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***Abstract*—In this paper, we propose a power encoding method for converting a high-resolution OFDM baseband signal into a signal that assumes only a finite number of values. This is achieved by the series interconnection of, optimally (co)designed, delta-sigma ($\Delta\Sigma$M) and digital pulse-width modulators (DPWM). Given an input signal class with a known amplitude distribution (e.g., OFDM), parameters of the multilevel DPWM are designed such that the mean squared error of the inherent DPWM quantization noise is minimized. Parameters of the $\Delta\Sigma$M are then chosen with respect to the designed DPWM, so to optimally shape the inherent quantization noise out of the spectral band of interest. The superior performance of the proposed novel power encoding scheme is demonstrated by Matlab simulations on several standardized LTE test signals.**

I. Introduction

Power efficiency of conventional RF power amplifiers (PA) suffers under modern communication standards that require signals with highly varying envelope (e.g., LTE) which, consequently, have large peak-to-average power ratio (PAPR). In order to satisfy the required linearity and spectral measures, the PA has to work at a larger power back-off resulting in lower efficiency of the PA. RF switched-mode power amplifiers (SMPA) have recently shown as a good alternative to traditional PAs, due to their high power efficiency mode of operation [1]-[3]. While traditional PAs are driven by conventional high resolution communication signals, the SMPAs are driven by piece-wise constant signals or pulse trains. Therefore, a reduction in amplitude resolution (i.e., quantization) of conventional signals is needed in order to adjust for the switched-mode operation. This mapping of high resolution signals into pulse trains is commonly called *power encoding*, and is typically done in the form of delta-sigma modulation ($\Delta\Sigma$M) [4], pulse-width modulation (PWM) [5]-[6], or a combination thereof [7].

Even though the electrical efficiency of the SMPA can be high (theoretically, close to 100% [3]), the overall power efficiency of the transmitter, employing the SMPA, mainly depends on the utilized power encoding method. Namely, after an encoded pulse train is amplified by the SMPA, the passband output signal has to be reconstructed by a bandpass filter. However, the pulse trains contain significant amount of out-of-band spectral power (i.e., quantization noise) which is dissipated at the filter, therefore reducing the overall power efficiency of the transmission system. For that reason, coding efficiency (CE) is introduced as a figure of merit to evaluate encoder performance in terms of power efficiency. Coding efficiency of a power encoder is defined as a ratio of the desired in-band power to the total power of the encoder output. It is, therefore, not a surprise that there were many attempts at optimizing coding efficiency of the most popular power encoding schemes. For example, in [8], coding efficiency of a delta-sigma modulator is improved by decreasing the energy of the quantization noise by appropriately choosing the $\Delta\Sigma$M quantization levels. Another important power encoder performance metric is linearity which can be measured in terms of in-band signal-to-noise ratio (SNR) of the encoder output. In general, delta-sigma modulators provide very good linearity but their coding efficiency suffers due to the high amount of out-of-band quantization noise (even in the optimized case like [8]). Pulse-width modulation has, in general, much better coding efficiency properties than $\Delta\Sigma$M, and there has been a significant work at optimizing the CE of various transmitter architectures employing PWM encoding schemes: two and multi-level continuous-time (CT) PWM for quadrature [9] and polar transmitter architectures [10], radio-frequency CT PWM [11], etc. PWM-based encoders can, in general, achieve high coding efficiency but the linearity suffers when PWM is digitally implemented (in which case it is abbreviated as DPWM). In DPWM, high in-band harmonic distortion (commonly called the *aliasing noise*) is present in the output which limits the usage of DPWM in transceivers realized for software defined radio [12]. In [13], coding efficiency of band-limited DPWM schemes was analyzed and a method was proposed in which linearity of the PA is sacrificed for achieving lower in-band harmonic noise. It was shown recently that this in-band harmonic noise in the DPWM output is a consequence of an inherent quantization process, dubbed the *hidden quantization*, that the DPWM input is subject to [14]. It was shown in [15], that this harmonic noise can be effectively mitigated by pre-conditioning the DPWM input with a carefully designed delta-sigma modulator ($\Delta\Sigma$M), whose quantization parameters (decision boundaries and quantization levels) correspond exactly to those of the hidden quantization. In [15], only digital pulse-width modulation schemes with equidistant output levels are considered, in which case the hidden quantization is uniform over the input signal dynamic range. This clearly limits the performance of such $\Delta\Sigma$M-

DPWM power encoders since sub-optimal performance is achieved when encoding the standard communication signals with non-uniform amplitude distributions like, e.g., OFDM signals [16].

