Abstract
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Topology Optimization of Electric Machines: A Review

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Abstract—: The development of high performance and power dense electric machines invariably requires exploration of the design space to identify promising designs. Conventional electric machine design optimization techniques aim at identifying optimal values for parameterized geometric variables by varying them within a specified range using an optimization algorithm. However, such approaches are limited by the parameterization which is typically determined by manufacturing constraints and the experience of the designer. Optimizing electric machine performance by using the material distribution as a design handle is known as topology optimization. This approach is enabled by the recent advances in additively manufactured metals that allow manufacturing complex geometries. While the application of topology optimization to structural mechanics has been widely studied, its application to identify optimal electric machine designs is an emerging area of research. In this paper, the state-of-the-art in topology optimization of electric machines is reviewed. First, the need for topology optimization is described, and the benefits and challenges of this technique over the conventional parametric optimization are identified. Next, the different topology optimization techniques reported in literature are described and their relative merits are discussed. Finally, a research outlook is provided for this technology.

Index Terms—Topology optimization, electric machine optimization, electric machines

I. INTRODUCTION

Electric machines are used in many applications including consumer appliances, industrial appliances, and transportation. Power dense, efficient, and cost effective electric machines are always desirable across all application domains. A recent technical roadmap report from the U.S. Department of Energy [1] targets 89% reduction in the electric machine volume by 2025 compared to 2020 values for 100 kW traction motors, evidencing the thrust towards improving power density.

The design of high performance electric machines requires investigating the trade-offs between competing objectives such as efficiency, power density, and cost. Several approaches to explore these trade-offs and optimize electric machines have been reported in literature. Bramerdorfer et al. reviewed the different techniques to optimize electric machine designs [2] and identified the salient features of the popular approaches to optimize electric machines. Most of these techniques use a parameterized geometry linked to population-based optimization algorithms, such as genetic algorithms (GA). However, parameterized geometry limits the design space since the components can only take shapes within the specified parameter range. Moreover, since this approach relies on the experience of the designer to come up with the parameterization, it is also susceptible to designer bias.

Topology optimization (TO) can overcome these limitations of parametric optimization (PO) by allowing a free form exploration of the design space. This technique was first proposed for solving compliance problems in structural engineering, and a large body of literature exists on the TO techniques for solving structural mechanics problems [3]–[8]. Recently, these structural topology optimization techniques have also been adapted for optimization of electric machines [9]–[12].

The core contribution of this paper is a review of the state-of-the-art on topology optimization of electric machines. Although topology optimization of electromagnetic devices has been reviewed in [13], this paper is one of the earliest papers to review topology optimization techniques in the specific context of electric machine design. This paper is organized into three main parts. First, the concept of topology optimization is introduced, and the benefits and challenges of this technique over the conventional parametric optimization method are discussed. Next, the different topology optimization techniques applied to optimize electric machines in literature are reviewed to analyze their relative merits. Finally, this paper concludes by providing a research outlook for topology optimization of electric machines.

II. TOPOLOGY OPTIMIZATION

This section introduces the concept of topology optimization and compares it with the ubiquitous parametric optimization technique in the context of optimizing electric machine designs. The material presented in this section will serve as the basis of the detailed review presented in the following sections.

Topology optimization aims to optimize the material distribution within a given design space to best meet the defined optimization objectives, subject to a given set of constraints. It offers more degrees of freedom compared to parametric optimization, since the design is free to take any shape within the design space.

To illustrate the difference between parameterized shape optimization and topology optimization, we consider an example design space for an interior permanent magnet machine (IPM) rotor shown in Fig. 1. A plausible parameterization for
optimization with five variables \((r_o, r_m, \alpha_m, t_m, w_m)\) is shown in Fig. 1a. In a typical optimization problem, the rotor outer radius \(r_o\) would be fixed based on the airgap and stator inner diameter. The remaining parameters would be allowed to vary subject to certain constraints necessary to realize valid geometry while optimizing the design objectives (e.g.: torque density, cost, and efficiency). The design space for this optimization problem is a range of valid values for the parameters indicated in Fig. 1a. The optimal design will have the same template shape as defined in Fig. 1a.

An example design space of the same rotor for topology optimization is shown in Fig. 1b. In this example implementation, the rotor outer radius \(r_o\) is first determined based on the design requirements (alternatively this can also be determined via a parametric optimization). The rotor is then sub-divided into several sub-domains or elements. Each element on the rotor is then assigned a material (air, PM, or iron) and the performance of the design is evaluated. The material distribution is varied subject to certain constraints until the design objectives are optimized.

