlinear programming problem, utilizing a generic branch-based radial load flow model that considers both wye and delta connections for loads and generations. For an upcoming scheduling horizon, the scheduler, based on forecasted generation, load, and price profiles, first identifies the time steps and loads/generations requiring phase swapping. Subsequently, the phase swapping actions for each determined time step are solved sequentially using a mixed-integer nonlinear optimization model with phase swapping at the determined locations and with determined number of swaps. Numerical experiments are conducted on the modified IEEE 13-node test feeder to validate the effectiveness of the proposed approach.

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***Abstract*—As renewable energy sources and plug-in electric vehicles increasingly penetrate power distribution systems, phase imbalance becomes more prevalent, posing significant challenges for electric utilities. This paper proactively tackles this issue by implementing phase swapping sequentially at strategic time steps and locations to improve long-term operational cost savings within a given scheduling horizon. An offline data-driven phase-swapping scheduler is developed, mimicking multi-step phase-swapping optimization through an imitation learning framework via supervised learning using random forest regression. This scheduler determines the time steps and associated loads and generations requiring phase swapping. The phase swapping over the finite scheduling horizon is modeled as a multi-step mixed-integer nonlinear programming problem, utilizing a generic branch-based radial load flow model that considers both wye and delta connections for loads and generations. For an upcoming scheduling horizon, the scheduler, based on forecasted generation, load, and price profiles, first identifies the time steps and loads/generations requiring phase swapping. Subsequently, the phase swapping actions for each determined time step are solved sequentially using a mixed-integer nonlinear optimization model with phase swapping at the determined locations and with determined number of swaps. Numerical experiments are conducted on the modified IEEE 13-node test feeder to validate the effectiveness of the proposed approach.**

Keywords—imitation learning, mixed integer nonlinear programming, phase imbalance, phase swapping, random forest regression.

I. Introduction

The increasing adoption of single-phase inverters in residential areas, driven largely by the rise in rooftop solar photovoltaic (PV) systems and electric vehicle charging, exacerbates phase imbalances in power distribution systems. These imbalances manifest as asymmetrical loads that challenge traditional power distribution paradigms, potentially reducing the efficiency of electrical equipment, increasing energy losses, causing overheating of conductors, and raising the likelihood of relay maloperations. Consequently, addressing these imbalances through innovative scheduling and operation strategies becomes increasingly critical in the pursuit of a sustainable and resilient energy future.

Phase balancing can be achieved in two ways: feeder reconfiguration [1] at the system level and load phase reconfiguration (i.e. phase swapping) at the feeder level. Feeder reconfiguration primarily aims at load balancing among feeders and is not effective in solving phase imbalance issues due to the limited number of sectionalization and tie switches. Therefore, electric utilities highly prioritize phase swapping optimization to maintain maximum three-phase balance by strategically assigning each load or generation to appropriate phases during a given scheduling horizon.

Several works have addressed the issue of phase swapping in systems, such as [2]~[4], most of which employ optimization approaches. For instance, [2] treats phase swapping as load assignment to lines and formulates the problem as a mixed-integer linear program with minimization of weighted sum of unbalanced branch current flows as its objective, and Kirchhoff's current laws serving as linear constraints. [3] formulates phase swapping as a mixed-integer non-linear programming problem to minimize the number of phase connection changes for single and two-phase transformers and laterals. It considers phase sequence constraints and aims for desired phase rebalancing, using geographical information system models of distribution circuits to create a reduced model. [4] proposes a look-ahead moving window optimization method to reduce phase imbalances, accepting a limited number of phase swapping operations. Despite addressing phase swapping problems with different focuses, these works have limitations that need to be overcome. For example, most models overlook voltage drop and power loss in their mathematical formulations. Additionally, the interdependence of phase swapping across time steps in the scheduling horizon is either not explicitly considered or not well-modeled. Another limitation is that approaches primarily focus on wye-connected generations and loads, mixed wye and delta-connected loads and generations are rarely considered.

In this regard, this paper proposes a proactive sequential scheduling method for implementing phase swapping across time steps within a given scheduling horizon. It consists of an offline training stage and an online scheduling stage. The offline stage learns the relationship between the total number of phase swaps and their locations for each time step with load, generation, and price profiles across the entire horizon. This learning is facilitated through imitation learning [5] of mixed-integer nonlinear programming solvers for multi-step phase swapping optimization throughout the horizon. The online stage determines phase swapping actions for the upcoming scheduling horizon by solving single-step phase swapping optimizations sequentially at strategic time steps corresponding to strategic locations of loads and generations. These are determined using the learned relationships against the upcoming load, generation, and price profiles for the upcoming horizon. A generic branch-based radial load flow model is utilized in phase swapping optimization, incorporating both wye-connected and delta-connected loads and generations.