In this paper, we propose an optimal $\Delta\Sigma$M-DPWM power encoding system which exploits statistical properties of the input signal to achieve better harmonic noise rejection for the input signal class for which it is optimized. Namely, given a fixed input signal class (with i.i.d. time-samples, e.g, normally distributed), the DPWM output levels are selected so that the expected hidden quantization error is minimized. Then the $\Delta\Sigma$M parameters are chosen so to match those of the optimal hidden quantization. Due to a mismatch between the optimized and actual hidden quantization levels in non-uniform DPWM, a compensation signal is generated and subtracted from the DPWM output, ensuring minimal harmonic noise in the output signal. We show, by Matlab simulations, that, by the above co-design of $\Delta\Sigma$M and DPWM, it is possible to increase power efficiency of the DPWM encoded LTE test signals by, roughly, 15% to 20%, in comparison to that of the unoptimized (i.e., uniform) $\Delta\Sigma$M-DPWM encoding scheme. This is achieved while preserving similar in-band SNR quality.

II. Background and Problem Formulation

A. Digital Pulse-Width Modulation

Pulse-width modulation is a method of mapping an input signal into a digital pulse train, where amplitude information of the input is encoded into a time-varying width of the output pulses. Forthcoming communication standards envision the whole transmitter front-end realized in digital domain [2] and, therefore, PWM has to be realized digitally as well (DPWM). Let M and N be positive integers such that $N > 1$. Let $\mathcal{A} = \{A_0, A_1, \ldots, A_M\}$ and $\mathcal{C} = \{c_1, \ldots, c_M\}$ be the sets of real numbers and N-periodic discrete sawtooth signals, respectively, such that $A_0 < A_1 < \cdots < A_M$, and $A_{m-1} \leq c_m[n] \leq A_m$ for all $n \in \mathbb{Z}$ and $m = \{1, \ldots, M\}$. Signals $c_m \in \mathcal{C}$ are called the DPWM reference signals, and N is called the oversampling ratio of the DPWM. In the rest of this paper, for simplicity, we assume that N is an even number and that reference signals c_m are symmetric double-edge discrete-time sawtooth signals (e.g., black colored signals in Fig. 1). Let $a = a[n]$ be a bounded real-valued scalar discrete-time signal. Digital pulse-width modulation (DPWM) system defined by $\mathcal{A}$, $\mathcal{C}$, and N, maps input signal $a = a[n]$ to output signal $y = y[n]$ as defined by the following formula

$$y[n] = \begin{cases} A_0, & a[n] \leq c_1[n], \\ A_m, & c_m[n] < a[n] \leq c_{m+1}[n], 1 \leq m \leq M-1, \\ A_M, & c_M[n] < a[n]. \end{cases} \tag{1}$$

Therefore, DPWM acts as a comparator, where the output y is generated by comparing input a to signals $c_m \in \mathcal{C}$. An example of DPWM output signal generation is shown in Fig.1, where the system parameters are given as $N = 4$, $M = 2$, $\mathcal{A} = \{0, 3/4, 1\}$. Without loss of generality, in the rest of this paper, we assume that the input signal is bounded in amplitude by A_0 and A_M, i.e., $a[n] \in (A_0, A_M)$ for all $n \in \mathbb{Z}$.

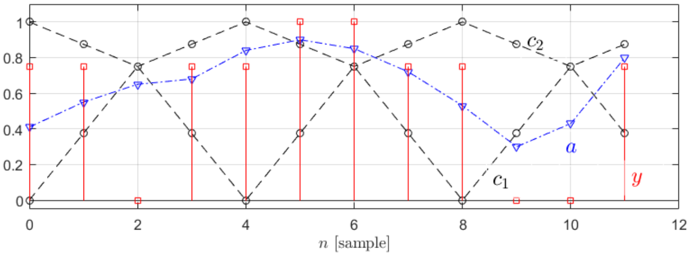

Fig. 1. An example of time-domain waveforms of the DPWM input (blue), reference (black), and output (red) signals.

