Decoupled Trajectory Planning for Monitoring UAVs and UGV Carrier by Reachable Sets

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I. INTRODUCTION

Coordinated operation of heterogeneous autonomous vehicles, such as unmanned ground and aerial vehicles (UGVs and UAVs, respectively) may automate many tasks that are time consuming, expensive, and tiring or dangerous for humans. One such tasks is monitoring [1] of distributed infrastructure, such as power lines, gas and oil pipelines, and of the environment, such as forests, water networks and farming areas, for preventive maintenance and risk mitigation. Monitoring often requires operating in remote or impervious areas, possibly for long periods of time, and the amount of collected information on the targets usually depends on how and for how long the targets are monitored.

Here, we consider motion planning for an UGV and multiple UAVs to monitor a sequence of targets. The plans of UGV and UAVs are correlated because the UAVs depart from and rendezvous with the UGV for long distance transport, re-charging, and possibly data dumping. Thus, while the UGV does not need to visit the targets itself, it needs to follow a plan that allows for recovery of UAV within the battery range and in a suitable time interval.

Even when the sequence of targets is assigned, e.g., by [2], and the references therein, the UGV and UAV planning problem presents multiple challenges: (i) the time scales, ranges, and capabilities of UGV and UAVs are significantly different; (ii) the computational resources required to solve the problem increase with the number of UAVs; (iii) for determining the UAV range, the energy consumption during flight can be estimated precisely given the flight profile, but the time and energy consumption during information acquisition and transmission, which are significant for small UAVs, are hard to predict.

In recent years [3], coordination of UGVs and UAVs is becoming an active research area. Prior research on coordinating task-executing UAVs with re-charging UGVs, see, e.g., [3]–[7] focuses on grid environments, uses graphs methods, e.g., the generalized traveling salesman problem (GTSP), by discretizing the space, and often ignores the UAVs and UGVs dynamics and constraints. The UGV is often considered only as a mobile charging station, and the energy/range depletion models are usually perfectly known.

In this work we consider the generation of trajectories for an UGV carrying and re-charging multiple UAVs tasked with acquiring monitoring information from multiple targets. We account for UGV and UAVs dynamics and operational constraints. We only use an UAV energy depletion model during flight, because during monitoring the energy consumption depends on sensing and transmission and is hard to predict. Since we do not rely on energy consumption models during monitoring and to address time-scale separation and computational burden, we decouple the UGV and UAV trajectory generation while ensuring satisfaction of constraints, energy budget, and rendezvous using reachable sets constructed from sublevel sets of Lyapunov functions [8]. The obtained UAV launch and recovery trajectories may be further refined by separately optimizing each one.

In what follows, Section II formulates the planning problem, Section III designs constraints that ensure feasible UGV-UAV rendezvous, Section IV and Section V describe UGV trajectory generation and optimization of each UAV launch and recovery trajectories, respectively. Section VI reports a case study and Section VII summarizes the conclusions.

Notation: \( \mathbb{R} (\mathbb{Z}) \), \( \mathbb{R}_0^+ (\mathbb{Z}_0^+) \), \( \mathbb{R}_+ (\mathbb{Z}_+) \) are the sets of real
the admissible state and input vectors, respectively.

We consider quadrotor UAVs with linear motion model

\begin{align}
\dot{x}_a(t) &= A_x a(t) + B u_a(t), \\
\dot{u}_a(t) &= C_a x_a(t),
\end{align}

where \( u_a(t) \in \mathbb{R}^m \) is the input vector and \( p_a(t) \in \mathbb{R}^2 \) is the position vector in global coordinates. The choices for the UAV motion model include linear double integrators, unicycle, and bicycle. The UAV is subject to constraints

\begin{align}
x_g \in X_g, \quad u_g \in U_g,
\end{align}

where \( X_g \subseteq \mathbb{R}^{n_g}, \ U_g \subseteq \mathbb{R}^{m_g} \), are sets defining the admissible state and input vectors, respectively.

The range of the UAV is assumed to be significantly longer than what needed for the task to be executed, while the UAVs are battery powered and have limited range. For energy consumption during flight, the spent battery energy changes according to

\[ \dot{E}(t) = c_2 \| v_a(t) \|^2 + c_1 \| a_a(t) \|^2, \]

where we model the dominant effects of air drag as proportional to the square of the velocity, and losses, mechanical and electrical, in accelerating as proportional to the squared acceleration.