The remainder of this paper is organized as follows: Section II presents a generic branch based nonlinear load flow model for radial distribution systems. Section III describes the formulation of multi-step phase swapping optimization over a scheduling horizon. Section IV discusses the use of imitation learning to mimic multi-step optimization and the related implementation strategy for achieving proactive sequential phase swapping. Section V presents the case studies, and Section VI concludes the paper.

II. Nonlinear Load Flow Formulation for Radial Distribution Systems

A nonlinear load flow model is used for a radial three-phase distribution system in term of branch currents and bus voltages. We use $\mathcal{N} = \{0,1,\cdots,n\}$ denoting the set of buses, in which 0 is the index of the substation fed powers to the distribution system, and $\mathcal{N}^+ = \mathcal{N}\backslash\{0\}$. We also use $\mathcal{E}$ denoting the set of branches connecting between a pair of buses. (i,j) represents a branch connecting an ordered pair of buses (i,j), where bus i lies upstream to bus j. We assume that all the buses $i \in \mathcal{N}$ and branches $(i,j) \in \mathcal{E}$ have three phases: a, b, c; and define $\Phi_Y := \{a,b,c\}$ and $\Phi_D := \{ab,bc,ca\}$. For $i \in \mathcal{N}$ and $\phi \in \Phi_Y$, let V_i^ϕ denote the complex voltage on phase ϕ of bus i, and define $V_i := [V_i^a \quad V_i^b \quad V_i^c]^T$. For $(i,j) \in \mathcal{E}$ and $\phi \in \Phi_Y$, let I_{ij}^ϕ and S_{ij}^ϕ denote the current and power flow on phase ϕ of branch (i,j), and define $I_{ij} := [I_{ij}^a \quad I_{ij}^b \quad I_{ij}^c]^T$, $S_{ij} := [S_{ij}^a \quad S_{ij}^b \quad S_{ij}^c]^T$. If any phase ϕ is missing, the corresponding voltage or current or power flow components will be set as zero. Loads and generations can also be differentiated by their connected locations at connected bus. We define $\mathcal{L}_Y := \{a,b,c\}$ and $\mathcal{L}_D := \{ab,bc,ca\}$ to represent the location sets for wye and delta connected loads or generations at any bus.

Fig. 1 gives a generic model for a branch between bus i and bus j, (i,j), in which bus i and bus j are regarded as a sending bus, and a receiving bus, respectively. The branch includes a voltage regulation component described by an ideal voltage amplifying matrix $A_{V_{i\to j}}$, and an ideal current amplifying matrix $A_{I_{i\to j}}$, a series impedance component z_{ij}, and two shunt admittance components, y_{Uij} and y_{Dij}. Bus j also connects with a set of downstream branches, (j,k), and a shunt admittance, y_{Cj}.

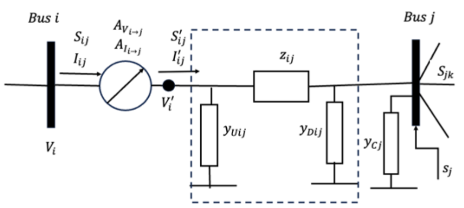

Fig. 1. Generic branch model for a radial distribution system

This generic branch model can be used to model different types of components for the distribution system, including a line segment, a voltage regulator, a switch, and a transformer.

For any branch (i,j), the load flow can be represented using an equation (1) for voltage drop between bus i and bus j, an equation (2) for current balance at bus j, and an equation (3) for power injection at bus j:

$$A_{V_{i\to j}}V_i - V_j = z_{ij}\left(A_{I_{i\to j}}I_{ij} - y_{Uij}A_{V_{i\to j}}V_i\right) \quad (1)$$

$$\sum_{i:(i,j)}\left(A_{I_{i\to j}}I_{ij} - y_{Uij}A_{V_{i\to j}}V_i - y_{Dij}V_j\right) - y_{Cj}V_j + I_j = \sum_{k:(j,k)} I_{jk} \quad (2)$$

$$diag(V_jI_j^H) = s_{G,Y_j} - s_{L,Y_j} + A_{S_{D\to Y}}s_{G,D_j} - A_{S_{D\to Y}}s_{L,D_j} \quad (3)$$