A plausible optimal design from a topology optimization is shown in Fig. 1c. It can be seen that the design has unconventional PM shape and irregular voids which are not captured by the parameterized geometry shown in Fig. 1a. Such optimization allows a thorough exploration of the design space and is not limited by the parameterization which usually relies on the experience of the machine designer. Instead, each element that can take 3 different materials is a variable for topology optimization. When used with a population based optimization algorithm (such as genetic algorithms), this presents a significant computation load compared to the parameterized design optimization that has just four independent variables. Techniques have been developed to utilize gradient based optimization algorithms and minimize the computation time. These techniques will be reviewed in Section III. Another challenge with topology optimized designs is the unconventional geometries which can pose serious manufacturing challenges using conventional manufacturing techniques. Filtering techniques to realize manufacturable designs from topology optimization have been reported in literature and will be reviewed in Section III. A qualitative comparison of parameterized shape optimization and topology optimization is presented in Table 1.

The recent advances in metal additive manufacturing (AM) allows the fabrication of electric machine components, e.g.: [14]–[19] and permanent magnets, e.g.: [20]–[22] with the unconventional geometries resulting from topology optimization. This has been demonstrated by [10] for soft magnetic components (rotor core of an SPM motor) and [23] for permanent magnets. However, the magnetic properties and performance of the additively manufactured structures remains a key concern. Wrobel and Mecrow presented a comprehensive review of additively manufactured electric machine components in [15], which indicates that additive manufacturing although a promising solution, has a varying level of technology maturity for fabricating different electric machine components. The most technology maturity has been in fabricating auxiliary components (that do not carry flux) such as in-slot heat exchangers using AM. Pham et al. in [17] reviewed additive manufacturing of magnetic materials for electric machine applications. The findings from this review indicate that the recent advances in AM has allowed the fabrication of soft-magnetic components with magnetic properties that are comparable to conventionally manufactured components. Li et al. in [20] proposed the big area additive manufacturing (BAAM) technique to manufacture bonded NdFeB magnets. The results show that permanent magnets with remanence of up to 0.51T can be fabricated using this process. The characterization results in [20] demonstrate that the magnetic and mechanical characteristics of the BAAM fabricated magnets are competitive and in some cases outperform the conventional injection molded magnets.

III. TOPOLOGY OPTIMIZATION TECHNIQUES

The primary objective of nearly all topology optimization studies that are focused on electric machines thus far has been to maximize the torque density of the machine or to shape the torque profile / cogging torque by changing the material distribution in the rotor. The reluctance machine (synchronous reluctance and switched reluctance) and IPM machine rotors are the most favored candidates for topology optimization (e.g.: [11], [12], [25]–[31]), since the reluctance torque in these machines has a strong coupling with the material distribution in the rotor. Topology optimization of electric machine stator geometry has received limited attention, but has also been reported (e.g.: [32], [33]).

Several topology optimization techniques have been presented in literature. Among them, the popular techniques applied to the optimization of electric machines and electromagnetic devices include i) ON-OFF method, ii) Bi-directional Evolutionary Structural Optimization (BESO), iii) Solid-Isotropic Material with Penalization (SIMP), and iv) Level-

![Fig. 1](image-url)
set based methods. Each of these techniques are introduced in this section.

A. ON-OFF Method

The ON-OFF method is the simplest topology optimization technique. This technique involves sub-dividing the geometry into multiple elements, which are typically finite-element analysis (FEA) mesh elements for electric machines, and varying the material assigned to each element. In the simplest case with two materials, each element can be assigned as either air (OFF) or iron (ON). This technique is relatively easy to implement with an evolutionary algorithm and can find reasonable approximations to global optimal solutions without the need for a sensitivity analysis [9].

One of the major drawbacks of the ON-OFF method is its susceptibility to result in discontinuous shapes that are not easily manufacturable. Several filtering techniques have been proposed to overcome this limitation. In [24], Otomo and Igarashi used a Gabor filter in conjunction with ON-OFF method based on Normalized Gaussian network (NGnet) to optimize a synchronous reluctance machine. The Gabor filter was shown to produce designs with significantly lower torque ripple compared to conventional NGnet as shown in Fig. 2. Watanabe et al. [34] proposed a filtering technique that checks the neighborhood of each mesh element prior to material assignment. This reduces the probability of checkerboard patterns and leads to manufacturable designs. This filtering scheme is illustrated in Figs. 3b, 3c and the results with and without filter are shown in Fig. 3d and Fig. 3a respectively. These results indicate the necessity of applying filtering techniques for ON-OFF method to realize manufacturable designs.