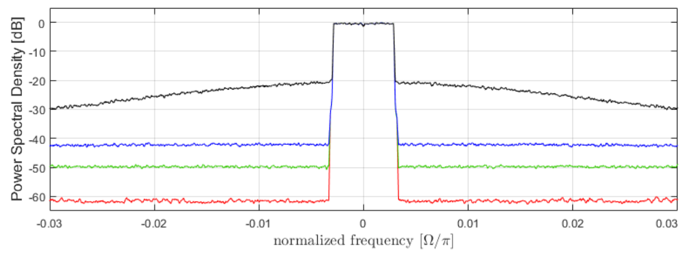

Fig. 2. In-band power spectra of the DPWM input signal (red), and DPWM output signals for three sets of parameters: $L = 6$, with $N = 4, M = 3$ (black); $L = 30$, with $N = 4, M = 15$ (blue); $L = 60$, with $N = 4, M = 30$ (green).

Performance of a DPWM based encoder is measured in terms of the achieved coding efficiency (CE) and signal-to-noise ratio (SNR) of the DPWM output signal. Let $BW \in (0, 2\pi)$ be the bandwidth of the DPWM input signal a, and let y be the response of DPWM to signal a. The CE and SNR of the DPWM output signal are defined as follows:

$$\text{CE}(y) = \frac{P_{BW}(y)}{P_{tot}(y)} \times 100, \quad \text{SNR}(y) = 10 \log \frac{P_{BW}(a)}{P_{BW}(y-a)},$$

where $P_{BW}(x)$ and $P_{tot}(x)$ denote the in-band and total power of signal x, respectively.

Let $\mathbf{Q}_u : (0,1) \to (0,1)$ be a uniform quantizer map such that

$$\mathbf{Q}_u \xi = \frac{2i-1}{N}, \text{ when } \xi \in \left(\frac{2i-2}{N}, \frac{2i}{N}\right], \forall i = \left\{1, \ldots, \frac{N}{2}\right\}.$$

For $m \in \{1, \ldots, M\}$, let $I_m = (A_{m-1}, A_m]$ and let $\Delta_m = 2(A_m - A_{m-1})/N$. For $i = \{1, \ldots, N/2\}$ let $I_m^i = (A_{m-1} + (i-1)\Delta_m,\ A_{m-1} + i\Delta_m]$. For $m \in \{1, \ldots, M\}$, let $\mathbf{Q}_m : I_m \to \mathbb{R}$ be a quantizer map such that

$$\mathbf{Q}_m \xi = A_{m-1} + i\Delta_m, \text{ when } \xi \in I_m^i, \quad \forall i = \left\{1, \ldots, \frac{N}{2}\right\}.$$

Therefore, $\mathbf{Q}_m$ has uniformly distributed decision boundaries over the interval I_m but is not a uniform quantizer since its quantization levels do not lie in the middle of the corresponding decision intervals I_m^i. In fact, the quantization level corresponding to I_m^i lies on the upper boundary of that interval, as can be seen from the definition of $\mathbf{Q}_m$.

Let $\mathbf{Q}_h : (A_0, A_M) \to \mathbb{R}$ be a quantizer map such that $\mathbf{Q}_h \xi = \mathbf{Q}_m \xi$ when $\xi \in I_m$, for all $m \in \{1, \ldots, M\}$. In the rest of this paper, we call $\mathbf{Q}_h$ the *hidden quantizer*.

Now the input-output model of DPWM can be written as ([17])

$$y[n] = \mathbf{Q}_h(a[n]) + \sum_{k=1}^{\frac{N}{2}} \frac{B_k[n] \sin\left(\pi k \mathbf{Q}_u(d[n])\right)}{N \sin\left(\frac{\pi k}{N}\right)} \cos\left(\frac{2\pi}{N} kn\right), \tag{2}$$

where

$$d[n] = \frac{a[n] - A_{m-1}}{A_m - A_{m-1}}, \quad B_k[n] = 2(A_m - A_{m-1}) \cdot (-1)^{k(m-1)}, \tag{3}$$

when $a[n] \in [A_{m-1}, A_m)$, for all $m \in \{1, \ldots, M\}$, all $k \in \{1, \ldots, N/2\}$ and $n \in \mathbb{Z}$. Let signal $a_q = a_q[n]$, be defined by $a_q[n] = \mathbf{Q}_h(a[n])$, for all $n \in \mathbb{Z}$. It can be seen from (2) that the DPWM output signal y is a sum of the baseband component a_q and the amplitude modulated harmonics oscillating at the positive integer multiples of the fundamental frequency equal to $2\pi/N$. Therefore, the baseband of the DPWM output signal mainly depends on the quantized version a_q of the high-resolution baseband input signal a, that is, not on a directly.