where V_i and V_j are the complex voltages at sending and receiving buses of branch (i,j). I_{jk} is the complex current flowing from bus j to bus k. I_j is the complex injection current at bus j, and I_j^H is the conjugate transpose of I_j. s_{G,Y_j} and s_{L,Y_j} are the wye-connected complex power generations and load demands at bus j. s_{G,D_j} and s_{L,D_j} are the delta-connected complex power generations and load demands at bus j. $A_{S_{D\to Y}}$ is a delta-to-wye conversion matrix to convert delta-connected complex powers into wye-connected ones which is determined by assuming voltages at any bus i are balanced, that is: $\frac{V_i^a}{V_i^b} \approx \frac{V_i^b}{V_i^c} \approx \frac{V_i^c}{V_i^a} \approx e^{j2\pi/3}$, and set as $A_{S_{D\to Y}} = \begin{bmatrix} \frac{1}{2} - j\frac{\sqrt{3}}{6} & 0 & \frac{1}{2} + j\frac{\sqrt{3}}{6} \\ \frac{1}{2} + j\frac{\sqrt{3}}{6} & \frac{1}{2} - j\frac{\sqrt{3}}{6} & 0 \\ 0 & \frac{1}{2} + j\frac{\sqrt{3}}{6} & \frac{1}{2} - j\frac{\sqrt{3}}{6} \end{bmatrix}$ when full phases are present.

III. Multi-Step Phase Swapping Optimizaztion Formulation

The optimal problem for determining phase swapping actions over a given horizon can be formulated to minimize the cumulative cost over the scheduling horizon of T time steps:

$$\text{Minimize} \quad \sum_{t=1}^{T} J(t) \quad (4)$$

where $J(t)$ represents the total sum of costs for power purchase at the substation along with load and generation curtailments $C_{tot}^{SUB}(t)$, penalties costs for branch current and bus voltage limit violations $C_{tot}^{OVR}(t)$, penalties costs for powers, current and voltage imbalances $C_{tot}^{IMB}(t)$, and costs for phase swapping actions $C_{tot}^{SWAP}(t)$ for the time step t:

$$J(t) = C_{tot}^{SUB}(t) + C_{tot}^{OVR}(t) + C_{tot}^{IMB}(t) + C_{tot}^{SWAP}(t) \quad (5)$$

where,

$$C_{tot}^{SUB}(t) = \sum_{\phi\in\Phi_Y}\left(C_P^{SUB}(t)Re\left(s_0^\phi(t)\right) + C_Q^{SUB}(t)Im\left(s_0^\phi(t)\right)\right) + \sum_{j\in\mathcal{N}^+} C_{SG}^{CURT}(t)\left(\sum_{l_Y\in\mathcal{L}_Y} B_{CURT,Y_j}^{l_Y}\left|s_{G,Y_j}^{l_Y}(t)\right| A_{LOC,Y_j}^{l_Y} + \sum_{l_D\in\mathcal{L}_D} B_{CURT,D_j}^{l_D}\left|s_{G,D_j}^{l_D}(t)\right| A_{LOC,D_j}^{l_D}\right) + \sum_{j\in\mathcal{N}^+} C_{SL}^{CURT}(t)\left(\sum_{l_Y\in\mathcal{L}_Y} B_{CURT,Y_j}^{l_Y}\left|s_{L,Y_j}^{l_Y}(t)\right| A_{LOC,Y_j}^{l_Y} + \sum_{l_D\in\mathcal{L}_D} B_{CURT,D_j}^{l_D}\left|s_{L,D_j}^{l_D}(t)\right| A_{LOC,D_j}^{l_D}\right) \quad (6)$$

$$C_{tot}^{OVR}(\mathrm{t}) = \sum_{(i,j)\in\mathcal{E}}\sum_{\phi\in\Phi_Y} C_I^{OVR}(t)\Delta I_{max,ij}^\phi(t) + \sum_{i\in\mathcal{N}^+}\sum_{\phi\in\Phi_Y}\left(C_{V_{max}}^{OVR}(t)\Delta V_{max,i}^\phi(t) + C_{V_{min}}^{OVR}(t)\Delta V_{min,i}^\phi(t)\right) \quad (7)$$

$$C_{tot}^{IMB} = \sum_{\phi\in\Phi_Y}\left(C_P^{IMB}(t)\left|Re\left(IMB_s^\phi(t)\right)\right| + C_Q^{IMB}(t)\left|Im\left(IMB_s^\phi(t)\right)\right| + C_I^{IMB}(t)IMB_I^\phi(t) + C_V^{IMB}(t)IMB_V^\phi(t)\right) \quad (8)$$

$$C_{tot}^{SWAP}(t) = C^{SWAP}n_{swap}(t) \quad (9)$$

where $C_P^{SUB}(t)$, $C_Q^{SUB}(t)$, $C_{SG}^{CURT}(t)$ and $C_{SL}^{CURT}(t)$ are the per-unit costs for active and reactive power purchase at the substation, per-unit costs for generation and load curtailments for time step t. $s_0^\phi(t)$ is the complex power fed from the substation on phase ϕ for time interval t and calculated as:

$$s_0^\phi(t) = \sum_{0:(0,i)} V_0^\phi(t)\left(I_{0i}^\phi(t)\right)^H, \phi \in \Phi_Y \quad (10)$$