Thus, the consumed energy evolves as

\[ \dot{E}(t) = x_a(t) \top Q_e x_a(t) + u_a(t) \top R_e u_a(t), \]

and the energy capacity constraint \( E(t) \leq E_{\text{max}} \) for \( t \in [0, T_{\text{max}}] \) must be satisfied, where \( E_{\text{max}} \) is the maximum usable energy. During monitoring operation, a major source of energy consumption is due to sensors, from processing acquired information, and possibly from communicating data to the UGV acting as base station. A prediction model for such consumption is hard to develop, and hence here we do not rely on it, as it will become clear later.

\textbf{Remark 2.} In model (5) we are ignoring the idle and stationary flight energy consumption, since we focus on the mission phases when the UAVs are moving. These may be included as constants or ignored if we allow the UGV to land and stop the propellers.

While the UAV has limited range, it is unaffected by ground obstacles, which impose constraints on the UGV. The exclusion constraints on the UGV are

\[ p_g \notin \mathcal{O}(\alpha), \alpha \in \mathbb{Z}_{[1,N_O]}, \]

where \( N_o \) is the number of obstacles, and \( \mathcal{O}(\alpha) \) is the collision region for obstacle \( \alpha \), i.e., the set of UGV positions \( p_g \) for which a collision may occur, accounting for the physical shapes of both, obstacle and UGV.

The objective of the UGV and UAVs is to acquire information about a set of targets, \( j \in [1,N_m] \), located a known positions \( p_m(j) = (p_{m,x}(j), p_{m,y}(j)) \). For that the UGV releases the UAVs carrying the sensors and communication to fly at the monitoring location, acquire data with sensors, process them and send the raw and/or processed data back to the UGV for storage or long distance transmission. We assume that the sequence in which targets are to be visited is assigned, e.g., by \( [2] \), and are clustered in groups with know maximum number of elements per cluster.

\textbf{Problem 1.} Given initial time instant \( T_0 \), \( N_m \) monitoring targets with positions \( p_m(j) = (p_{m,x}(j), p_{m,y}(j)) \), \( j \in [1,N_m] \), \( N_O \) obstacle collision sets \( \mathcal{O}(\alpha) \subseteq \mathbb{R}^2, \alpha \in [1,N_O] \), a UGV with motion model (1) subject to (2), (6), initial state \( x_g(0) = x_{g,s} \), and desired final state \( x_{g,f} \), and \( N_a \) UAVs with motion model (3), energy depletion model (5), subject to (4) and initial states \( x_a(i) = x_{a,s} \) for \( i \in [1,N_a] \).

\text{determine}

\begin{itemize}
  \item[i)] a terminal time instant \( T_f > T_0 \)
  \item[ii)] time instants \( t_{i,j}^l, t_{i,j}^r \), \( j \in [1,N_m]; \ i \in [1,N_a] \;
  \item[iii)] commands \( u_g(t) \) for \( t \in [T_0,T_f] \), \( u_a(t) \) for \( t \in \bigcup_{j=1}^{N_m} [t_{i,j}^l, t_{i,j}^r] \cup [t_{i,j}^r, T_f], i \in [1,N_a] \),
\end{itemize}

such that:

\begin{enumerate}
  \item (Timing) \( t_{i,j}^l \leq t_{i,j}^r \leq t_{f,j} \) for all \( i, j, t_{i,j}^l+1 \geq t_{i,j}^r \) for all \( j \in [1,N_m-1]; \ i \in [1,N_a]; \ t_{i,j}^l \geq T_0, t_{i,j}^r \leq T_f, t_{j,f} \leq T_{f,i} \) for all \( i \in [1,N_a] \)
  \item (UAV trajectory) \( x_a(t_{i,j}^l) = x_g(t_{i,j}^l), \dot{x}_a(t_{i,j}^l) = 0, \)
\end{enumerate}

\( x_a(t_{i,j}^r) = x_g(t_{i,j}^r), \dot{x}_a(t_{i,j}^r) \leq E_{\text{max}}, \) for all \( j \in [1,N_m]; \ i \in [1,N_a]; \) and (4) is satisfied.
3) (Monitoring) for all \( j \in \mathbb{Z}_{[1,N_p]} \), there exists \( i \in \mathbb{Z}_{[1,N_a]} \) and \( t_{i,j}^m \) such that \( p^{(i)}_a(t) = p^{(j)}_m \), for all \( t \in [t_{i,j}^m, t_{i,j}^e] \).
4) (UGV trajectory) \( x_g(T_f) = x_{g,f} \) and (2), (6) are satisfied.

