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SHORT LATTICE-BASED SHAPING APPROACH EXPLOITING NON-BINARY CODED MODULATION

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Abstract

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1 Introduction

In order to meet the demand for high-speed fiber-optic communications, high-order modulations combined with capacity-approaching forward error correction (FEC) are essential. Bit-interleaved coded modulation (BICM) is the simplest approach for coded modulation, but has a performance loss in terms of achievable information rates. By using non-binary FEC codes instead of binary counterparts, this fundamental penalty of BICM can be avoided. In fact, it has been experimentally demonstrated in [1] that the non-binary coded system based on turbo trellis-coded modulation (TTCM) [2] outperforms BICM by 0.4 dB for a 1000 km transmission at 100 Gbit/s.

Although TTCM can approach the constellation-constrained capacity of standard quadrature-amplitude modulation (QAM) signaling, still there is a gap from the Shannon limit due to the uniform distribution of signal constellations. To compensate for this gap, constellation shaping has been actively studied in the community [3–11]. Although their shaping gains typically increase as the block length increases, implementation of shaping operations, i.e., distribution matching, for long block lengths will be computationally cumbersome. Therefore, shaping methods that achieve high gain at short block lengths with low computational operations are desirable in practice.

One of such low-dimensional shaping techniques based on lattice codes has been proposed for wireless communication systems [12]. Since short dimensional lattices enable efficient quantization, i.e., closest point search algorithms [13], low-complexity shaping can be realized. Furthermore, since lattices are known to achieve the densest sphere packing for some short dimensions [14], short lattice shaping would achieve good trade-off between shaping gain and computational complexity.

In this paper, we propose the E_8 lattice-based shaping approach for high data-rate fiber-optic communication systems employing capacity-approaching TTCM as FEC. We use lattice decoding, i.e., quantization to generate Gaussian-distributed integers from uniform data, which we refer to as "Voronoi integers". Systematic non-binary TTCM is employed to achieve high power efficiency, while keeping their distribution Gaussian-like. It is demonstrated that the proposed coded modulation scheme significantly outperforms conventional TTCM-based system by approximately 0.5 dB and performs very closely to the Gallager's achievable performance bound at finite block lengths.

2 Encoding and Indexing of Lattice Codes

We first describe how "Encoding" and "Indexing" of lattice codes are performed. "Encoding" of lattices is to map information integers $\mathbf{u} = (u_1, u_2, \ldots, u_n)$ to lattice points $\mathbf{c} = (c_1, c_2, \ldots, c_n)$. "Indexing" means the reverse mapping, which finds the corresponding $\mathbf{u}$ for a given $\mathbf{c}$. The mapping should be bijective to recover information from a given lattice point.

A quotient group $\mathbb{Z}^n/\Lambda$ is n-dimensional integer vectors in the lattice Λ. The coset leader of $\mathbb{Z}^n/\Lambda$ is given by $\mathbb{Z}^n \cap \mathcal{F}$, where $\mathcal{F}$ is any fundamental region of lattice Λ. Specifically, the coset leader with the zero-centered fundamental Voronoi region $\mathcal{V}$, i.e., $\mathbb{Z}^n \cap \mathcal{V}$, is of practical interest to satisfy the power constraint. Finding the lattice $\mathbb{Z}^n \cap \mathcal{V}$ is performed by quantization of lattice Λ. We denote the shortest distance quantization of $\mathbf{x} \in \mathbb{R}^n$ by $Q_\Lambda(\mathbf{x}) = \min_{\lambda \in \Lambda}(\mathbf{x} - \lambda)$. We assume that the lattice used for shaping is the scaled version of well-known lattice Λ', i.e., $\Lambda = K\Lambda'$, where $K \in \mathbb{Z}$ is a scaling factor. The quantization of the scaled lattice is given by $Q_\Lambda(\mathbf{x}) = Q_{\Lambda'}(\mathbf{x}/K) \cdot K$.

Let $\mathbf{G}$ denote the $n \times n$ generator matrix for Λ. In this work, we assume that $\mathbf{G}$ is lower triangular, where $g_{ij} = 0$ for $j > i$ and the diagonal elements are positive integers. Also for each column j, g_{ij}/g_{jj} is an integer for all elements i in that column. We note that these conditions are satisfied by well-known lattices such as D_n and E_8. Conway and Sloane [13] described hardware-efficient encoding and indexing algorithms for such lattices. The generalization to other lattices are referred to [16].