Multi-material (air, steel, and other materials) TO based on the ON-OFF technique has also been investigated. For example, a multi-material TO with 3 materials (air, iron and PM) was employed by Sato et al. in [9] for optimizing the rotor shape of an IPM motor to maximize the average torque with constraints on the permanent magnet volume. This technique is based on the ON-OFF method using NGnet. The optimal designs had smooth and manufacturable shapes and approximately 17% torque improvement compared to the baseline design.

B. BESO Method

BESO method, first proposed by Querin et al. in [35] and reviewed in [36], [37] is considered an improvement over the conventional ON-OFF method. In this technique, the sensitivity of the optimization objectives to the material assigned to each element in the design space is used to identify the elements where materials need to be updated (for example from iron to air). This sensitivity is quantified as the BESO sensitivity number for each element. The use of the BESO sensitivity number speeds up the optimization process compared to the conventional ON-OFF method which does not consider the topological sensitivity and only relies on heuristic algorithms.

Garibaldi et al. in [10] applied 3D BESO method to maximize the torque output of an surface permanent magnet (SPM) machine by optimizing the rotor core structure. It was shown in [10] that maximizing the output torque is equivalent to minimizing the stored magnetic energy $W_r$ in the rotor core. The magnetostatic optimization problem is formulated as (1) where, $V_o$ is the target volume, $w_e$ and $v_e$ are respectively the elemental magnetic energy density and elemental volume, $e$ is the mesh element, $N$ is the total number of mesh elements, and $x_e$ is an identifier. $x_e = 1$ implies that the element $e$ is
The BESO sensitivity number is then computed as

\[ p = \frac{\partial W}{\partial x_e} \]

is a penalization factor, and formulated as follows [12]:

\[ \text{bound on the torque ripple, the optimization problem can be removing material from the rotor and subject to an upper} \]

\[ \text{density of material in each element.} \]

\[ \text{such that with respect to solving the optimization problem, which requires the sensitiv-} \]

\[ \text{continuous function. A gradient based optimizer is used for} \]

\[ \text{technique.} \]

\[ \text{using high silicon steel and the selective laser melting (SLM) technique.} \]

\[ \text{approximation 24% lower rotor copper losses compared to a} \]

\[ \text{density, and material distribution (to ensure that intermediate} \]

\[ \text{variable, it can take intermediate values which are invalid} \]

\[ \text{solution to overcome this drawback and avoid intermediate material densities is} \]

\[ \text{valid (non-physically realizable materials). A solution to overcome} \]

\[ \text{variable,} \]

\[ \text{SIMP method is one of the most popular techniques for} \]

\[ \text{SIMP method define the interfaces between material} \]

\[ \text{A concern with the SIMP technique is intermediate material} \]

\[ \text{Since the material density} \]

\[ \text{A concern with the SIMP technique is intermediate material} \]

\[ \text{Guo et al. presented a multi-material magneto-structural} \]

\[ \text{optimized for wound field synchronous machine (WFSM) rotors in [38]. The optimization objective} \]

\[ \text{approximately 24% lower rotor copper losses compared to a} \]

\[ \text{Baseline design obtained from a parametric optimization.} \]

\[ \text{The optimal rotor geometry from [10] is shown in Fig. 4.} \]

\[ \text{It can be seen that this rotor core design does not have a} \]

\[ \text{This core was fabricated using high silicon steel and the selective laser melting (SLM) technique.} \]

\[ \text{C. SIMP Method} \]

\[ \text{SIMP method is one of the most popular techniques for} \]

\[ \text{SIMP method define the interfaces between material} \]

\[ \text{A concern with the SIMP technique is intermediate material} \]

\[ \text{Guo et al. presented a multi-material magneto-structural} \]

\[ \text{optimized for wound field synchronous machine (WFSM) rotors in [38]. The optimization objective} \]

\[ \text{approximately 24% lower rotor copper losses compared to a} \]

\[ \text{Baseline design obtained from a parametric optimization.} \]

\[ \text{D. Level-Set Methods} \]

\[ \text{Level-set method define the interfaces between material} \]

\[ \text{Baseline design obtained from a parametric optimization.} \]
first material, the negative set \( \Omega_2 = \{ x \in \Omega | \phi < 0 \} \) defines the domain of the second material or void, and the zero set \( \Gamma = \{ x \in \Omega | \phi = 0 \} \) defines the interface between the two materials. It thereby avoid any intermediate material densities, which is a major concern with the SIMP method; it can also largely avoid the check board pattern that is a problem for ON-OFF method. An evolution of the level-set function is used to update the zero level-set interface and the material shape throughout the optimization process, which typically done by solving for the Hamilton-Jacobi equation

\[
\frac{\partial \phi}{\partial t} + \mathbf{V} \cdot \nabla \phi = 0,
\]

where \( \mathbf{V} \) is the velocity field vector for the change of the interface.