In Problem 1 the time instants \( t_{i,j}^l, t_{i,j}^b, t_{i,j}^e, t_{i,j}^r \) are the launch, beginning of the monitoring, end of the monitoring and recovery time of UAV \( i \) to target \( j \), respectively. For notational simplicity we determine such time instants for all UAVs to each target, but we only require one UAV to actually reach the target for monitoring, that means that the launch, monitoring (beginning and ending), and release time for all the others may be set equal, and hence ignored. We determine UAV commands only between launch and beginning of monitoring, and between end of monitoring and recovery. Between recovery and the next launch the UAVs will be docked with the UGV, and during monitoring the UAVs are considered stationary, though practically they will use a separate motion strategy to optimize data acquisition.

In practice, to solve Problem 1 as a whole may be challenging to compute, and imposing the resulting constraint (8) in an optimal control problem may make its solution challenging. Thus, we build sets that are more conservative but easier to compute and use in optimization.

A. Reachable set construction

Reachability can be computed efficiently for several set classes, e.g., polyhedra, ellipsoids, zonotopes [9]–[11]. Ellipsoids have a compact representation as a single convex constraint, which makes them suitable for optimization. To implement (7) by ellipsoids, we consider a fixed linear UAV control law

\[
u = K\delta x_a, \quad \delta x_a = (x_a - x_s),\]

where \( x_s = (p_a, 0) \) is the desired equilibrium, resulting in the asymptotically stable closed-loop dynamics

\[
\dot{\delta} x_a(t) = (A + BK)\delta x_a = A\delta x_a. \quad (10)
\]

Constructing the reachable set proceeds in two steps: (i) building a stabilizing control law (9) and Lyapunov function \( \gamma_e \) that results in a sublevel set \( S_e \) where the constraints are satisfied; (ii) building a Lyapunov function \( \gamma_e \) that results in the reachable set within a given energy budget \( \gamma_e \) as a sublevel set \( S_e \) for the obtained closed-loop dynamics.

We construct and \( K \) in (9) such that the closed-loop (10) is exponentially stable and \( \dot{\gamma}_e(\delta x_a) = \delta x_a^TPe\delta x_a, \quad Pe \succ 0 \) is a corresponding Lyapunov function (10),

\[
\dot{\gamma}_e(\delta x_a(t)) \leq -\alpha \gamma_e(\delta x_a(t)), \quad (11)
\]

with a decay rate \( \alpha \in \mathbb{R}_+ \). \( P_e \) and \( K \) can be easily computed by a semidefinite program [12]. Let the sublevel set \( S_e = \{ \delta x_a : \gamma_e(x_a) \leq 1 \} \) satisfy \( S_e \subseteq \{ \delta x_a : \delta x_a \in X_a, K\delta x_a \in U_a \} \), then

\[
\mathcal{R}_c(p_m) = \{ x_a = (p_a, 0) : (x - x_a)^TP_e(x - x_a) \leq 1, \quad x = (p_m, 0) \}, \quad (12)
\]

is the set of equilibria \( (p_a, 0) \) that are reachable from initial state \( (p_m, 0) \) by control law (9) without violating state or input constraints.\(^2\)

For computing the reachable set within a given energy budget, the energy consumed by the closed-loop dynamics (9) to reach a given equilibrium \( x_a \) from initial state \( x_a \) is

\[
\gamma_e(x_a) = \int_0^\infty \dot{\gamma}(t)dt \leq \gamma_e. \quad (13)
\]

Then, the sublevel set \( S_e = \{ \delta x_a : \gamma_e(\delta x_a) \leq \gamma_e \} \) includes trajectories with energy usage less than \( \gamma_e \). It is

