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Abstract

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ROBUST PARAMETER ESTIMATION OF CONTAMINATED DAMPED EXPONENTIALS

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ABSTRACT

Parameter estimation of damped exponential signals has wide applications including fault detection and system parameter identification, etc. However, existing methods for estimating parameters of damped exponentials are either sensitive to noise or restricted to dealing with a certain type of noise such as Gaussian noise. In this paper we aim to estimate parameters of damped exponentials from contaminated signal, *i.e.*, a mixture of damped exponentials, random Gaussian noise, and spike interference. We propose two robust approaches, a convex one solved by the alternating direction method of multipliers (ADMM) and a non-convex one solved by coordinate descent, to recovering a low-rank Hankel matrix of damped exponentials from noisy measurements for further parameter estimation using the matrix pencil technique. Numerical experiments show that our proposed methods outperform classical ones in detecting small damped fault signatures from noisy measurements. While the convex approach is amenable to theoretical analysis and global convergence guarantees, the non-convex one exhibits more robustness and computational efficiency.

***Index Terms*—** Signal decomposition, parameter estimation, damped exponential, matrix pencil, fault detection

1. INTRODUCTION

The model of damped exponentials occurs naturally in a wide range of applications including fault detection [1,2], structural health monitoring [3], and system identification [4], etc.

1.1. Parameter Estimation of Damped Exponentials

Mathematically, the system observes a time-domain signal

$$y(t) = \sum_{j=1}^{M} A_j e^{\alpha_j t} e^{i\left(2\pi f_j t + \theta_j\right)} + \eta(t) \quad (1)$$

where $y(t)$ is composed of M damped exponentials but contaminated by noise $\eta(t)$. Parameters $A_j > 0$, $\alpha_j \le 0$, $f_j > 0$ and $\theta_j \in \mathbf{R}$ represent the corresponding amplitude, the damping coefficient,the frequency, and the phase of the $j^{th}(j = 1, ..., M)$ damped exponential component, respectively. In many practical applications, *e.g.*, signal analysis of electric circuits [5] and fault detection of induction machines [6], these parameters as well as the number M are typically unknown and to be identified for either analyzing the system status or evaluating the machine fault severity, *etc*. Furthermore, the noise $\eta(t)$ may include not only white Gaussian noise but also spike interference, which is caused by external interference such as switching operations or internal defects such as mechanical faults. Therefore, it is of great interest to identify damped exponentials as well as spikes from contaminated measurements.

The goal of this paper is to provide robust solutions to decompose a mixture signal of damped exponentials, spikes, and random Gaussian noise, and further to estimate all unknown parameters of the exponentials.

1.2. Related Work

Parameter estimation of damped exponentials has been extensively studied in the noiseless setting [7, 8]. Well-established methods for solving this problem include Prony's method [9, 10], which contains a polynomial root-finding operation, and the matrix pencil method [11], which forms a matrix pencil based on the input signal and solves a generalized eigenvalue problem. According to [12, 13], the matrix pencil method is computationally more efficient and has better statistical properties compared to the Prony's method. However, both methods are very sensitive to noise.

To combat noise, data pre-processing methods based on singular value decomposition (SVD) have been proposed for the matrix pencil method, and demonstrated very effective for Gaussian noise [8, 14]. For example, the total-least square matrix pencil (TMP) [8] truncates the singular values of a Hankel matrix constructed from the noisy observation. The underlying principle is that the true noise-free Hankel matrix of damped exponentials is low-rank [15, 16]. However, in the presence of ubiquitous spike interference [5, 17], a few grossly corrupted entries severely affect the result of SVD [18], resulting poor performance in parameter estimation. Although one may use robust principle component analysis (RPCA) [18] to effectively extract a low-rank matrix despite of spike interference, this low-rank matrix typically cannot preserve the Hankel structure required for further parameter estimation.