A level-set TO method for synchronous reluctance machines using body-fitted mesh representation was proposed by Kuci et al. in [41]. The geometry was modeled as a bounded domain \( \Omega \) consisting of two materials distributed in domains \( \Omega_1 \) and \( \Omega_2 \) without any overlap, i.e., \( \Omega_1 \cap \Omega_2 = \emptyset \). Level-set method has also been used to optimize the stator coil layout of electric machines in [42] and permanent magnets in [43].

**E. Comparison of Topology Optimization Techniques**

In this section the topology optimization techniques are compared in the electric machine optimization context. This qualitative comparison is expected to help electric machine designers select a suitable technique to optimize electric machine topologies.

ON-OFF technique and its variants are popular for electric machine topology optimization. These techniques are usually steered by heuristic algorithms that require several evaluations. The optimization design space usually has variables on the order 1000-10,000, requiring significant computation power and time. In addition, filtering techniques are necessary to realize manufacturable shapes.

The BESO method was proposed for electromagnetic TO in [10]. While a simple analytical expression was obtained, enabling the computation of BESO sensitivity using FEA results [10], that simplicity may be attributed to the objective function defined. It is not guaranteed that every objective can be reduced to simple analytical expressions for BESO.

The SIMP technique requires access to the FEA stiffness matrix which is not available in most commercial FEA tools. While some tools do provide access to the stiffness matrix, it is not very straightforward to perform the necessary modifications to implement the SIMP technique.

The level-set based methods have been used with both evolutionary algorithms and gradient based algorithms to optimize electric machine topologies. Adjoint method is often used for computationally efficient sensitivity analysis of gradient based algorithms (eg: [44]). However, since the electric machine optimization is a non-convex problem, all gradient based methods although fast, can get stuck at local minima. Therefore, two-level techniques such as the one proposed in [40] are necessary to effectively search the design space.

Several commercial multi-physics analysis tools have SIMP and level-set methods implemented in built-in libraries. However, most of these have limited functionality and are more suited for structural compliance problems. Modifications are required to be successfully used to optimize electric machines.

The popular TO techniques are qualitatively compared in Table II. It should be noted that even though BESO is an evolutionary method, it is considered as gradient-assisted, since the gradient information is used to steer the optimization. The level-set method can be implemented using either gradient based or evolutionary optimizers.

Due to the strengths and weaknesses of each method, a promising research direction is to combine different methods in the optimization process. Hidaka et al. proposed a two-step TO technique in [40] that uses the level-set method in conjunction with the ON-OFF method. A graphical depiction of this technique is presented in Fig. 6 The ON-OFF method is used to perform a global search and the level-set method is used to perform a local search. Genetic algorithm and gradient-based algorithms are employed for the former and latter optimizations, respectively. This technique was used to maximize the average torque and minimize torque ripple by affecting the material distribution in an IPM rotor. The results showed approximately 40% reduction in torque ripple and 0.5% increase in average torque over design optimized using the ON-OFF method alone. Similar two-step methods have been developed by Otomo et al. in [45] for TO of the 3D structure in a claw-pole alternator.

**IV. CHALLENGES AND RESEARCH OPPORTUNITIES**

Although topology optimization of electric machines is very promising to realize innovative designs, it involves several challenges. In this section, these challenges are identified and a research outlook towards helping improve the adoption of these techniques in the design of electric machines is presented.

**TABLE II**

<table>
<thead>
<tr>
<th>Metric</th>
<th>ON-OFF</th>
<th>BESO</th>
<th>SIMP</th>
<th>Level Set</th>
</tr>
</thead>
<tbody>
<tr>
<td>Type of Optimizer</td>
<td>Evolutionary</td>
<td>Gradient assisted</td>
<td>Gradient based</td>
<td>Flexible</td>
</tr>
<tr>
<td>Local minima</td>
<td>Less susceptible</td>
<td>Yes</td>
<td>Yes</td>
<td>Optimizer dependent</td>
</tr>
<tr>
<td>Popularity</td>
<td>Medium</td>
<td>Low</td>
<td>Very high</td>
<td>High</td>
</tr>
<tr>
<td>Computation time</td>
<td>Extremely slow</td>
<td>Fast</td>
<td>Fast</td>
<td>Optimizer dependent</td>
</tr>
<tr>
<td>Available in commercial tools?</td>
<td>No</td>
<td>No</td>
<td>Yes</td>
<td>Yes</td>
</tr>
</tbody>
</table>
A. Manufacturability