$I_{0i}^\phi(t)$ is the complex current flowing through the substation at bus 0 to downstream bus i on phase ϕ, $V_0^\phi(t)$ is the complex voltage at the substation on phase ϕ. $B_{CURT,Y_j}^{l_Y}$ and $B_{CURT,D_j}^{l_D}$ are binary variables to represent if wye-connected and delta-connected load or generation at location l_Y or l_D for bus j is shed at time step t. Those variables for curtailment statuses are constrained by the availability statuses of location l_Y, and l_D, $A_{LOC,Y_j}^{l_Y}$,and $A_{LOC,D_j}^{l_D}$ at bus j:

$$B_{CURT,Y_j}^{l_Y}(t) \leq A_{LOC,Y_j}^{l_Y}, \quad l_Y \in \mathcal{L}_Y, j \in \mathcal{N}^+ \qquad (11)$$

$$B_{CURT,D_j}^{l_D}(t) \leq A_{LOC,D_j}^{l_D}, \quad l_D \in \mathcal{L}_D, j \in \mathcal{N}^+ \qquad (12)$$

$C_I^{OVR}(t)$, $C_{V_{max}}^{OVR}(t)$ and $C_{V_{min}}^{OVR}(t)$ are per-unit costs for branch current limit violation, maximum and minimum bus voltage limit violations at time step t. $\Delta I_{max,ij}^{\phi}(t)$ is branch current limit violation on phase ϕ of branch (i,j) and determined as:

$$\left|I_{I_{ij}}^{\phi}\right| \leq \bar{I}_{ij}^{\phi} + \Delta I_{max,ij}^{\phi}(t), \qquad \phi \in \Phi_Y, (i,j) \in \mathcal{E} \quad (13)$$

where $\bar{I}_{ij}^{\phi}$ is the current limit on phase ϕ of branch (i,j). $\Delta V_{max,j}^{\phi}(t)$ and $\Delta V_{min,j}^{\phi}(t)$ are the voltage upper and lower limit violations at bus j and determined as:

$$V_{min,j} - \Delta V_{min,j}^{\phi}(t) \leq \left|V_j^{\phi}(t)\right| \leq V_{max,j} + \Delta V_{max,j}^{\phi}(t), \quad j \in \mathcal{N}^+ \qquad (14)$$

$V_{max,j}$ and $V_{min,j}$ are the upper and lower limits of voltage on phase ϕ of bus j. $C_P^{IMB}(t)$, $C_Q^{IMB}(t)$, $C_I^{IMB}(t)$ and $C_V^{IMB}(t)$ are per-unit costs for penalizing active power, reactive power, current deviations at the substation among phases, and maximum voltage phase deviation among all buses. $IMB_s^{\phi}(t)$ is the load imbalance at substation on phase ϕ of time step t, and determined as:

$$IMB_s^{\phi}(t) = s_0^{\phi}(t) - \tfrac{1}{3}\textstyle\sum_{\phi\in\Phi_Y} s_0^{\phi}(t), \; \phi \in \Phi_Y \quad (15)$$

$IMB_I^{\phi}(t)$ is the maximum current imbalance at the substation for phase ϕ of time step t, and determined as:

$$IMB_I^{\phi}(t) = \max_{i\in\mathcal{N}_0}\left|\left|I_{0i}^{\phi}(t)\right| - \tfrac{1}{3}\textstyle\sum_{\phi\in\Phi_Y}\left|I_{0i}^{\phi}(t)\right|\right|, \; \phi \in \Phi_Y \quad (16)$$

$\mathcal{N}_0$ is the set of downstream buses of the substation, i.e. bus 0. $IMB_V^{\phi}(t)$ is the maximum voltage imbalance on phase ϕ over all buses downstream to the substation:

$$IMB_V^{\phi}(t) = \max_{j\in\mathcal{N}^+}\left|\left|V_j^{\phi}(t)\right| - \frac{1}{\sum_{\phi\in\Phi_Y} A_{BUS_j}^{\phi}}\textstyle\sum_{\phi\in\Phi_Y}\left|V_j^{\phi}(t)\right| A_{BUS_j}^{\phi}\right|, \quad \phi \in \Phi_Y \quad (17)$$