B. ΔΣM-PWM Based Power Encoders

The DPWM input-output model (2) suggests that the leading in-band harmonic distortion in the output comes from the 'baseband' quantization $\mathbf{Q}_h$ of the input signal (see the red and black colored signals in Fig. 2). This quantization is dubbed the *hidden* quantization in order to distinguish it from the quantization operation of the DPWM itself [14]. Consequently, we will call the in-band harmonic noise, caused by the action of $\mathbf{Q}_h$, the hidden quantization noise. It follows from the definition of $\mathbf{Q}_h$ that it has $L = MN/2$ quantization levels. It is clear that by increasing either M or N, the number of quantization levels of $\mathbf{Q}_h$ increases and, therefore, the amount of quantization noise in the DPWM output decreases (and, equivalently, the amount of in-band harmonic noise reduces). The effects of increasing L on the in-band output signal power spectrum are depicted in Fig. 2. Unfortunately, the upper limits on M and N are specified by hardware constraints and it is not possible to satisfactorily decrease the hidden quantization error while using feasible values for parameters M and N. It was shown in [15], that the hidden quantization noise can be shaped to out-of-band frequencies by pre-quantizing the DPWM input with a carefully designed delta-sigma modulator (ΔΣM). In order to have no in-band spectral regrowth once such pre-quantized signal is passed through the DPWM, it is necessary that the parameters of the underlying quantizer in ΔΣM match those of the hidden quantizer $\mathbf{Q}_h$. That is, the output signal, denoted $\tilde{a}_q$, of the delta-sigma modulator driven by the input signal a, should assume the same amplitude levels as signal $\mathbf{Q}_h(a)$. When driven by such a signal $\tilde{a}_q$, the DPWM would generate an output signal y such that its baseband component a_q satisfies $a_q[n] = \mathbf{Q}_h(\tilde{a}_q[n]) = \tilde{a}_q[n]$. Therefore, the total quantization noise in the baseband signal a_q will correspond to that of the ΔΣM output $\tilde{a}_q$, which is shaped to out-of-band

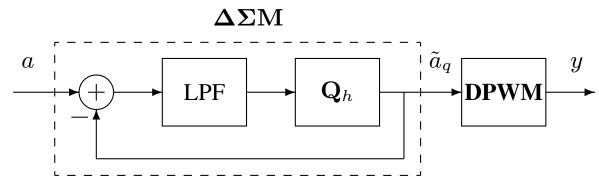

Fig. 3. Block diagram of the ΔΣM-DPWM power encoder proposed in [15].

frequencies, ensuring low in-band noise in the DPWM output y. A block diagram of such a ΔΣM-DPWM power encoder is shown in Fig. 3.

It is not hard to see, from the input-output model (2), that in the case of equidistant DPWM output levels, i.e., when $A_m = (A_M - A_0)m/M$ for all m, the hidden quantizer $\mathbf{Q}_h$ becomes a uniform quantizer. It should be noted that only this type of DPWM was investigated in [14]-[15]. Clearly, DPWM power encoding schemes with uniform hidden quantization (i.e., with equidistant output levels) have simplistic appeal from a hardware perspective. On the other hand, modern baseband communication signals (e.g., LTE) have amplitude distributions which are highly non-uniform. For example, the I/Q components of an OFDM signal have approximately Gaussian distribution [16]. It follows that power encoders with uniform quantization do not fully exploit the available information about the underlying communication signals and might lead to sub-optimal performance of the transmitters employing them. Namely, the number L of the hidden quantization levels of $\mathbf{Q}_h$ is commonly relatively small leading to uniform hidden quantizers of low resolution which cannot achieve satisfying out-of-band noise rejection for signals with highly non-uniform amplitude distribution. In the next section we show how ΔΣM-DPWM power encoder can be optimally designed to incorporate information about the input signal statistics in order to not just reduce the hidden quantization (i.e., in-band) noise but also minimize the out-of-band harmonic noise and, therefore, improve coding efficiency.