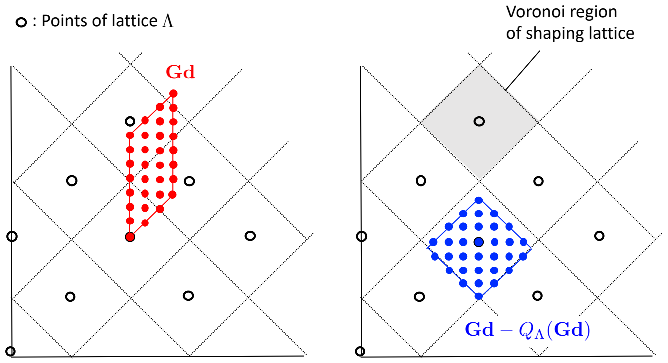

Fig. 1 Example of lattice shaping using a lattice $\Lambda = 4D_2$. Lattice points before shaping (left) and after shaping (right).

Letting $u_i \in \{0, 1, \ldots, g_{ii} - 1\}$ denote the information vector, we define a new vector as

$$\mathbf{d} = (u_1/g_{11}, u_2/g_{22}, \ldots, u_n/g_{nn}), \tag{1}$$

where all element satisfies $0 \leq u_i/g_{ii} < 1$. The cardinality of information vector u_i and the scaling factor g_{ii} is chosen such that the condition of bijective mapping between $\mathbf{u}$ and $\mathbf{c}$ is satisfied [16]. The Voronoi integer is then obtained by lattice quantization as $\mathbf{c} = \mathbf{Gd} - Q_\Lambda(\mathbf{Gd})$. The cardinality of the codebook is $|\det \mathbf{G}| = \prod_{i=1}^{n} g_{ii}$, since $\mathbf{G}$ is triangular. The resulting spectral efficiency is $R = |\det \mathbf{G}|/n$ bits per dimension.

An example of lattice encoding with a $\Lambda = 4D_2$ lattice is shown in Fig. 1. The generator matrix of $4D_2$ lattices is given by

$$\mathrm{G} = \begin{bmatrix} 4 & 0 \\ 4 & 8 \end{bmatrix}. \tag{2}$$

The left figure in Fig. 1 shows the lattice points $\mathbf{Gd}$ for information vector $u_1 = \{0, 1, 2, 3\}$ and $u_2 = \{0, 1, 2, 3, 4, 5, 6, 7\}$. Subsequently for each point of $\mathbf{Gd}$, a closest $4D_2$ lattice point is subtracted in order to generate Voronoi integers as in the right figure. We employ the computationally efficient closest point search algorithms proposed in [13] for low-dimensional lattices. Note that we use the D_2 lattice as an example for simplicity of explanation, while it has no shaping gain. Fig. 2 shows the achievable shaping gain by some well-known lattices. One can see that the lattice shaping achieves excellent gain close to sphere packing bounds. For 24-dimensional Leech lattice, shaping gain greater than 1.0 dB is achievable. Although the increase of dimensionality can improve the shaping gain towards the asymptotic limit of 1.53 dB, the computational complexity of lattice encoding and indexing may increase. Therefore, the present paper focuses relatively short lattice based on E_8 which has a maximum gain of 0.654 dB.

Consider indexing of lattice points $\mathbf{c}$ into information $\mathbf{u}$. We first rewrite the given lattice point as $\mathbf{c} = \mathbf{Gb} + \mathbf{Gd}$, where $\mathbf{b} = \{b_1, b_2, \ldots, b_n\}$ and $\mathbf{d} = \{d_1, d_2, \ldots, d_n\}$ are integer and fractional vectors, respectively. More specifically, $\mathbf{Gb}$ corresponds to $Q_\Lambda(\mathbf{Gd})$, and our aim is to find vector $\mathbf{d}$ from $\mathbf{c}$, which is the scaled information. Taking advantage of the lower triangular structure of $\mathbf{G}$, the first low element is expressed as $c_1 = g_{11}(b_1 + d_1)$, which has a unique solution. For $i = \{2, 3, \ldots, n\}$, re-encoding is recursively performed as $c_i = g_{ii}(b_i + d_i) + \sum_{j=1}^{i-1} g_{ij}(b_j + d_j)$, where b_i and d_i are found uniquely at each step. Finally, information u_i is retrieved from d_i as $u_i = g_{ii} d_i$ for $i = \{1, 2, \ldots, n\}$.