$A_{BUS_i}^{\phi}$ is the availability of phase ϕ at bus j. C^{SWAP} is the cost per phase swapping action, $n_{swap}(t)$ is the total number of phase swapping at time step t and defined based on phase balancing actions for each swappable bus j, $n_{swap,j}(t)$ using assignment matrices of locations to phases $\{B_{Y_j}^{\phi_Y,l_Y}(t), \phi_Y \in \Phi_Y, l_Y \in \mathcal{L}_Y\}$ or $\{B_{D_j}^{\phi_D,l_D}(t), \phi_D \in \Phi_D, l_D \in \mathcal{L}_D\}$ as:

$$n_{swap}(t) = \textstyle\sum_{j\in\mathcal{N}^+} n_{swap,j}(t), \qquad (18)$$

$$n_{swap,j}(t) = \Big(\textstyle\sum_{l_Y\in\mathcal{L}_Y} A_{LOC,Y_j}^{l_Y} - \sum_{l_Y\in\mathcal{L}_Y} B_{CURT,Y_j}^{l_Y} - \sum_{\phi_Y\in\Phi_Y}\sum_{l_Y\in\mathcal{L}_Y} B_{Y_j}^{\phi_Y,l_Y}(t) B_{Y_j}^{\phi_Y,l_Y}(t-1)\Big) + \Big(\sum_{l_D\in\mathcal{L}_D} A_{LOC,D_j}^{l_D} - \sum_{l_D\in\mathcal{L}_D} B_{CURT,D_j}^{l_D} - \sum_{\phi_D\in\Phi_D}\sum_{l_D\in\mathcal{L}_D} B_{D_j}^{\phi_D,l_D}(t) B_{D_j}^{\phi_D,l_D}(t-1)\Big), \quad j \in \mathcal{N}^+ \qquad (19)$$

$$B_{Y_j}^{\phi_Y,l_Y}(t) = \{0,1\}, \qquad j \in \mathcal{N}^+, \phi_Y \in \Phi_Y, l_Y \in \mathcal{L}_Y \quad (20)$$

$$B_{D_j}^{\phi_D,l_D}(t) = \{0,1\}, \qquad j \in \mathcal{N}^+, \phi_D \in \Phi_D, l_D \in \mathcal{L}_D \quad (21)$$

$B_{Y_j}^{\phi_Y,l_Y}(t)$ is a binary variable representing assigning phase $\phi_Y \in \Phi_Y$ to location $l_Y \in \mathcal{L}_Y$ at bus j and time step t. $B_{D_j}^{\phi_D,l_D}(t)$ is a binary variable representing assigning phase-pair $\phi_D \in \Phi_D$ to location $l_D \in \mathcal{L}_D$ at bus j and time step t. Those two sets of variables are used to represent phase swapping actions.

The phase swapping optimization should satisfy technical constraints for each time step t, include load flow equations, substation voltage settings, and feasibility requirements for phase swapping actions.

Integrated with phase swapping actions, the radial load flow equations at time step t are expressed as:

$$A_{V_{i\to j}} V_i(t) - V_j(t) = z_{ij}\left(A_{I_{i\to j}} I_{ij}(t) - y_{Uij} A_{V_{i\to j}} V_i(t)\right), \quad (i,j) \in \mathcal{E} \qquad (22)$$

$$\textstyle\sum_{i:(i,j)}\left(A_{I_{i\to j}} I_{ij}(t) - y_{Uij} A_{V_{i\to j}} V_i(t) - y_{Dij} V_j(t)\right) - y_{Cj} V_j(t) + I_j(t) = \sum_{k:(j,k)} I_{jk}(t), \qquad (i,j) \in \mathcal{E} \quad (23)$$

$$V_j^{\phi}(t)\left(I_j^{\phi}(t)\right)^* = \textstyle\sum_{l_Y\in\mathcal{L}_Y} B_{Y_j}^{\phi,l_Y}(t)\left(s_{G,Y_j}^{l_Y}(t) - s_{L,Y_j}^{l_Y}(t)\right) + \sum_{\phi_D\in\Phi_D} A_{S_{D\to Y}}^{\phi,\phi_D}\sum_{l_D\in\mathcal{L}_D} B_{D_j}^{\phi_D,l_D}(t)\left(s_{G,D_j}^{l_D}(t) - s_{L,D_j}^{l_D}(t)\right), \quad \phi \in \Phi_Y, j \in \mathcal{N}^+ \quad (24)$$

The substation voltage at time step t is fixed:

$$V_0(t) = V_0^{ref}, \qquad (25)$$

where V_0^{ref} is pre-set three-phase voltages at the substation.