III. Optimal Co-Design of ΔΣM and DPWM

Let a be a real-valued scalar discrete-time signal whose amplitude samples are i.i.d. according to the probability density function $p_a = p_a(x)$, where $p_a : \mathbb{R} \to [0, \infty)$. The mean squared error of quantizing a by an arbitrary quantizer $\mathbf{Q} : \mathbb{R} \to \mathbb{R}$ is given by

$$J_p(\mathbf{Q}) = \int_{-\infty}^{\infty} \left(x - \mathbf{Q}(x)\right)^2 p_a(x) dx. \tag{4}$$

In practice, the utilized baseband communication signals have finite dynamic range due to hardware or some other constraints (e.g., amplitude clipping due to PAPR reduction). For that reason, in the rest of this paper, we assume that $p_a(x) = 0$ for all $x \notin [a_0, a_M]$, for some fixed $a_0, a_M \in \mathbb{R}$.

For all $m \in \{1, \ldots, M\}$, let b_m^n and q_m^n be the decision boundaries and quantization levels, respectively, of the m-th sub-quantizer $\mathbf{Q}_m$. Dependence of b_m^n and q_m^n on the elements of $\mathcal{A} = \{A_0, \ldots, A_M\}$ is given by the following expressions

$$b_m^n = A_{m-1} + \frac{2n}{N}(A_m - A_{m-1}), \quad 0 \le n \le N/2, \tag{5}$$

$$q_m^n = b_m^n, \quad 1 \le n \le N/2. \tag{6}$$

For $L = MN/2$, let $\mathcal{B} = \{b_0, \ldots, b_L\}$ and $\mathcal{Q} = \{q_1, \ldots, q_L\}$ be the decision boundaries and quantization levels of $\mathbf{Q}_h$, respectively. By the definition of $\mathbf{Q}_h$ we have

$$b_k = b_m^n, \text{ when } k = (m-1)\frac{N}{2} + n, \tag{7}$$

for all $n \in \{0, \ldots, N/2\}$ and $m \in \{1, \ldots, M\}$, and

$$q_k = b_k, \text{ for all } k \in \{1, \ldots, L\}. \tag{8}$$

The mean-squared error cost (4) for $\mathbf{Q} = \mathbf{Q}_h$ is, therefore, a function of $\mathcal{A}$ and can be written as

$$J_p(\mathbf{Q}_h) = \sum_{m=1}^{M} \sum_{n=1}^{N/2} \int_{b_m^{n-1}}^{b_m^n} (x - q_m^n)^2 p_a(x) dx. \tag{9}$$

Assume now that DPWM is driven by the above described signal a with amplitude distribution p_a. It follows that the out-of-band harmonic noise in the DPWM output can be decreased by choosing the output levels A_m such that the hidden quantizer $\mathbf{Q}_h$ minimizes the above defined mean squared quantization error (MSQE). Unfortunately, the structure of $\mathbf{Q}_h$ is significantly restricted: decision boundaries b_k are piecewise uniformly distributed and quantization levels q_k are at the boundary of each individual decision interval $(b_{k-1}, b_k]$. This implies that, in general, one can expect very poor MSQE performance of such a quantizer (even in the case of optimal MSQE!). This problem can be mitigated in the following way.

Let $\tilde{\mathbf{Q}} : [A_0, A_M] \to \mathbb{R}$ be a quantizer whose decision boundaries $\tilde{b}_k$, for $k \in \{0, \ldots, L\}$, and quantization levels $\tilde{q}_k$, for $k \in \{1, \ldots, L\}$, satisfy the following

$$\tilde{b}_k = b_k \qquad \tilde{q}_k \in (\tilde{b}_{k-1}, \tilde{b}_k]. \tag{10}$$