\(^2\)The constraint sets \( X_a, U_a \) do not change when considering the error state \( \delta x_a \) because they do not include constraints on position. Otherwise, an augmented state vector will be needed.
straightforward that for (5), (10), \( \mathcal{V}_c(\delta x_a) = \delta x_a^T P_c \delta x_a \)
where \( P_c > 0 \) is the solution of \( P_c (A + BK) + (A + BK)^T P_c = -(Q_c + K^T R_c K) \). Thus,
\[
\mathcal{R}_c(p_m, \gamma_c) = \{ x_a = (p_a, 0) : (x - x_a)^T P_c (x - x_a) \leq \gamma_c, x = (p_m, 0) \},
\]
is the set of equilibria reached from initial state \((p_m, 0)\) by control law (9) with energy expenditure less than \( \gamma_c \).

Finally, consider the set
\[
\mathcal{R}_K(p_m, \gamma_c) = \mathcal{R}_c(p_m, \gamma_c) \cap \mathcal{R}_c(p_m),
\]
which is invariant, since for any \( \delta x_a \in S_c \cap S_c \), \( \mathcal{V}_c(A_d \delta x_a) \leq \mathcal{V}_c(\delta x_a) \), and \( \mathcal{V}_c(A_d \delta x_a) \leq \mathcal{V}_c(\delta x_a) \) due to the integral in (13), so that \( A_d \delta x_a \in \mathcal{R}_c(p_m, \gamma_c) \cap \mathcal{R}_c(p_m) \). Then, the constraint
\[
(p_g, 0) \in \mathcal{R}_K(p_m, \gamma_c),
\]
determines a set of rendezvous positions for the UAV with the UGV, such that the UAV trajectories from stationary flight at the monitoring target position \( p_m \) to equilibria in such positions satisfy flight envelope constraints and energy budget. In (16) we consider only the rendezvous position and ignore velocity, as the UGV velocity may briefly stop to allow landing, or the UAV may simply perform a landing on a (slowly) moving platform.

Using the exponential stability of \( \mathcal{V}_c \) from (11), we can bound the time for the UAV to reach a neighborhood of the equilibrium, where the rendezvous occurs. Since \( \delta x_a(t)^T P_c \delta x_a(t) \leq 1 \) for all \( t \in \mathbb{R}_{0^+} \), \( \mathcal{V}_c(\delta x_a(t)) \leq \mathcal{V}_c(\delta x_a(0)) e^{-\alpha t} \leq e^{-\alpha t} \). Then, for any \( \epsilon \in \mathbb{R}_{(0,1)} \)
\[
T_{rec} = -\alpha^{-1} \ln \epsilon,
\]
is the upper bound to the time to achieve \( \mathcal{V}_c(\delta x_a) \leq \epsilon \), that defines the acceptable rendezvous region for \( \epsilon \) small enough.

IV. TRAJECTORY OPTIMIZATION FOR UGV

Leveraging the reachable set constraints (16) developed in Section III, we can formulate the UGV trajectory generation separately from the UAVs, while ensuring that rendezvous can occur within the range constraints.

As UGV model (1), we use the kinematic bicycle model
\[
\begin{align*}
\dot{p}_g^x &= v_g \cos \theta_g \quad \tag{18a} \\
\dot{p}_g^y &= v_g \sin \theta_g \quad \tag{18b} \\
\dot{\theta}_g &= \frac{v_g \tan(\delta_g)}{L} \quad \tag{18c} \\
\dot{v}_g &= a_g, \quad \tag{18d}
\end{align*}
\]
where \((p_g^x, p_g^y)\) is the position, \( \theta_g \) is the yaw angle, \( v_g \) is the velocity, \( a_g \) is the acceleration, \( \delta_g \) is the front-steering angle, and \( L \) is the wheelbase. The state and input vectors are \( x_g = [p_g^x, p_g^y, \theta_g, v_g]^T \), \( u_g = [a_g, \delta_g]^T \), respectively.