To maintain the feasibility of the assignment matrix for loads or generations, it is required that the assignment matrix for wye-connected loads or generations must satisfy that each available location is only assigned to one phase, and each phase is assigned no more than 1 of available location. Similarly, the assignment matrix for delta-connected loads or generations must satisfy that each available location is only assigned to one phase pair, and each phase pair is assigned no more than 1 of available location:

$$B_{CURT,Y_j}^{l_Y} + \textstyle\sum_{\phi_Y\in\Phi_Y} B_{Y_j}^{\phi_Y,l_Y}(t) = A_{LOC,Y_j}^{l_Y}, l_Y \in \mathcal{L}_Y, j \in \mathcal{N}^+ \quad (26)$$

$$\textstyle\sum_{l_Y\in\mathcal{L}_Y} B_{Y_j}^{\phi_Y,l_Y}(t) \leq 1, \; \phi_Y \in \Phi_Y, i \in \mathcal{N}^+ \qquad (27)$$

$$B_{CURT,D_j}^{l_D} + \textstyle\sum_{\phi_D\in\Phi_D} B_{D_j}^{\phi_D,l_D}(t) = A_{LOC,D_j}^{l_D}, l_D \in \mathcal{L}_D, j \in \mathcal{N}^+ \quad (28)$$

$$\textstyle\sum_{l_D\in\mathcal{L}_D} B_{D_j}^{\phi_D,l_D}(t) \leq 1, \; \phi_D \in \Phi_D, i \in \mathcal{N}^+ \qquad (29)$$

As described above, the multi-step phase swapping optimization can be formulated as a mixed integer nonlinear programming problem (30), according to:

$$\text{Minimize } \textstyle\sum_{t=1}^{T} J(t) \qquad (30a)$$

$$\text{Subject to: (5)\textasciitilde(29)}, \; t = 1,2,\ldots,T \qquad (30b)$$

Solving this problem, we can determine an optimal scheme for phase balancing for each time step. The solution for $n_{swap}(t)$ and $n_{swap,j}(t)$ provide the indications if the system and more specifically bus j need phase connection changes at time step t. If $n_{swap}(t) > 0$, a phase balancing is required for time step t, otherwise no phase balancing is needed. The detailed connection changes for any bus $j \in \mathcal{N}^+$ for time step t are given by the instance differences of phase-location-assignment matrices, $\{B_{Y_j}^{\phi_Y,l_Y}(t), \phi_Y \in \Phi_Y, l_Y \in \mathcal{L}_Y\}$ and $\{B_{D_j}^{\phi_D,l_D}(t), \phi_D \in \Phi_D, l_D \in \mathcal{L}_D\}$ between two consecutive steps t and (t-1). Besides, the generation or load curtailment statuses for any bus $j \in \mathcal{N}^+$ can be provided by $B_{CURT,Y_j}^{l_Y}(t)$ and $B_{CURT,D_j}^{l_D}(t)$. Therefore, we can get a set of recommended actions to conduct phase swapping at strategical time steps and at strategical determined buses by implicitly considering the interdependence between time steps and long-term cost savings.

For practical power distribution systems, (30) is a large-scale mixed integer nonlinear programming (MINLP) problem which is ideally solved as a whole, and phase-swapping decisions for all time steps are obtained simultaneously via a one-time solution. However, due to the substantial computational burden, this solution strategy may not be feasible for real-time applications. An alternative approach commonly used is solving the single-step phase swapping problem for each step within the horizon sequentially. While sacrificing some optimality significance, this approach allows for obtaining a tractable solution step by step. It is difficult to find a good compromise between solution optimality and computation speed using either of these two approaches. To addresses the challenge of phase swapping scheduling for larger systems, we proposed a proactive strategy to schedule phase swapping over a finite scheduling horizon. To enable this strategy, we proposed a data-driven scheduler of sequential phase-swapping based on the decisions of multi-step MINLP optimization. The scheduler as a learned regressor imitates the MINLP expert which would decide when and where the phase swapping should happen.

IV. Imitative Learning-based Proactive Sequential Phase Swapping Scheduling

As shown in Fig. 2, the phase swapping scheduling for a finite horizon T includes two stages, an offline training stage to model the relationship between phase swapping actions and system states given by load, generation and price profiles based on imitation learning, and an online scheduling stage to generate full decisions on phase swapping according to real-time load, generation and price forecasts. The offline stage serves as a phase swapping scheduling agent utilizes imitation learning to mimic a multi-step MINLP solver, and the policy demonstrated by the MINLP expert is approximated with a random forest model. The offline stage begins by identifying the phase swapping time steps and locations through the utilization of a phase swapping scheduling agent, and subsequently, an optimal phase swapping solution is determined for each time step that necessitates phase swapping.