In this paper, we propose two robust approaches, *i.e.*, a convex one which we refer to as Convex Robust Parameter Estimation (CRPE) and a non-convex one which we refer to as Non-convex Robust Parameter Estimation (NRPE). Both methods take into consideration the low-rank property and the structure of Hankel matrices, as well as the sparse property of spike interference. We solve the former problem using the alternating direction method of multipliers (ADMM) [19], and the latter one using coordinate descent. By solving these problems, we can robustly decompose the Hankel matrix constructed from the noisy observations into a low-rank noise-free Hankel matrix of damped exponentials, a sparse matrix of spike interference, and a residual matrix of Gaussian noise. It is then straightforward to estimate parameters of damped exponentials from the recovered low-rank Hankel matrix by applying the classical matrix pencil algorithm [8,11]. Our numerical experiments show that both approaches outperform classical ones in recovering small fault signatures, at similar computational cost. While the convexity of CRPE makes it amenable to theoretical analysis and global

Youye Xie performed this work as an intern at MERL.

convergence guarantees, NRPE exhibits better robustness and computational efficiency.

Our paper is organized as follows. In Section 2 we propose two approaches to decomposing the Hankel matrix of noisy measurements. We then develop two optimization algorithms respectively in Section 3. Details of our numerical experiments are described in Section 4 with conclusion drawn in Section 5.

2. CONVEX AND NON-CONVEX ROBUST PARAMETER ESTIMATION

Without loss of generality, we define a Hankel matrix $\mathbf{H}_p(\boldsymbol{x}) \in \mathbf{C}^{(N-p)\times(p+1)}$ of a sampled signal $\boldsymbol{x} \in \mathbf{C}^N$ as

$$\mathbf{H}_p(\boldsymbol{x}) = \begin{bmatrix} \boldsymbol{x}(1) & \boldsymbol{x}(2) & \dots & \boldsymbol{x}(p+1) \\ \boldsymbol{x}(2) & \boldsymbol{x}(3) & \dots & \boldsymbol{x}(p+2) \\ \vdots & \vdots & & \vdots \\ \boldsymbol{x}(N-p) & \boldsymbol{x}(N-p+1) & \dots & \boldsymbol{x}(N) \end{bmatrix}.$$

If the sampled signal $\boldsymbol{x} \in \mathbf{C}^N$ is a sum of $M (M << N)$ damped exponentials, by choosing $p \in [M, N-M]$, the Hankel matrix is proved to be a rank $M (M \leq p)$ matrix [15, 16], or a low-rank matrix if $M << p$. In the noiseless case, the matrix pencil algorithm exploits this low-rank property to accurately estimate parameters of the exponentials by eigen analysis. Following this idea, in this paper we aim to extract such a low-rank Hankel matrix, $\mathbf{H}_p(\boldsymbol{x})$, where $\boldsymbol{x}$ is the estimated sum of damped exponentials, using the Hankel matrix of noisy observation $\mathbf{Y} = \mathbf{H}_p(\boldsymbol{y}) \in \mathbf{C}^{(N-p)\times(p+1)}$, where $\boldsymbol{y} \in \mathbf{C}^N$ is the sampled noisy observation. Since p is fixed during the optimization process, we simplify the notation of Hankel matrix as $\mathbf{H}(\boldsymbol{x})$ by dropping the subscript p.

2.1. CRPE Optimization Problem

Inspired by the success of the robust principal component analysis [18] and the blind signal decomposition work in the compressive sensing community [20–22], we formulate the Hankel matrix demixing problem as a convex optimization problem

$$\min_{\boldsymbol{x},\mathbf{S}} \frac{1}{2}||\mathbf{Y} - \mathbf{H}(\boldsymbol{x}) - \mathbf{S}||_2^2 + \lambda_1||\mathbf{H}(\boldsymbol{x})||_* + \lambda_2||\mathbf{S}||_1, \quad (2)$$

where we apply the nuclear norm $||\cdot||_*$ to relax the low-rank constraint on $\mathbf{H}(\boldsymbol{x})$, use the ℓ_1 norm to impose sparsity on matrix $\mathbf{S}$ caused by spike interference, and assume the residual is Gaussian noise.