Clearly, quantizers $\tilde{\mathbf{Q}}$ and $\mathbf{Q}_h$ have identical decision boundaries (for a fixed choice of $\mathcal{A}$), while the quantization levels of $\tilde{\mathbf{Q}}$ are unrestricted unlike those of $\mathbf{Q}_h$. Now we want to find $\tilde{b}_k$ and $\tilde{q}_k$ such that the quantizer $\tilde{\mathbf{Q}}$ minimizes mean squared error (4). This problem can be formulated as follows

$$\begin{aligned}
&\min_{A_1,\ldots,A_M,\tilde{q}_1,\ldots,\tilde{q}_L} \quad J_p(\tilde{\mathbf{Q}}) \\
&\text{s.t.} \quad a_0 = A_0 < A_1 < \cdots < A_M = a_M, \\
&\qquad b_{(m-1)N/2+n} = A_{m-1} + \frac{2n}{N}(A_m - A_{m-1}), \\
&\qquad \forall m \in \{1, \ldots, M\}, \quad \forall n \in \{0, \ldots, N/2\}, \\
&\qquad \tilde{q}_k \in (b_{k-1}, b_k], \quad \forall k \in \{1, \ldots, L\}.
\end{aligned} \tag{11}$$

Let $\tilde{\mathbf{Q}}^*$ be the argument of minimum of (11) (more precisely, let $\tilde{\mathbf{Q}}^*$ be a quantizer whose parameters are the arguments of minimum of (11)).

Remark 1: It should be noted that, in general, the optimal solution $\tilde{\mathbf{Q}}^*$ of the above optimization problem is not a Lloyd-Max quantizer [18], since the decision boundaries of $\tilde{\mathbf{Q}}^*$ are fixed to be uniformly distributed on sub-intervals $(A_m, A_{m+1}]$.

Remark 2: It is easy to see that, in general, $J_p = J_p(\tilde{\mathbf{Q}})$ is a non-convex function of $A_0, A_1, \ldots, A_M, \tilde{q}_1, \ldots, \tilde{q}_L$, and the optimal problem (11) cannot be solved explicitly. Furthermore, there is no guarantee that a global optimum would be achieved by applying any of the standard non-convex optimization algorithms. In practice, one calculates off-line the parameters of $\tilde{\mathbf{Q}}^*$ and then programs digital hardware that the power encoder should be implemented on. For that reason, we find an approximate optimal solution of (11) by performing a grid search. The number M of output levels of DPWM is commonly low (for high power amplifiers to be used in base stations it is typically not larger than 5 [19]), and performing a grid search to find an approximate optimal solution of (11) commonly imposes just a mild computational burden.

The optimal quantization parameters, as defined above, cannot, in general, match the quantization parameters of $\mathbf{Q}_h$ for any DPWM. That is, quantizer $\tilde{\mathbf{Q}}^*$ is, in general, not equivalent to $\mathbf{Q}_h$. Hence, if the optimal quantizer $\tilde{\mathbf{Q}}^*$ was used in the $\Delta\Sigma$M pre-quantizer system of a $\Delta\Sigma$M-DPWM encoder, then amplitude values of the $\Delta\Sigma$M output would not match those of the hidden quantization. This implies that the DPWM system would then generate significant in-band harmonic noise in the output and any benefit of the $\Delta\Sigma$M pre-quantization would be lost. This problem can be mitigated by introducing a *compensation signal* that should be added to the DPWM output to compensate for the difference between the quantization levels of $\tilde{\mathbf{Q}}^*$ and $\mathbf{Q}_h$. A block diagram of such an optimal power encoder is shown in Figure 4. The $\Delta\Sigma$M subsystem utilizes the optimal quantizer $\tilde{\mathbf{Q}}^*$ and maps baseband input signal a to a quantized signal a_q. Since $\tilde{\mathbf{Q}}^* \neq \mathbf{Q}_h$, the output $\tilde{y}$ of the DPWM subsystem will be equal to $\tilde{y} = a_q + e + harmonics$, where e is the error signal $e = \mathbf{Q}_h(a_q) - a_q$. Compensator $\mathbf{C}$ takes input signal a and generates the error signal e, as defined above. More precisely, the compensator system $\mathbf{C}$, mapping signal a into signal e is defined as follows:

$$e[n] = q_k - \tilde{q}_k^*, \text{when } a[n] \in (\tilde{b}_{k-1}^*, \tilde{b}_k^*] \equiv (b_{k-1}, b_k], \tag{12}$$

for all $k \in \{1, \ldots, L\}$ and all $n \in \mathbb{Z}$. The compensator $\mathbf{C}$ is such that

$$e[n] = (\mathbf{C}a)[n] = \mathbf{Q}_h(a_q[n]) - a_q[n].$$

In other words, signal e should cancel the hidden quantization error that is caused by the difference in the quantization level values of $\tilde{\mathbf{Q}}^*$ and $\mathbf{Q}_h$.