For the UGV constraints (2), we consider
\[
\begin{align*}
\mathcal{X}_g &= \{ x_g : 0 \leq v_g \leq v_{max} \}, \quad \tag{19a} \\
\mathcal{U}_g &= \{ u_g : |a_g| \leq a_{max}, |\delta_g| \leq \delta_{max} \}, \quad \tag{19b}
\end{align*}
\]
The achievable rendezvous constraint that enforces the UGV to remain inside the reachable set \( \mathcal{R}_K(p_m, \gamma_c) \) of the UAV for a time interval is
\[
\exists t_1^{(j)}, t_2^{(j)} \in [T_0, T_f] \quad \text{s.t.} \quad (p_g(t), 0) \in \mathcal{R}_K(p_m^{(j)}, \gamma_c^{(j)}),
\]
\[
\gamma_c^{(j)} \leq \gamma_{max}, t_2^{(j)} - t_1^{(j)} \geq T_{min}, \forall t \in [t_1^{(j)}, t_2^{(j)}],
\]
where \( T_{min} \in \mathbb{R}_{+} \) is the minimum duration of the rendezvous window. The upper bound to the energy budget for rendezvous \( \gamma_{max} < \mathcal{E}_{max}/2 \) is chosen to leave enough energy for launch and monitoring.

Summarizing, the free-final-time optimal trajectory generation problem for the UGV that ensures feasibility of the rendezvous is formulated as
\[
\begin{align*}
\max_{u_g(\cdot), \gamma_c^{(j)}, T_f} \int_{T_0}^{T_f} J(x_g(t), u_g(t)) dt + w_{g_y} \sum_{j=1}^{N_m} \gamma_c^{(j)} \quad \tag{21a} \\
\text{s.t.} \quad (18), (19), (6), (20), \quad j \in \mathbb{Z}_{[1,N_m]}, \quad \tag{21b} \\
x_g(T_0) = x_{g,s}, \quad x_g(T_f) = x_{g,f}. \quad \tag{21c}
\end{align*}
\]
The cost function \( J : \mathbb{R}^{n_g} \times \mathbb{R}^{m_g} \rightarrow \mathbb{R} \) includes the completion, i.e., final, time and the input energy,
\[
J(x_g) = w_{gt} + w_{g_y} u_g^T u_g,
\]
where \( w_{gt}, w_{g_y}, w_{g_y} \in \mathbb{R}_{+} \) are user-defined weights.

In order to solve (21b) numerically, we discretize it with time scaling. We use a multiple shooting parameterization [13] with the adaptive time-mesh
\[
T_0 = t_0 < t_1 < \cdots < t_N = T_f,
\]
where \( N \in \mathbb{N} \) is the number of sub-intervals \([t_k, t_{k+1}]\) for \( k \in \mathbb{Z}_{[0,N-1]} \), and we introduce time-scaling variables
\[
s_k = t_{k+1} - t_k. \quad \tag{22}
\]
By parametrizing the control as a first order hold and integrating in the intervals, the dynamics are
\[
x_{k+1} = f_g^{(j)}(x_k, u_k, u_{k+1}, s_k). \quad \tag{23}
\]
and the constraints (2), (6) are enforced at node points,
\[
x_k \in \mathcal{X}_g, \quad u_k \in \mathcal{U}_g, \quad p_{g,k} = h_g(x_k, u_k) \notin \mathcal{O}^{(o)}, \quad \tag{24}
\]
where \( x_k \triangleq x_g(t_k), u_k \triangleq u_g(t_k) \). To implement the rendezvous constraint (20), we first assign consecutive node points \( \{k_1^{(j)}, k_2^{(j)}, \ldots, k_{l_2^{(j)}}^{(j)} \} \) to each monitoring target \( j \in \mathbb{Z}_{[1,N_m]} \). Then, we formulate (20) as
\[
\sum_{k=k_1^{(j)}}^{k_{l_2^{(j)}}} s_k \geq T_{min}, \quad \gamma_c^{(j)} \leq \gamma_{max}, \tag{25}
\]
In (20), even if \( k_1^{(j)}, k_2^{(j)} \) are specified, the time instants \( t_1^{(j)} \)
and \( t_2^{(j)} \) are not fixed, since \( s_k \) is a decision variable.
Thus, the discrete-time formulation of (21) is

$$\max_{u_k, e_k, \{\gamma^e_{i,j}\}} w_{gt} \sum_{k=0}^{N-1} s_k + w_{ge} \sum_{k=0}^{N} u_k u_k^T + w_{gy} \sum_{j=1}^{N_m} \gamma^e_{i,j}$$

(26a)

subject to

$$\begin{align*}
(23), (24) & \\
(25), k \in \mathbb{Z}_{[k_1^{(i)}, k_2^{(i)}]}, j \in \mathbb{Z}_{[1, N_m]} & \\
x_0 = x_{g,s}, x_N = x_{g,f}
\end{align*}$$

(26b)

which can be solved by algorithms such as [14].