2.2. NRPE Optimization Problem

Alternatively, we can also perform a non-convex optimization by replacing the nuclear norm regularization in (2) with an explicit constraint on the rank of $\mathbf{H}(\boldsymbol{x})$ as follows

$$\min_{\boldsymbol{x},\mathbf{S}} \frac{1}{2}||\mathbf{Y} - \mathbf{H}(\boldsymbol{x}) - \mathbf{S}||_2^2 + \lambda_2||\mathbf{S}||_1, \text{ s.t. } \text{Rank}(\mathbf{H}(\boldsymbol{x})) \leq r, \quad (3)$$

where r denotes the maximum number of damped exponentials we expect to recover. In practice, r can be set according to an initial estimate of r or a prior knowledge based on the nature of the practical application. The main advantage of this non-convex method is that the constraint on rank is more intuitive and relatively easier to set than the convex one in (2). However, due to its non-convexity, the optimization algorithm could get trapped in local minima.

Algorithm 1 Solving CRPE via ADMM

Input: $\boldsymbol{y}$, λ_1, λ_2, p, μ, tol, MaxIter

Initialization: $\mathbf{Y} = \mathbf{H}_p(\boldsymbol{y})$, $\boldsymbol{x}_0 = \mathbf{0}$, $\mathbf{S}_0 = \mathbf{Z}_0 = \mathbf{V}_0 = \mathbf{0}$, $Loss_0 = 0$
for $k = 0, 1 \cdots$, MaxIter **do**
  Update $\boldsymbol{x}$:
    $\boldsymbol{x}_{k+1} = \frac{1}{1+\mu}$ RevDM($\mathbf{Y} - \mathbf{S}_k + \mu\mathbf{Z}_k - \mathbf{V}_k$)
  Update $\mathbf{Z}$:
    $\mathbf{Z}_{k+1} = \mathcal{D}_{\lambda_1\mu^{-1}}\left(\mathbf{H}_p(\boldsymbol{x}_{k+1}) + \mu^{-1}\mathbf{V}_k\right)$
  Update $\mathbf{S}$:
    $\mathbf{S}_{k+1} = \mathcal{S}_{\lambda_2}\left(\mathbf{Y} - \mathbf{H}_p(\boldsymbol{x}_{k+1})\right)$
  Update $\mathbf{V}$:
    $\mathbf{V}_{k+1} = \mathbf{V}_k + \mu[\mathbf{H}_p(\boldsymbol{x}_{k+1}) - \mathbf{Z}_{k+1}]$
  Calculate the Loss:
    $Loss_{k+1} = f_{CRPE}(\boldsymbol{x}_{k+1}, \mathbf{S}_{k+1})$

  if $|Loss_{k+1} - Loss_k|/|Loss_{k+1}| \leq$ tol **then**
    $Break$
Output: $\mathbf{x}_{k+1}, \mathbf{S}_{k+1}$

3. OPTIMIZATION ALGORITHM

3.1. ADMM for CRPE

To solve the CRPE optimization problem (2), we introduce an auxiliary variable $\mathbf{Z}$ with constraint $\mathbf{Z} = \mathbf{H}(\boldsymbol{x})$. Then the augmented Lagrangian function of (2) can be expressed as

$$\mathcal{L}_\mu(\boldsymbol{x}, \mathbf{S}, \mathbf{Z}, \mathbf{V}) = \frac{1}{2}||\mathbf{Y} - \mathbf{H}(\boldsymbol{x}) - \mathbf{S}||_2^2 + \lambda_1||\mathbf{Z}||_* + \lambda_2||\mathbf{S}||_1 + \langle \mathbf{H}(\boldsymbol{x}) - \mathbf{Z}, \mathbf{V}\rangle_{\mathbf{R}} + \frac{\mu}{2}||\mathbf{H}(\boldsymbol{x}) - \mathbf{Z}||_2^2,$$

