Abstract
In single photon lidar (SPL), the laser repetition rate sets the maximum distance that can be recovered unambiguously. Conventional SPL extends this maximum recordable depth by reducing the repetition rate; however, the slower acquisition speed limits the number of received photons, which may be insufficient to track fast-moving objects. Inspired by recent successes in modulo sensing, we leverage the smoothness of typical trajectories to achieve long-range tracking beyond the unambiguous range. Although SPL naturally acquires modulo time-of-flight measurements, it introduces several challenges—including random sampling times, multiple noise sources, and absolute distance uncertainty—that are not addressed by the current modulo sensing literature. Hence, we propose an interpolation and denoising method that operates directly over the modulo samples. We further disambiguate the absolute distance based on the changing reflectivity fall-off. Monte Carlo simulations considering realistic trajectories under practical conditions show that, when properly unwrapped, the normalized mean squared error of our depth estimate decreases by over 20 dB with respect to a lidar setup whose repetition period leads to no ambiguity.

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TRACKING BEYOND THE UNAMBIGUOUS RANGE WITH MODULO SINGLE-PHOTON LIDAR

S. Fernández-Menduiña*,†, J. Rapp*, H. Mansour*, M. Greiff*, K. Parsons*

*Mitsubishi Electric Research Laboratories (MERL), Cambridge, MA 02139, USA
†University of Southern California, Los Angeles, CA 90089, USA

ABSTRACT

In single photon lidar (SPL), the laser repetition rate sets the maximum distance that can be recovered unambiguously. Conventional SPL extends this maximum recordable depth by reducing the repetition rate; however, the slower acquisition speed limits the number of received photons, which may be insufficient to track fast-moving objects. Inspired by recent successes in modulo sensing, we leverage the smoothness of typical trajectories to achieve long-range tracking beyond the unambiguous range. Although SPL naturally acquires modulo time-of-flight measurements, it introduces several challenges—including random sampling times, multiple noise sources, and absolute distance uncertainty—that are not addressed by the current modulo sensing literature. Hence, we propose an interpolation and denoising method that operates directly over the modulo samples. We further disambiguate the absolute distance based on the changing reflectivity fall-off. Monte Carlo simulations considering realistic trajectories under practical conditions show that, when properly unwrapped, the normalized mean squared error of our depth estimate decreases by over 20 dB with respect to a lidar setup whose repetition period leads to no ambiguity.

Index Terms—Lidar, modulo sensing, single-photon detection, non-uniform sampling, modulo single-photon lidar

1. INTRODUCTION

Conventional single-photon lidar (SPL) systems operate by illuminating a scene with a periodically pulsed laser and detecting the photons scattered back using a time-resolved single-photon avalanche diode (SPAD). Photon detection times are measured with respect to the most recent pulse emission time, which enables direct time-of-flight depth imaging from as little as one photon per scene pixel [1, 2]. Importantly, the periodic illumination causes a tradeoff: a higher repetition rate results in more photon detections but decreases the depth range that can be measured unambiguously. This trade-off becomes critical in tracking applications [3] such as autonomous driving [4] that require both a high frame rate and long-distance tracking capability.

Most SPL approaches set the unambiguous range to be larger than an expected upper bound on the depth [2, 5], or they measure only relative depths for a scene that is at a long distance but lies entirely within one unambiguous range [6, 7]. Alternatively, some methods aim to make absolute distance measurements by modifying the hardware to illuminate with either non-periodic pulse trains [8, 9], multiple repetition rates [7], or different repetition rates for each pixel [10]. Unfortunately, these approaches are designed for imaging static scenes and are unsuitable for single-point tracking of a moving object.

In this paper, we explore the feasibility of modulo SPL operation, i.e., maximizing the illumination rate at the expense of the depth aliasing that occurs when recording detection times that are modulo the repetition period. To track targets beyond the unambiguous range, we can take advantage of the continuity of realistic trajectories to unwrap the modulo by applying existing methods from the modulo sensing literature [11, 12, 13]. However, modulo SPL measurements introduce several key challenges: 1) samples of the trajectory are non-uniformly spaced in time, 2) not all samples are informative due to ambient light, and informative samples are themselves noisy, and 3) modulo sensing methods unwrap sequences up to a constant ambiguity equal to an integer multiple of the modulo threshold. In this work, we address these problems through the following contributions:

1. We generalize the SPL acquisition model to account for target motion and modulo detection times;
2. We propose an interpolation and denoising algorithm that operates over modulo samples, allowing recovery of uniform, denoised modulo samples of the trajectory for use with existing unwrapping methods;
3. We demonstrate recovery of the absolute target position by finding the global offset that best fits the inverse-squared fall-off of reflectivity with distance.

Monte Carlo simulations show that for signal-to-background ratio (SBR) as low as 1 and targets moving at up to 40 m/s, our algorithm unwraps the shape of the underlying signal in 99.75% of trials and finds the absolute depth in 99.5% of trials. Conditioned on correct unwrapping, our method reduces the normalized mean-squared error (NMSE) by over 20 dB compared to a full-range lidar whose repetition period leads to no ambiguity.

2. DATA ACQUISITION AND MODELING

We consider a target moving in one dimension and aim to acquire its position over time. Our basic setup is depicted in Fig. 1(a).