One of the major challenges with topology optimization is the manufacturability of the optimized designs. Topology optimized designs nearly always have irregular features, jagged surfaces, and discontinuous material distribution, which are unrealistic to manufacture with conventional techniques. Although additive manufacturing can overcome these limitations, with the current technology maturity, it is cost prohibitive. Moreover, it is difficult to achieve the desired magnetic and structural properties with additively manufactured structures. For example, irregular PM shapes are difficult to manufacture economically, and additively manufacturing high performance PMs is not feasible with the current technology maturity [46]. Therefore, the electric machine TO process needs to consider manufacturability of the design, which limits the flexibility that the TO techniques offer.

B. Computation Requirement

Topology optimization techniques such as the ON-OFF method based on population-based evolutionary optimization, can present a significant computational requirement based on the geometry being optimized. Although using a gradient based techniques such as SIMP and level-set can overcome this limitation and reduce the computational burden, since electric machine optimization is a non-convex problem, the gradient based techniques are not always successful in identifying the global optima. Techniques based on machine learning and deep learning are being developed (e.g.: [47]–[51]), which can potentially reduce the computational burden of topology optimization while identifying globally optimal designs.

C. Mechanical Concerns

Topology optimized electric machine designs with only electromagnetic performance improvement and volume reduction as objectives can result in structurally weak designs. For example, when optimizing an IPM or a reluctance machine rotor, the resulting rotor shapes may introduce several airgaps/voids which can significantly reduce the structural strength of the rotor and also cause rotordynamic concerns. Therefore, structural compliance requirements must be included in the topology optimization problem [12] to ensure that the optimized geometry is mechanically robust.

D. Multiple Materials

Most electric machine parts have multiple materials, e.g.: copper, iron, PMs. While it is relatively straightforward to optimize electric machine parts that are made of a single material, such as rotors of switched / synchronous reluctance motors, it is challenging to optimize parts that contain two or more materials (e.g.: PM rotors, stators). While multi-material topology optimization studies have been reported in literature (e.g.: [9], [38], [52], [53]), it is not as straightforward to implement as parametric optimization. Furthermore, when dealing with non-isotropic materials such as permanent magnets, additional constraints such as the magnetization vector also need to be considered.

E. System Optimization

Electric machines are typically designed for specific applications. This necessitates the consideration of the complete machine drive-cycle for optimization. However, most topology optimization studies in literature have been limited to maximizing the torque output of the electric machine at a given rotor position, maximizing the average torque over one rotation, and minimizing the torque ripple. None of the studies consider system-level objectives such as efficiency, power density and cost of the overall system.

Hybrid optimization techniques that integrate parametric optimization with topology optimization, e.g.: [11], [54] can present a potential solution to optimize system-level parameters and address this limitation. Credo et al. in [11] proposed a hybrid optimization technique (Fig. 7) that integrates parametric and topology optimization to optimize the rotor shape of high speed synchronous reluctance machines. In this framework, the topology optimization based on the SIMP method is first used to identify the rotor topology. This topology is used to derive a parametric model. The parametric model is then used for a final magneto-structural optimization (run as a parametric optimization). This ensures that the final optimal design is manufacturable. Moreover, multiple system-level objectives and additional physics (thermal, structural, rotor dynamics) can be included in the final parametric optimization. However, the biggest challenge in this technique is automating the parametric model creation from the topology optimization results. This presents an opportunity for further research.
V. CONCLUSION

This paper presented a review of topology optimization techniques applied to electric machine design optimization. First, topology optimization was introduced and compared with parametric optimization. This comparison showed that while topology optimization can potentially enable discovering unconventional designs and is free of designer bias, implementation of topology optimization algorithms and manufacturability of the optimal designs remain a key challenge. Next, four popular topology optimization techniques were presented and compared. Finally, the challenges in applying topology optimization techniques for electric machine design optimization were identified along with avenues for future research.

Ultimately, the review presented in this paper shows that although topology optimization allows fundamentally rethinking the design of electric machines, several challenges including manufacturability, computational complexity, limit on the number of materials, and implementation concerns must be overcome for it to be successfully used in realizing practical machine designs and optimizing the complete electric drive system performance. Future work will consider performance evaluation of the different topology optimization techniques on benchmark electric machine designs.

REFERENCES