where $\mathbf{V} \in \mathbf{C}^{(N-p)\times(p+1)}$ is the Lagrange multiplier matrix, μ is the penalty parameter associated with the augmented term, and $\langle \mathbf{A}, \mathbf{B}\rangle_{\mathbf{R}} = Re(\text{Tr}(\mathbf{B}^H\mathbf{A}))$. The update steps of ADMM [19] are summarized in Algorithm 1, where the symbols are explained as follows. The Reverse Diagonal Mean operator (RevDM : $\mathbf{C}^{(N-p)\times(p+1)} \longmapsto \mathbf{C}^N$) is defined as

$$\text{RevDM}(\mathbf{A}) = \begin{bmatrix} \mathbf{A}(1,1) \\ \frac{1}{2}\left[\mathbf{A}(2,1) + \mathbf{A}(1,2)\right] \\ \frac{1}{3}\left[\mathbf{A}(3,1) + \mathbf{A}(2,2) + \mathbf{A}(1,3)\right] \\ \vdots \\ \mathbf{A}(N-p, p+1) \end{bmatrix},$$

for $\mathbf{A} \in \mathbf{C}^{(N-p)\times(p+1)}$ and $\mathbf{A}(i,j)$ is the entry of $\mathbf{A}$ in the i^{th} row and j^{th} column. $\mathcal{S}_\tau(\mathbf{A}) = \text{sign}(\mathbf{A})\max\{|\mathbf{A}| - \tau, 0\}$ is the complex element-wise soft thresholding operator with threshold τ [23], where $\text{sign}(\mathbf{A}) = \mathbf{A}/|\mathbf{A}|$ for the non-zero entry and 0 otherwise. $\max\{\cdot,\cdot\}$ is the element-wise maximum operator. Moreover, $\mathcal{D}_\nu(\mathbf{A}) = \mathbf{U}\,\text{diag}(\max\{\sigma - \nu, 0\})\mathbf{W}^H$ is the singular value soft thresholding operator [19] with threshold ν, where the singular value decomposition of $\mathbf{A} = \mathbf{U}\,\text{diag}(\sigma)\mathbf{W}^H$. f_{CRPE} is the objective function of the CRPE optimization problem defined in (2).

3.2. Coordinate Descent for NRPE

We solve the NRPE optimization problem (3) by coordinate descent with projection. The details of this solver are summarized in Algorithm 2, where $\mathcal{T}_r(\mathbf{A})$ is the singular value truncation operator,

Algorithm 2 Solving NRPE via coordinate descent

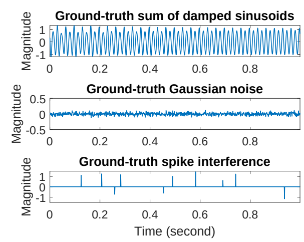
Input: $\boldsymbol{y}$, λ_2, p, r, tol, MaxIter

Initialization: $\mathbf{Y} = \mathbf{H}_p(\boldsymbol{y})$, $\boldsymbol{x}_0 = \hat{\boldsymbol{x}}_0 = \mathbf{0}$, $\mathbf{S}_0 = \mathbf{L}_0 = \mathbf{0}$, $Loss_0 = 0$
for $k = 0, 1 \cdots$, MaxIter **do**
  Update $\hat{\boldsymbol{x}}$:
    $\hat{\boldsymbol{x}}_{k+1} = \text{RevDM}(\mathbf{Y} - \mathbf{S}_k)$
  Project $\mathbf{H}_p(\hat{\boldsymbol{x}}_{k+1})$ onto the low-rank space:
    $\mathbf{L}_{k+1} = \mathcal{T}_r(\mathbf{H}_p(\hat{\boldsymbol{x}}_{k+1}))$
  Update $\boldsymbol{x}$ by projecting $\mathbf{L_{k+1}}$ onto the Hankel space:
    $\boldsymbol{x}_{k+1} = \text{RevDM}(\mathbf{L_{k+1}})$
  Update $\mathbf{S}$:
    $\mathbf{S}_{k+1} = \mathcal{S}_{\lambda_2}\left(\mathbf{Y} - \mathbf{H}_p(\boldsymbol{x}_{k+1})\right)$
  Calculate the Loss:
    $Loss_{k+1} = f_{NRPE}(\boldsymbol{x}_{k+1}, \mathbf{S}_{k+1})$