2.1. Acquisition system

Illumination. The laser illuminates the target with pulses at times \( n t_r, n = 0, \ldots, n_0 - 1 \). The laser pulse shape \( s(t) \) is approximately Gaussian with root mean square (RMS) duration \( t_p \ll t_r \). The unambiguous range is \( z_t = c t_r / 2 \), where \( c \approx 3 \times 10^8 \text{ m/s} \) is the speed of light.

Detection. For each photon the SPAD detects, the absolute detection time \( T_i \), for \( i = 0, \ldots, N_t - 1 \), is recorded with time-stamping...
consider a Lambertian surface albedo, which implies that the light reflects isotropically, producing fall-off proportional to the inverse-square of the distance [17]. Then, the reflectivity for the \( n \)th pulse is
\[
\alpha_n := \alpha_{\text{ref}} / (\bar{t}_n)^2, \tag{1}
\]
where \( \alpha_{\text{ref}} \) includes constant attenuation effects from the albedo, view angle, detector efficiency, and fall-off measured with respect to reference TOF, \( \bar{t}_{\text{ref}} \). In some scenarios, e.g., with a known target, it may be possible to calibrate \( \alpha_{\text{ref}} \) in advance of tracking.

### 2.3. Generalized probabilistic measurement model

Photons arrive at the detector as a realization of a Poisson process, but existing modeling e.g., [2, 5]) assumes a static scene and \( z < z_r \). Accounting for aliasing and motion, the illumination
\[
\sum_{t=0}^{\infty} \Phi(t) dt = \alpha_n S + B, \quad q \leq n,
\]
where \( B \) is the background intensity that combines ambient light and dark counts, i.e., false detections due to thermal noise in the SPAD. Assuming the target moves sufficiently slowly so there is approximately one (potentially wrapped) pulse \( s(t) \) in each repetition period, then during the \( n \)th illumination period, the photon detection rate is
\[
\Phi_n := \int_{nt_n}^{(n+1)t_n} \phi(t) dt = \alpha_n S + B, \quad q \leq n,
\]
where \( S := \int_0^{t_n} s(t) dt, B := \bar{t}_n \), and \( \alpha_n S / B \) is the SBR. Note that a detection in period \( n \) may be produced by a photon from a previous illumination \( q \leq n \). We assume operation in the low-flux regime, i.e., \( \Phi_n \ll 1 \forall n \), so that SPAD dead times can be ignored [14]. Thus, photon detection constitutes an inhomogenous Poisson process with time-varying rate function \( \phi(t) \). Wrapped detection times are random variables with density \( f^{(w)}(t) = \phi(t) / \Phi_n \), for \( t \in [nt_n, (n+1)t_n) \). Both reflectivity and TOF vary as a function of \( n \), so these random variables are not identically distributed, but we assume they are independent.

Unlike conventional modeling, the formulation (2) allows not only for the case when \( \bar{t}_n > t_n / 2 \), but also when the pulse shape \( s(t) \) lies across a wrapping boundary. These events represent the points when the detector introduces the modulo discontinuity. Although the distribution of the detection times has to be modified to account for the wrapping effect [18], since we assume \( t_p \ll t_n \), the effect of the pulse wrapping onto itself is negligible.

### 3. MODULO SENSING

Modulo sensing aims to acquire samples of a signal minimizing quantization noise [12, 19] and clipping distortion [11]. Since the noise introduced by the detection-time quantization is negligible compared to other sources of uncertainty, we focus on the latter. Consider a signal \( g(t) \) and an analog-to-digital converter with dynamic range \( [-\lambda, \lambda] \). The goal is to recover \( g(t) \) from its uniform and modulo reduced samples \( M_\lambda (g[n]) := (g[n] + \lambda \mod 2\lambda) \mod \lambda \). The recovery process comprises two steps: 1) recovering the unwrapped samples from the wrapped samples and 2) applying classical reconstruction techniques to recover \( g(t) \). Theoretical results show that recovery of noiseless bandlimited signals is...
shown by the purple dots in Fig. 1(d). This sequence represents repeating the process for histogram of the photons received during the frame. Since $I$, where $B$ matched filter assuming the log-matched filter becomes we update the censoring to wrap around the modulo boundary, and censoring window length may differ. We obtain a sequence of values $\alpha_j$, for $j = 0, \ldots, n_α - 1$, and interpolate the unwrapped TOF to the same temporal positions, which we denote by $U_j$ with some abuse of notation. We then define a set of candidate TOFs as $U_{j,k} = U_j + k't_τ$, where $k \in \mathbb{N}$ sets the global offset of the signal. For each $k$, we compute the reflectivity decay as $\theta_j,k = 1/U_{j,k}$.
In this paper, we addressed the problem of using an SPL system to track a target in one dimension beyond the unambiguous range imposed by the repetition period of the laser. We introduced a method to denoise and interpolate samples that leverages the scenario where the oversampling factor is high. We demonstrated that computational reconstruction of the trajectory from its modulo samples is possible, and that using shorter repetition periods can reduce the distortion in the depth estimates at the expense of slightly more complicated processing. Future work may focus on improving the offset recovery success rate.

6. CONCLUSION

In this paper, we addressed the problem of using an SPL system to track a target in one dimension beyond the unambiguous range imposed by the repetition period of the laser. We introduced a method to denoise and interpolate samples that leverages the scenario where the oversampling factor is high. We demonstrated that computational reconstruction of the trajectory from its modulo samples is possible, and that using shorter repetition periods can reduce the distortion in the depth estimates at the expense of slightly more complicated processing. Future work may focus on improving the offset recovery success rate.

7. REFERENCES