V. LAUNCH AND RECOVERY TRAJECTORY OPTIMIZATION FOR UAVS

Besides for generating the UAV recovery trajectories, the control law (9) in Section III can be used to compute the launch instants and launch trajectories, by defining the energy budget to travel to the target and using the monitoring target position \(p_{m}^{(j)}\) as equilibrium of the Lyapunov function. However, we see that as less critical because at launch the UAVs have full energy.

In practice, it is convenient to further optimize both launch and recovery the UAV trajectories which provide additional time/energy for the monitoring activities, and/or less time for re-charging at the UGV. Thus, we formulate optimal control problems for launch and recovery that are guaranteed to be feasible since (16) holds the trajectory generated by (9) is feasible, and it can be used both as initial guess and as backup, should the optimization not converge in the available time or encounter numerical issues.

A. Optimization of UAV recovery trajectory

The recovery trajectory planning computes the trajectory of the UAV for returning to the UGV from the monitoring target position. Due to the construction in Section III such trajectory can be generated easily using the linear control law (9). Let \(t_r^{i,j} \in [t_1^{(i)}, t_2^{(i)}]\), where \(t_1^{(i)}, t_2^{(i)}\) are as in (20), the recovery trajectory is generated as the output of

$$\begin{align*}
x^e_a(t) &= (A + BK)x^e_a(t) - BK(p_g(t_r^{i,j}), 0) \\
u^e_a(t) &= BK(x^e_a(t) - (p_g(t_r^{i,j}), 0)) \\
x^e_a(t_r^{i,j}) &= (p_m^{(j)}, 0)
\end{align*}$$

(27a)

(27b)

(27c)

until \(t = t_r^{i,j}\), where \(t_r^{i,j} = t_r^{e,i,j} - T_{rec}\) is computed according to (17). The monitoring operation continues until either \(t < t_r^{e,i,j}\) or the remaining energy reaches \(\gamma^e_{i,j}\), where, if the latter occurs sooner than the former, the UAV hovers until \(t_r^{e,i,j}\) at the monitoring target, or starts the return trajectory earlier than \(t_r^{e,i,j}\) and hovers, or wait on the ground, at the rendezvous location \(p_g(t_r^{i,j})\).

Due to the construction of the control law (9) and imposing of constraint (20) on the UGV, the recovery trajectory is guaranteed to achieve the rendezvous condition \(\mathcal{N}_c(x_a - (p_g, 0)) \leq \epsilon\), while satisfying the flight envelope constraints and within the energy budget \(\gamma^e_{i,j}\).

Further optimization of the recovery trajectory can be achieved to possibly delaying the departure time and/or minimizing the used energy, hence leaving more time/energy for the monitoring operation. We can generate the optimized trajectory by solving

$$\begin{align*}
\min_{u_i, e_i, \{\gamma^e_{i,j}\}} & \ w_{gt}^e t_r^{e,i,j} + w_{ge}^e \mathcal{E}(t_r^{i,j}) \\
\text{s.t.} & \quad \tilde{e} \geq \epsilon \quad (3), (4), (5) \\
& \quad x^e_a(t_r^{i,j}) - x_g(t_r^{i,j}) \leq \epsilon \\
& \quad \mathcal{E}(t_r^{i,j}) = 0, \quad \mathcal{E}(t_r^{i,j}) \leq \gamma^e_{i,j}.
\end{align*}$$

(28a)

(28b)

(28c)

(28d)

(28e)

(28f)

where \(w_{gt}^e, w_{ge}^e \in \mathbb{R}_{++}\) are user-defined weights. Problem (28) is guaranteed to be feasible because the solution from (27) is feasible, with \(t_r^{i,j} = t_r^{e,i,j}\). With the newly computed departure time \(\tilde{e}\) and energy \(\mathcal{E}(\tilde{e})\), the UAV can operate for longer at the monitoring location.