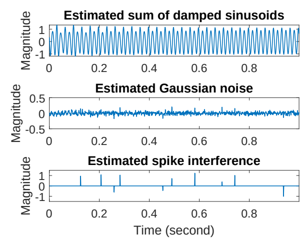
  if $|Loss_{k+1} - Loss_k| / |Loss_{k+1}| \le$ tol **then**
    $Break$

Output: $\mathbf{x}_{k+1}, \mathbf{S}_{k+1}, \mathbf{L}_{k+1}$

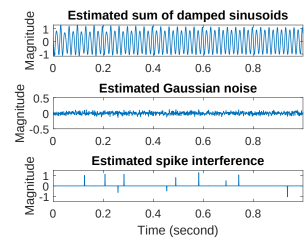
which implements the singular value decomposition on the input matrix $\mathbf{A}$ and returns the matrix constructed using $\mathbf{A}$'s r largest singular values. f_{NRPE} is the objective function of the NRPE optimization problem in (3).



(a) The demixing ground truth.



(b) The demixing result of CRPE.



(c) The demixing result of NRPE.

Fig. 1: The demixing results of CRPE and NRPE methods.

4. NUMERICAL EXPERIMENTS

4.1. Robust Parameter Estimation in Fault Detection

In the first experiment, we consider the bearing fault detection problem of induction machines [6], where the machine current includes a 60Hz operating signal and a 90Hz sideband wave related to its rotational frequency component in the presence of Gaussian noise and spike interference. When a bearing fault or defect occurs, a damped frequency component in the current will be generated with parameters related to the fault location and the bearing size. For example, in our case a 73Hz frequency component is caused by the cage defect of an outer ring. The magnitude of this defect frequency component is typically very small compared to the 60Hz operating current signal, making bearing fault detection a very challenging problem. Still, its parameters, and sometimes the spike interference, are useful to evaluate the fault severity and operating condition of the machine.

To evaluate our approaches in this application, we simulate a noisy fault observation as follows

$$\begin{aligned} y(t) =& e^{0t} 1.0 \cos(2\pi \cdot 60t + 1.3) + e^{-4.2t} 0.1 \cos(2\pi \cdot 73t + 0.2) \\ &+ e^{-1.3t} 0.3 \cos(2\pi \cdot 90t + 1.7) + g(t) + s(t). \end{aligned}$$

We collect 1 second of current signal y with $N = 1000$ samples. The signal to Gaussian noise $g(t)$ ratio is 25 dB and spike interference $s(t)$ has 1% cardinality whose non-zero entries are randomly selected with magnitudes uniformly sampled in $[0.5, 1.5]$, as shown in Fig. 1 (a). By fixing $p = 167 \approx (N/6)$, $\lambda_1 = 4$, and $\mu = 10$ for CRPE, $r = 10$ for NRPE, and fine tuning λ_2, we obtain the demixing results of CRPE and NRPE recorded in Fig. 1 (b) and (c). We observe that both CRPE and NRPE can demix y into the sum of damped exponentials, Gaussian noise, and spikes accurately. The consequent parameter estimation results are shown in Fig. 2. For comparison, we also plot the results of RPCA [18] and TMP [8], where the objective function of RPCA is

$$\min_{\mathbf{X},\mathbf{S}} ||\mathbf{X}||_* + \lambda ||\mathbf{S}||_1, \quad \text{s.t. } \mathbf{X} + \mathbf{S} = \mathbf{Y}, \tag{4}$$