Remark: It should be noted that the compensation signal takes at most L amplitude values and has much smaller power than the main signal. Therefore, its power added cost to the overall system design is minimal since a low-power linear PA can be used to amplify this signal.

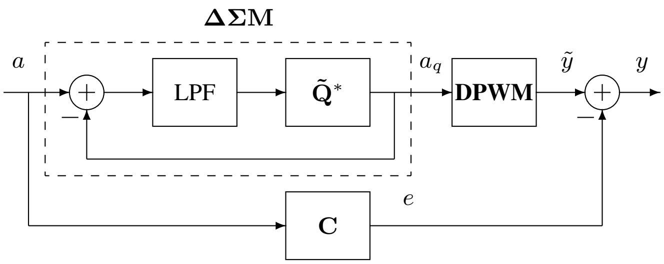

Fig. 4. Block diagram of the proposed optimal power encoder.

IV. Simulation Results

Performance of the proposed optimal ΔΣM-DPWM power encoding scheme was evaluated by Matlab simulations, where CE and SNR of the encoder output were used as a measure of performance. The proposed encoder was compared to the non-optimized ΔΣM-DPWM power encoder (i.e, the one with uniform quantization in ΔΣM [15]) and the regular DPWM encoder (i.e., without ΔΣM pre-conditioning of the input).

Simulations were performed for two types of input signals, both E-UTRA test models as specified in [20]. The simulation parameters, as well as the test signals' parameters, for each case, are presented in Table I. As can be seen, the DPWM parameters N and M take relatively small values and, as a consequence, the number L of the hidden quantization levels is also relatively small (see Table I). In both cases, test signals in FDD mode were used, and the order of the ΔΣM was set to 1, so to simplify the power encoder design.

The signal flow of the simulation is as follows: the I and Q components of the input LTE signal, generated through Matlab's LTE System Toolbox [21], are fed into the above described power encoders, and their outputs combined to get the complex baseband output signal. For simplicity, the input signals are normalized so that their amplitude values fully span the DPWM dynamic range which was set to $(-1, 1)$ (i.e., $A_0 = -1$ and $A_M = 1$). For the optimal ΔΣM-DPWM encoder, the input signal amplitude pdf parameters are first estimated and then used to calculate the optimal decision boundaries and quantization levels (both of these tasks are done off-line).

The performance results of the compared encoding methods are reported in Tables II and III. The wideband and in-band output spectra, for each power encoding method, are depicted in Figs. 5, 7 and 6, 8, respectively. As can be seen, the in-band harmonic noise of both ΔΣM-DPWM encoders is significantly lower than that of the regular DPWM encoder, which was to be expected from [15]. It should be noted that the optimal encoder is slightly better than the uniform one in terms of SNR (by 1-3 dB). On the other hand, in terms of coding efficiency, the proposed optimal power encoder significantly outperforms (by 15%-20%) the other two methods. This can also be inferred from the wideband output spectra plots in Figs. 5 and 7.

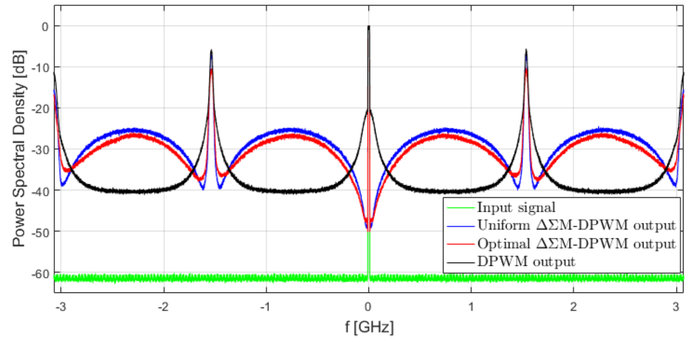

Fig. 5. Wideband output spectra for E-TM3.1 test signal.