B. Optimization of UAV launch trajectory

The launch trajectory for the UAV starts from the UGV position \(p_g\), reaches the target position \(p_{m}^{(j)}\) and its energy usage must be smaller than \(\tilde{e}_n^{max} - \gamma_{max}\), to save some energy for monitoring. This may be achieved by choosing \(t^l\) to avoid releasing the UAV when too far from target, e.g., according to the scenario and UGV trajectory, or by the results of Section III.

Given the launch time \(t_l^{i,j}\), the optimization of the launch trajectory is formulated as

$$\begin{align*}
\min_{x_a(\cdot), u_a(\cdot), t_l^{i,j}} & \ w_{lt}^l t_l^{i,j} + w_{lt} e \int_{t_l}^{t_l^{i,j}} \tilde{e}(t)dt \\
\text{s.t.} & \quad (3), (4), (5) \\
& \quad x_a(t_l^{i,j}) = (p_g(t_l^{i,j}), v(t_l^{i,j})), \\
& \quad x_a(t_l^{i,j}) = (p_m^{(j)}, 0).
\end{align*}$$

(29a)

(29b)

(29c)

(29d)

where \(v = (v_g \sin \theta_g, v_g \cos \theta_g)\) is the UGV velocity vector. The cost function (29a) aims at minimizing the flight time and the energy with user-defined weights \(w_{lt}^l, w_{lt}^e \in \mathbb{R}_{++}\). The problem in (29) is a free-final-time optimal control problem subject to convex state and input constraints.

C. Trajectory generation and execution summary

The approach for solving the UGV-UAVs trajectory generation Problem 1 with the method proposed in this paper and the execution of the UGV and UAVs trajectories is summarized in Algorithm 1.

VI. CASE STUDY

We consider a case study for validating the proposed method, where we use the following parameters:

$$\begin{align*}
c_1 &= 0.2, c_2 = 1, \alpha = 0.025, \epsilon = 10^{-4}, \\
v_{max} &= 25 m/s, a_{max} = 10 m/s^2, \gamma_{max} = 6000, \\
T_{min} &= 600s, w_{gt} = 1, w_{ge} = 0.02, w_{gy} = 200, \\
w_l^l &= 1, w_l^e = 0.1, w_l^e = 1, \gamma_l^e = 0, L = 2, N = 34, \\
u_{max} &= 10 m/s, a_{max} = 1 m/s^2, \delta_{max} = 5 \text{deg}.
\end{align*}$$
Algorithm 1 Decoupled UGV and UAVs Trajectory Planning

Parameters: $\gamma_{\text{max}}, c_1, c_2, a_g, \mathcal{U}_g, \mathcal{K}, \mathcal{U}_a$

Data: $p_m^{(j)}$ for all $j \in \mathbb{Z}_{[0,N_m]}$, $O^{(a)}$, for all $a \in \mathbb{Z}_{[0,N_a]}$, $x_g, f, T_0, x_g(T_0) = x_a^{(i)}(T_0) = x_{g,s}$ for all $i \in \mathbb{Z}_{[0,N_n]}$.

Trajectory Generation:
1. Construct $\mathcal{R}_K, T_{\text{rec}}, K$ by (15),(17)
2. Compute $T_T, \gamma^{(j)}_{\text{rec}}, t_{r_i,j}^{(j)}$ for all $j \in \mathbb{Z}_{[1,N_m]}$, $i \in \mathbb{Z}_{[1,N_n]}$, $(x_g(t), u_g(t))$ by (21) for $t \in [T_0, T_T]$
3. Compute updated $\gamma^{(j)}_{\text{rec}}, t_{r_i,j}^{(j)}$ and $(x_a^{(i)}(t), u_a^{(i)}(t))$ for $t \in [t_{r_i,j}^{(j)}, t_{r_i,j}^{(j)}]$ for all $j \in \mathbb{Z}_{[1,N_m]}$, $i \in \mathbb{Z}_{[1,N_n]}$ by (28)
4. Compute $t_{r_i,j}^{(j)}$ and $(x_a^{(i)}(t), u_a^{(i)}(t))$ for $t \in [t_{r_i,j}^{(j)}, t_{r_i,j}^{(j)}]$ for all $j \in \mathbb{Z}_{[1,N_m]}$, $i \in \mathbb{Z}_{[1,N_n]}$ by (29)