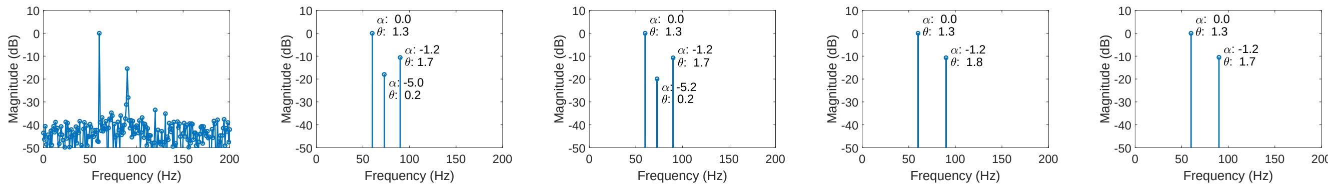
with λ selected based on [18]. From Fig. 2 we note that all the parameters are precisely recovered by CRPE and NRPE except that the damping coefficient α of the fault signature component is a little smaller than the ground truth. Although RPCA and TMP also succeed in recovering the parameters of 60Hz and 90Hz components, both methods fail to identify the fault signature component for detection purpose.

To further investigate the performance of different approaches under different noise conditions, we generate noisy observations using the same exponentials as the first experiment but with only 25dB random Gaussian noise or 1% cardinality spike interference whose magnitudes are uniformly sampled in $[1, 3]$. Results are recorded in Figs. 3 and 4 respectively. We observe that both CRPE and NRPE recover all exponentials and related parameters accurately, no matter with only Gaussian noise or with only spike interference, exhibiting robust performance. In both cases, the relative error of any estimated parameter with respect to the ground truth is less than 7.1%. In contrast, RPCA failed to identify the weak exponential with the existence of Gaussian noise, because RPCA is not capable of preserving the Hankel structure when recovering a low rank matrix. TMP failed in the spike interference case because those grossly corrupted non-Gaussian entries distort the result of SVD, the most critical operation in TMP.

As regarding to computational time, it takes 2.5 seconds and 0.7 second for CRPE and NRPE, respectively, to finish parameter estimation on an i7-6700 CPU, comparable to 2.0 seconds and 0.1 second for RPCA and TMP, respectively.

4.2. The Effect of Sparse Constraint

To examine the effect of hyper-parameter λ_2 which controls the sparse constraint, we randomly generate a mixture of 6 complex damped exponentials of $N = 300$ samples in 0.3 second, with their frequencies, magnitudes, phases, and damping coefficients uniformly random chosen in $[60, 180]$ with 10Hz separation, $[1, 2]$,



(a) Fourier spectrum of noisy observation. (b) Estimation using CRPE. (c) Estimation using NRPE. (d) Estimation using RPCA [18]. (e) Estimation using TMP [8].

Fig. 2: Comparison of frequency spectra using different methods. α denotes the damping coefficient and θ denotes the phase.

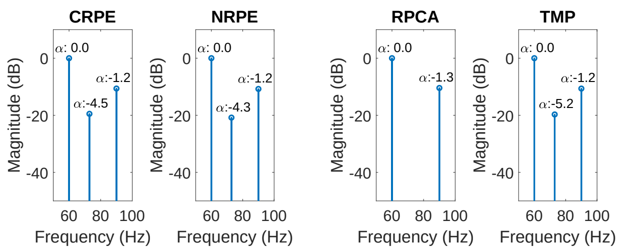


Fig. 3: Estimation results with random Gaussian noise only.