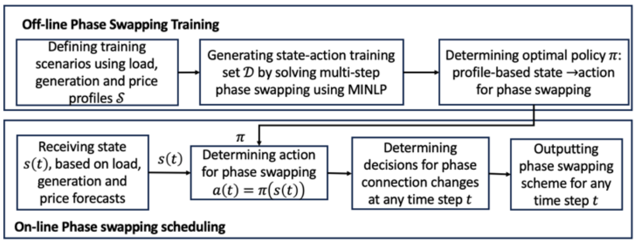

Fig. 2. Steps for proactive sequential phase swapping scheduling

In the context of phase swapping scheduling, since the optimal decisions (including the phase swap actions) can be acquired directly by solving MINLP problem assuming the availability of all information, the MINLP solver is a qualified expert and the optimal solutions of MINLP form perfect demonstrations. We then train an agent to mimic the MINLP solver to map a state to its optimal action. Given a dataset of ω state-action pairs $\mathcal{D} = \{(s_i, a_i)\}_{i=1}^{\omega}$ extracted from expert demonstrations, a policy π that generates a desired action π(s) can be learnt to approximate the expert's optimal decision for a given state s. By doing so, the sequential decision-making problem for phase swapping is reduced to a supervised learning problem, or a regression problem in particular: we try to train a function approximator on $\mathcal{D}$ as the policy π. In this paper, we choose a random forest regression model as the function. We use net power injection derived from corresponding generations and loads, and power purchase prices to define the input features for training samples. We divide the set of buses with non-zero net power injections, $\mathcal{N}_{inj}$ into two sub-sets, $\mathcal{N}_{inj-fix}$ and $\mathcal{N}_{inj-swap}$. $\mathcal{N}_{inj-fix}$ is the set of buses that have power injections but phase connections non-swappable, and $\mathcal{N}_{inj-swap}$ is the set of buses with non-zero power injections and swappable phase connections. The input features for each time step may contain total net active and reactive power injections for all fixed-connection buses for each wye-connected phase, and each delta-connected phase pair, net active and reactive power injections for each swappable-connection bus on each wye-connected location, and each delta-connected location, and per unit purchase costs for active and reactive powers (all other per-unit costs are assumed to be fixed). The output features for each time step may contain the total number of phase swapping, and binary values representing if phase swapping is required for each swappable bus with load and generation.

The Random Forest Regression model is chosen for its robustness, scalability, and ability to handle high-dimensional data with ease. It determines phase swapping statuses and locations for each time step within the scheduling horizon by creating numerous decision trees during training. Each tree outputs the mode of the classes, achieved through random sampling of training data with replacement. For each sample, a decision tree is built where each node considers a random subset of features for splitting. The final prediction aggregates "votes" from all trees to determine the most probable class.

When a time step necessitates phase swapping, we can execute a single-step phase swapping optimization with additional constraints to regulate the count and locations of phase swaps determined by the regressor. If multiple consecutive steps require phase swapping, we can merge them together to formulate a multiple-step optimization, enabling detailed phase swapping decisions for each consecutive time step as:

$$\text{Minimize} \sum_{t=T_b}^{T_e} J(t) \tag{31a}$$

$$\text{Subject to: (5)\textasciitilde(29)}, \quad t = T_b, \dots, T_e \tag{31b}$$

$$n_{swap}(t) = n_{swap}^*(t), \quad t = T_b, \dots, T_e \tag{31c}$$

$$n_{swap,j}(t) > 0, j \in \mathcal{N}_{inj-swap}^*(t) \ \ , t = T_b, \dots, T_e \tag{31d}$$

$n_{swap}^*(t)$ and $\mathcal{N}_{inj-swap}^*(t)$ are the total number of phase swapping actions, and the set of buses with loads or generations having a need for phase swapping that determined by the regressor for time step t. T_b and T_e are the beginning time step and the ending time step, respectively. For a single time step, the beginning and ending time steps are set as the same.

V. Case Studies

In this section, a modified IEEE 13 node test feeder as shown in Fig. 3 is introduced to validate the effectiveness of our proposed approach.

The system includes 3 non-swappable (i.e., connection-fixed) loads: delta-connected loads at bus 671, and wye-connected loads at buses 634 and 675. Additionally, there are 2 swappable generations and 5 swappable loads. The swappable generations are situated at buses 680 and 684, with the delta-connected generations initially connected at phases CA and AB, respectively. Bus 680 hosts a wind power plant, while bus 684 accommodates a solar power plant. The swappable single-phase loads connected to buses 611, 645, and 652 are wye-connected, initially at phases C, B, and A, respectively. Furthermore, the swappable single-phase loads connected to buses 646 and 692 are delta-connected, with their initial connections between phases BC and CA, respectively. The scheduling horizon includes 24 time-steps, and each step lasts one hour. Fig. 4 shows the normalized profiles for load demands, solar generation, wind generation, and active power purchase prices for the test system.

Four different phase swapping scenarios have been tested, including

- Scenario I: No phase swapping.
- Scenario II: Sequential single-step phase swapping optimization.
- Scenario III: Simultaneous multi-step phase swapping optimization.
- Scenario IV: Proactive sequential phase swapping optimization (i.e. the proposed approach).