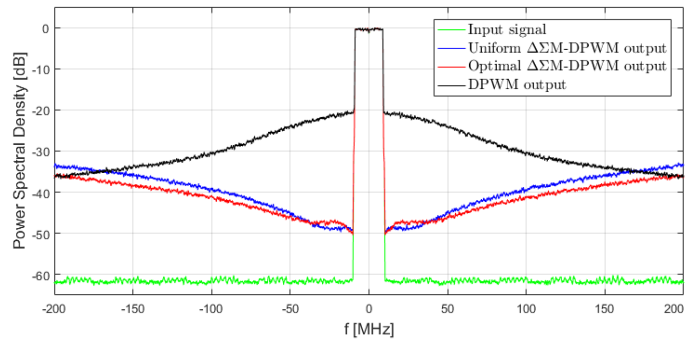

Fig. 6. Baseband (zoom-in) output spectra for E-TM3.1 test signal.

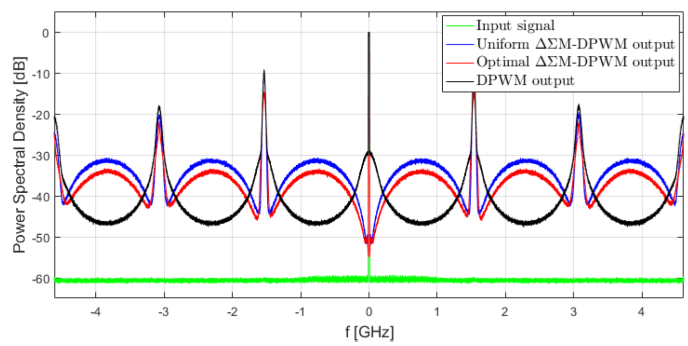

Fig. 7. Wideband output spectra for E-TM3.1a test signal.

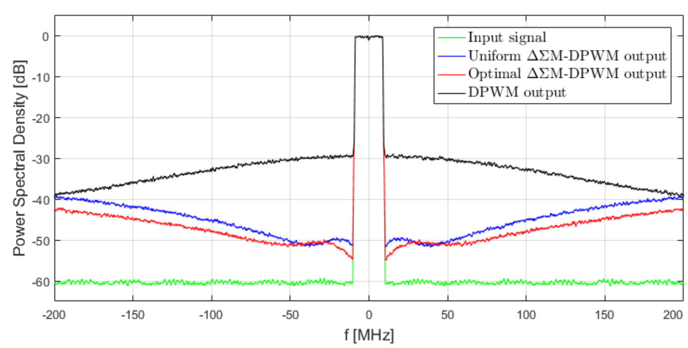

Fig. 8. Baseband (zoom-in) output spectra for E-TM3.1a test signal.

TABLE I
SIMULATION PARAMETERS AND TEST SIGNALS

	Test Signal	Bandwidth	Modulation	PAPR	f_{samp}	N	M	L
Case 1	E-TM 3.1	20 MHz (100 RB)	64QAM	11.1 dB	6.14 GS/s	4	3	6
Case 2	E-TM 3.1a	20 MHz (100 RB)	256QAM	11 dB	9.21 GS/s	6	4	12

TABLE II
PERFORMANCE COMPARISON FOR E-TM 3.1 TEST SIGNAL

	DPWM	$\Delta\Sigma$M-DPWM [15]	New Model
CE	30.33%	29.94%	45.26%
SNDR	19.62 dB	43.2 dB	46.32 dB

TABLE III
PERFORMANCE COMPARISON FOR E-TM 3.1a TEST SIGNAL

	DPWM	$\Delta\Sigma$M-DPWM [15]	New Model
CE	46.47%	46.1%	66.13%
SNDR	28.63 dB	42.73 dB	43.15 dB

V. CONCLUSION

In this paper, we propose a power encoding method for converting a high-resolution baseband communication signal into a piece-wise constant signal, of low resolution, which is suitable for efficiently driving a switched-mode PA. We consider power encoders in the form of a series interconnection of delta-sigma and pulse-width modulators. In the proposed method, information about the input signal amplitude distribution is exploited in order to jointly optimize $\Delta\Sigma$M and DPWM subsystems and maximize the output coding efficiency of the overall encoder system. Output levels of the digital pulse-width modulator are chosen such that the mean squared error of the inherent DPWM quantization noise is minimized. Parameters of the $\Delta\Sigma$M are then chosen with respect to the designed DPWM, so to optimally shape the inherent quantization noise out of the spectral band of interest. We illustrate the performance gains of the proposed power encoding method by Matlab simulations with several standardized LTE test signals.

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