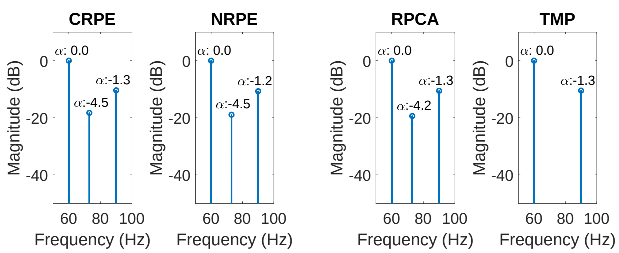
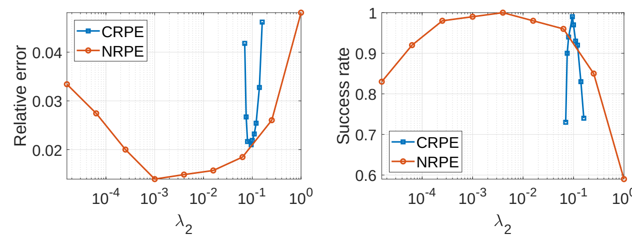


Fig. 4: Estimation results with spike interference only.

$[1, 2]$, and $[-5, -1]$ respectively. The signal to complex Gaussian noise ratio is 30dB and the complex spike interference has 10% cardinality and for each spike the real and imaginary parts are uniformly sampled in $[-1, 1]$. Fixing $p = 50$, $\lambda_1 = 1$, and $\mu = 10$ for CRPE, and $r = 10$ for NRPE, we record in Fig. 5 (a) the average relative error of damped exponentials $\frac{||\tilde{\boldsymbol{x}}-\boldsymbol{x}_0||_2}{||\boldsymbol{x}_0||_2}$ (over 100 trials) versus different values of λ_2, where $\tilde{\boldsymbol{x}}$ is the estimated sum of damped exponentials and $\boldsymbol{x}_0$ is the ground-truth. We also record the recovery success rate of parameters, as depicted in Fig. 5 (b), where a success is counted when the difference between the estimated frequency and the corresponding ground-truth is smaller than 1Hz and at the same time the relative errors of all other associated parameters are no larger than 15%. We observe that to achieve the same success rate, the NRPE method has a much wider range of λ_2 for selection than CRPE does, meaning much less sensitive to the adjustment of λ_2.

4.3. The Effect of Low-rank Constraint

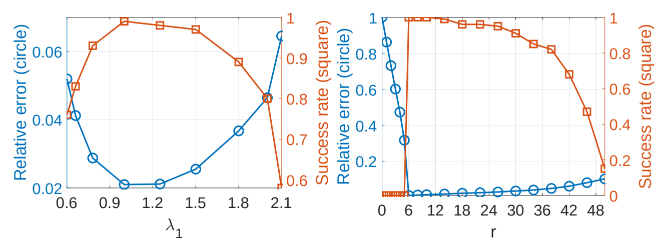
We run the same experiment but with fixed $\lambda_2 = 0.095$ for CRPE and $\lambda_2 = 10^{-3}$ for NRPE based on Fig. 5 and varying λ_1 and r for CRPE and NRPE respectively. The relative error of the damped exponentials and the recovery success rate of the exponentials parameters are recorded in Fig. 6. To achieve above 90% recovery success rate, λ_1 should be set within $[0.8, 1.7]$ for CRPE and r can be set



(a) The relative error of exponentials. (b) The recovery success rate.

Fig. 5: Effect of the sparse constraint for CRPE and NRPE.

in the range of $[6, 30]$ for NRPE. Note that 6 is the true number of exponentials. Similarly, we can observe that NRPE is less sensitive to the adjustment of its hyper-parameter of the low-rank constraint than CRPE in terms of the relative range of parameters.



(a) The CRPE method. (b) The NRPE method.

Fig. 6: Effect of the low-rank constraint for CRPE and NRPE.

5. CONCLUSION

In this paper, we propose two novel approaches, named CRPE and NRPE, to decomposing damped exponentials contaminated by Gaussian noise and spike interference, considering the low-rank property of the Hankel matrix as well as the sparsity of spike interference. Numerical experiments demonstrate that our proposed approaches outperform classical ones in detecting small fault signatures, exhibiting robust performance in different noise situations. While the CRPE method is amenable to theoretical analysis and global convergence guarantees, the NRPE method is less sensitive to hyper-parameters and computationally more efficient.

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