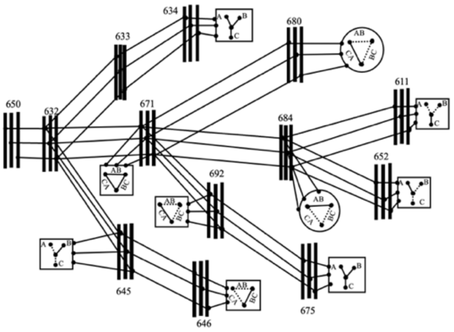

Fig. 3. Modified IEEE 13 node test feeder

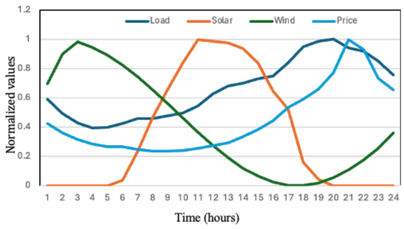

Fig. 4. Load, solar & wind generation, and power purchase price profiles

Table I lists the summaries of phase swapping results for scenarios. Fig. 5 and Fig. 6 show the corresponding variations in costs and active powers at the feeder head for each scenario over the scheduling horizon.

TABLE I.COMRASIONS OF PHASE SWAPPING RESULTS FOR 4 SCENARIOS

Scenario	I	II	III	IV
Optimization formulation	None	24 single-step optimizations	1 multi-step optimization	6 single-step optimizations
Cost(k$)	24.03	8.22	6.80	
Total swaps	0	2	10	
Swap actions	None	t=0: 692: CA→AB, 652: A→ B.	t=0: 692: CA→AB,652:A→ B; t=1: 680: CA→AB; t=2: 684:AB→ BC, 692:AB→ BC; t=7: 646:BC→ AB, 684:BC→ CA; t=13: 684:CA→ AB,t=16, 646:AB→ BC, 692:BC→ AB.	

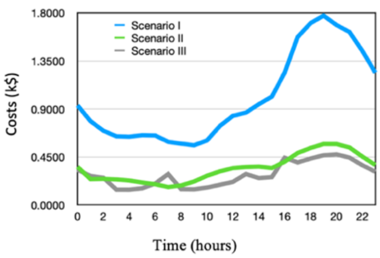

Fig. 5. Total costs for 4 different scenarios

As shown in Table I, Scenario I, which involves no phase swapping, incurs the highest cost among all scenarios. Although more phase swapping actions are required, the costs of the multi-step phase swapping scenario (Scenario III) and the proactive sequential phase swapping scenario (Scenario IV) are lower than that of the sequential single-step phase swapping strategy (Scenario II). Scenario IV, the proactive sequential phase swapping strategy, requires the same number of phase swaps and costs, but involves less computational burden compared to Scenario III, the multi-step phase swapping scenario.

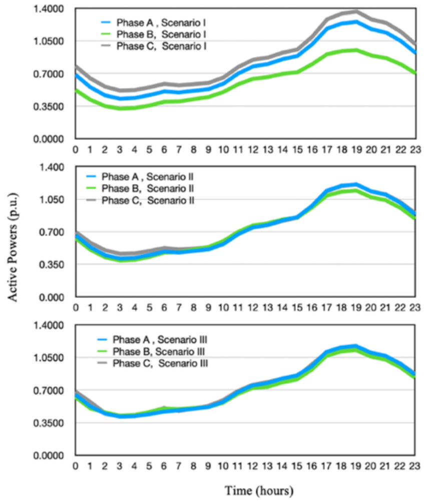

Fig. 6. Active powers at the feeder head for different scenarios

Examining all these figures and table, we observe that compared to scenarios without phase swapping or sequential single-step phase swapping strategies, the proposed proactive sequential phase swapping optimization approach not only reduces operational costs but also enhances power quality through proactive phase rebalancing. Additionally, it requires less computational effort compared to simultaneous multi-step phase swapping optimization.

VI. CONCLUSION

Phase imbalance, typically caused by single-phase loads, presents a significant challenge to the management of distribution systems, exacerbated by the growing penetration of distributed energy resources. This paper addresses the phase imbalance mitigation problem by optimally redistributing loads and renewables across the three phases within a finite horizon. The proposed approach accurately models distribution systems, considering phase-wise voltage drops, power losses, and both wye-connected and delta-connected loads and generation. It determines the optimal phase swapping schedule by mimicking global solutions of phase swapping problem defined by minimizing total costs, including phase swap costs, power purchase and shedding expenses, penalties for operational limit violations, and penalties for phase imbalances, all while ensuring feasible load flows throughout the scheduling horizon.

Our future work will focus on testing with larger systems to enhance algorithm scalability and tractability.

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