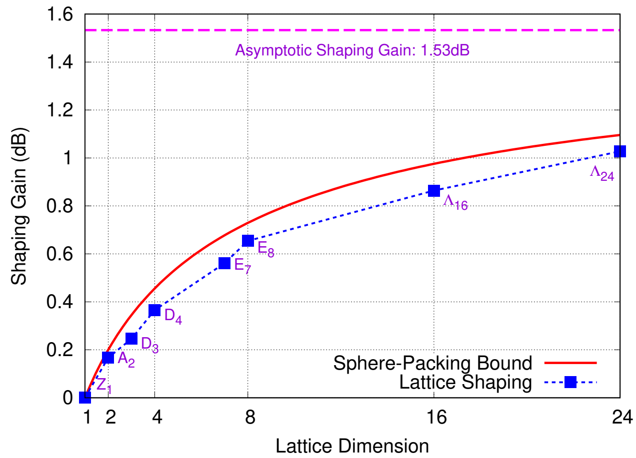

Fig. 2. Sphere-packing bound and lattice shaping gain.

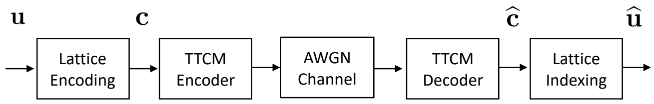

Fig. 3. The proposed system model.

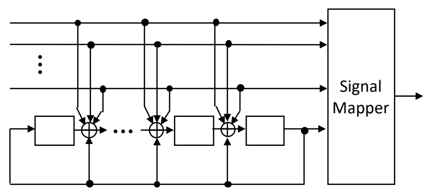

Fig. 4. Constituent TCM encoder.

3 Lattice-shaped Non-binary TTCM System

The proposed system model is shown in Fig. 3, where lattice encoding and indexing are concatenated with the conventional TTCM-based system. Each element of $\mathbf{c}$ is converted into a binary sequence by natural labeling and fed into the subsequent TTCM encoder. Punctured TTCM [2] is employed in order to keep the spectral efficiency of the component TCM encoder constant, where outputs of upper and bottom TCM encoders are punctured alternately after deinterleaving. We note that since the cardinality of each element of $\mathbf{c}$ is not necessarily power-of-two, the input to TTCM encoder may not be uniformly distributed. Each constituent TCM encoder generates just one parity bit, which is mapped onto the least significant bit (LSB) with natural labeling to keep the distribution of the input

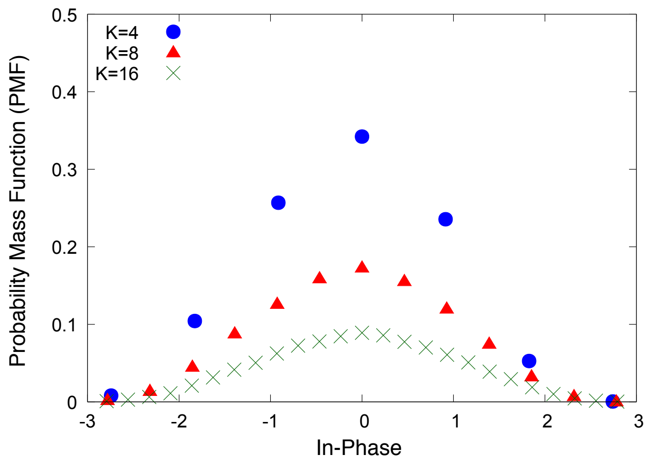

Fig. 5 Probability mass function (PMF) of E_8-lattice shaped constellations with $K = 4, 8, 16$ (2, 3, 4 bits/dim, respectively).

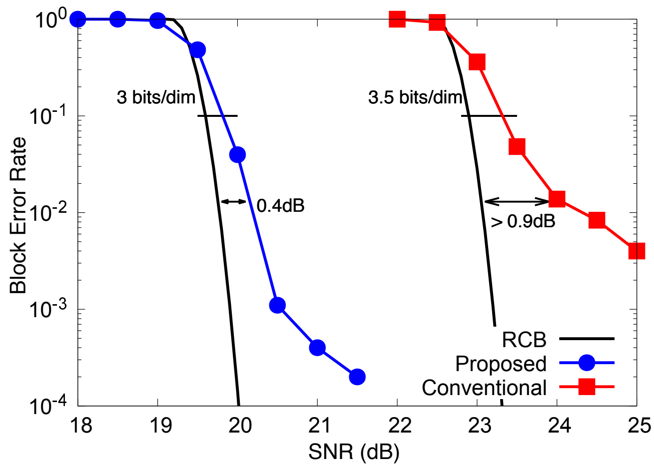

Fig. 7 BLER performance of the proposed lattice shaping and conventional TTCM [2] with block length of 1024 real symbols.