Trajectory Execution:
5. Execute UGV trajectory $x_g(t)$ for $t \in [T_0, T_T]$
6. for $i = 1 : N_a$ do
7. for $j = 1 : N_m$ do
8. Launch UAV $i$ to target $j$ at $t_{l_i,j}^{(j)}$ and execute $x_a^{(i)}(t)$ for $t \in [t_{l_i,j}^{(j)}, t_{r_i,j}^{(j)}]$.
9. while $t \in [t_{l_i,j}^{(j)}, t_{r_i,j}^{(j)}]$ and $\gamma^{(j)}_{\text{rec}} \leq \gamma_{\text{max}} - \gamma^{(j)}_{\text{rec}}$ do
10. Monitor target $j$ with UAV $i$.
11. end while
12. Return UAV $i$ to UGV by $x_a^{(i)}(t)$ for $t \in [t_{r_i,j}^{(j)}, t_{l_i,j}^{(j)}]$
13. end for
14. end for

We construct the sets using MOSEK to obtain $P_e$ and $K$, and solve the optimal trajectory generation for UGV (26) and UAVs (28), (29) by the PTR method [15] using GUROBI from MATLAB. The computation of the UGV trajectory from (26) takes less than 6s and the computation of the optimized recovery and launch trajectories from (28) and (29), respectively, take less than 0.5s on an 2023 14" MacBook Pro M2 laptop with 64GB Ram running Matlab 2021b (non-native for M2).

The reachable set $\mathcal{R}_K$ of the UAV is shown in Fig. 2, where we confirm the constraint satisfaction of the reachable set by showing the projections of $\mathcal{R}_e$ in the velocity and acceleration planes. Fig. 2 also shows the recovery trajectory by linear control (27) and the optimized one (28) with the highlighted corresponding linear ($T_{\text{rec}}$) and optimized ($\tilde{T}_{\text{rec}}$) recovery instants. The comparisons between energy consumption of linear (27) and optimal (28) recovery trajectory are obtained from an initial point on the border of $\mathcal{R}_K$. Clearly, the energy consumption and the recovery time $T_{\text{rec}}$ are improved by the refinements in Section V, which provides longer time for monitoring, or a shorter re-charge period at the UGV carrier.

Fig. 3 shows the results for the UGV trajectory, where the optimized completion time is $T_f = 105s$, 47s. In the environment, there are 18 monitoring targets ($N_m = 18$), clustered in groups of at most 5, which determines the maximum number of UAVs operating concurrently, and 3 ellipsoidal obstacles ($N_o = 3$). For the rendezvous constraint (25), we specify $(k_1^{(j)}, k_2^{(j)})$ such that $k_2^{(j)} - k_1^{(j)} = 2$, i.e., three consecutive nodes are assigned to each target $j$ in $\mathbb{Z}_{[1,N_m]}$, and hence two arcs of the UGV trajectory remain within the set $\mathcal{R}_K$ centered at the target, see in Fig. 3.

The launch and recovery trajectories of the UAVs are also shown in Fig. 3. While in an actual mission the end time of monitoring $t'$ varies due to factors such as acquired information and remaining energy, in the simulations $t'$ is such that the rendezvous time $t'$ occurs when the UGV is at the exiting border of the $\mathcal{R}_K$ sets, i.e., $t_2$ in (20). This allows the UAV to stay the longest at the monitoring target. Fig. 3 shows that rendezvous always occur in the reachable sets of the UAVs, and hence, according to Fig. 2, flight envelope and range constraints are satisfied. Fig. 4 shows some zoomed-in views of the operations for two targets, and
that allows for decoupling UGV and UAVs planning, while guaranteeing recovery satisfying flight envelope and energy range constraints. The method uses Lyapunov functions to build reachable sets where the constraints are satisfied, and then use those within the UGV trajectory generation problem to ensure feasibility of UAVs rendezvous, possibly extended to launch. The method provides candidate UAV trajectories for recovery and launch, which may be improved by optimal trajectory generation. Future works will consider the recharge period and possibly modifications to the monitoring targets sequence to further optimize the overall mission, the usage of different sets for reachability, and the optimization of motion for information acquisition during monitoring [17].

REFERENCES