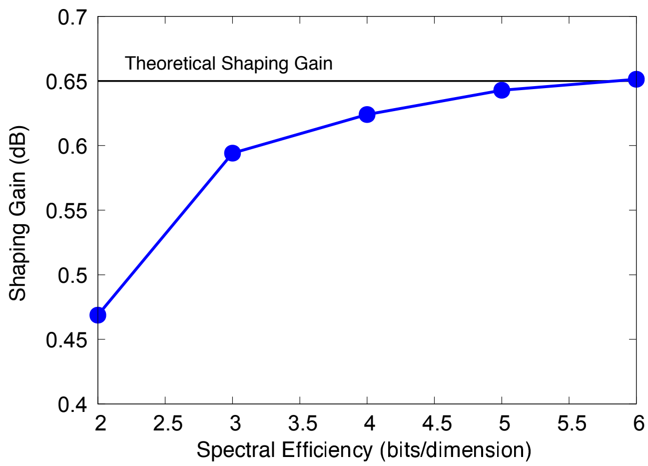

Fig. 6 Achievable shaping gain for E_8 lattice with respect to the spectral efficiency.

to TCM encoder as shown in Fig. 4. At the receiver side, lattice indexing estimates information vector $\hat{\mathbf{u}}$ for a given TTCM decoder output $\hat{\mathbf{c}}$ as described in the previous section.

4 Simulation Results

Fig. 5 shows the probability mass function (PMF) of the shaped constellations by the proposed E_8 lattice with $K = 4, 8, 16$, where a signal power is normalized to 1. We can confirm that the signal distribution of Voronoi integers becomes Gaussian-like. Fig. 6 shows the achievable shaping gain of the E_8 lattice Voronoi integers, according to the normalized second moment of a lattice [15]. The shaping gain depends on the lattice scaling factor K, i.e., the spectral efficiency. It is observed that the gain achieved by E_8 lattice shaping increases as a spectral efficiency increases and reaches the theoretical maximum value for E_8 lattices, which is 0.65 dB [14] at around 6 bits per dimension.

We then evaluate block error rate (BLER) performance of the proposed system employing E_8 lattice shaping with $K = 8$, corresponding to the spectral efficiency of 3 bits per dimension. The BLER performances of the proposed system and conventional Robertson's TTCM employing 256-QAM (equivalently, 16-PAM per dimension) are shown in Fig. 7. The code length is set to 1024 real symbols. The generator polynomial of the constituent TCM encoders of the proposed system is optimized by exhaustive computer search. For both schemes, we use a randomly chosen interleaver, and logarithmic maximum *a posteriori* (Log-MAP) decoding with an iteration count of 10 is employed, where the decoding trellis has 8 states.

As a benchmark of the BLER performance with finite code block lengths, Gallager's random coding bounds (RCB) [17] are also plotted. From Fig. 7, it is demonstrated that the proposed system approaches the finite-length RCB within 0.4 dB at a BLER of 10^{-2}. Considering the fact that conventional TTCM without shaping has more than 0.9 dB gap from the bound, the proposed shaping method based on E_8 lattice has a significant benefit by approximately 0.5 dB gain, which is almost as predicted from Fig. 6.

5 Conclusions

We have proposed a new shaping approach based on short-dimensional lattice shaping combined with non-binary TTCM scheme for high-spectral efficiency fiber-optic communications. It has been demonstrated that the E_8-based shaping approach offers 0.65 dB shaping gain as a spectral efficiency increases. We have also demonstrated that the proposed TTCM scheme combined with E_8 shaping outperforms conventional TTCM without shaping by about 0.5 dB in terms of the gap from the finite block length bound.

As a final remark, our proposed system has a room for performance improvement by interleaver optimization. Furthermore, the application of higher dimensional lattices such as 24-dimensional Leech lattice would be the subject of further investigation.

6 References

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