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### Abstract

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# Last-Mile Scheduling Under Uncertainty

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**Abstract.** Shared mobility is revolutionizing urban transportation and has sparked interest in optimizing the joint schedule of passengers using public transit and last-mile services. Scheduling systems must anticipate future requests and provision flexibility in order to be adopted in practice. In this work, we consider a two-stage stochastic programming formulation for scheduling a set of known passengers and uncertain passengers that are realized from a finite set of scenarios. We present an optimization approach based on decision diagrams. We obtain, in minutes, schedules for 1,000 known passengers that are robust and optimized with respect to scenarios involving up to 100 additional uncertain passengers.

**Keywords:** Decision diagrams · Scheduling · Stochastic programming

## 1 Introduction

The transportation industry is transforming due to recently introduced mechanisms for shared mobility [9, 16]. Transportation systems are a key element of integrative smart city operations, leading to a host of complex optimization problems [12, 19–21]. Of critical importance is the joint scheduling of mass transportation systems with last-mile vehicles, which when scheduled in unison can lead to significant operational improvements [15, 17, 18, 22].

This paper studies the joint scheduling of passengers on mass transit systems and last-mile vehicles under uncertainty. Passengers arrive by train to a central terminal and board limited-capacity pods called *commuter vehicles* (CVs) which are automated or otherwise operated, where some passengers are known and other passengers are uncertain (and thus may or may not request service). The goal is to assign passengers to trains and then to group passengers traveling together on a CV so as to minimize a combination of total travel time and number of CV trips.<sup>5</sup> This objective models a tradeoff between quality of service

<sup>5</sup> We assume a single destination per CV trip [17, 18] and only a few destinations [15], which is operationally favorable since destination batching leads to efficiency.

(passenger travel time) and operational costs related to fuel consumption and maintenance requirements (number of trips). In the absence of uncertainty, this problem is known as the *integrated last-mile transportation problem* (ILMTP).

The uncertain setting is applicable to systems where a central scheduler takes requests from passengers and assigns them to trains and CV trips, while building in flexibility for passengers that might request transportation services but are yet to submit an associated request. This leads to a significant increase in problem complexity with respect to previous work [15, 17, 18], but also makes for a more realistic setting where the initial scheduling of passengers must account for additional demand from late requests that also needs to be accommodated.

Optimization under uncertainty, or *stochastic optimization*, defines a broad class of challenging problems [8]. A relatively recent and popular technique for handling uncertainty is robust optimization [2], where an uncertainty set is defined and worst-case operational decisions are employed. It is well known that this can lead to highly conservative solutions, since unlikely outcomes can drive decisions. A more classical approach is simulation-based optimization, where algorithms such as sample average approximation methods [11] are employed, which consider a finite set of possible realizations described as a sample of scenarios which are optimized over in order to maximize the expected value over the sampling.

This paper presents a two-stage stochastic programming formulation for the ILMTP under additional passenger uncertainty (ILMTP-APU). In addition to a set of *known* passengers, we model *uncertain passengers* through a finite set of scenarios. The first-stage decision is the scheduling of known passengers and the second-stage decision schedules the additional passengers in each scenario. Our approach relies on decision-diagram (DD)-based optimization (DDO) [1, 3, 6], and more specifically on decompositions based on DDs [4, 5, 13, 7] and is inspired by the model presented in [17]. Specifically, a DD is built for known passengers going to each building and separately for unknown passengers in each scenario and for each building. The DDs are then integrated through channeling constraints that can be optimized with an integer programming (IP) formulation. This results in a large model. However, due to the tightness of the formulation, we obtain a reliable approach for optimally solving the problem. The resulting solutions are significantly better than what could be obtained by solving the problem for the known passengers to optimality and then scheduling the unknown passengers with the remaining capacity when a scenario is realized.

This paper adds to the recent literature on DDO for stochastic optimization, where BDDs have been used for determining decision-dependent scenario probabilities [10] and, more closely related to the current study, a study of a class of two-stage stochastic IP problems [14]. Our proposed approach differs from that of [14] in that we model both the first-stage and second-stage decisions using DDs and link them through assignment constraints. This provides an additional mechanism by which decision making under uncertainty can be tackled through DDs. The main contributions of this paper are therefore (1) an extension of the ILMTP to incorporate uncertain passenger arrivals; (2) structural results on

families of optimal solutions; and (3) the development of a novel DDO stochastic programming modeling framework for solving the ILMTP-APU based on these structural results. An experimental evaluation on synthetic instances shows great promise. In particular, the solutions obtained are far superior to a basic heuristic extension of the work in [17] and the algorithm scales to 100 uncertain passengers per scenario when 1,000 confirmed passengers are scheduled.

## 2 Problem Description

We first describe the elements of the problem, including the mass transit system, last-mile vehicles, destinations, passenger requests, and associated parameters.

**Mass transit system:** We assume that the mass transit is a train system. Let  $\mathbf{T}_0$  be the *terminal station* that links a mass transit system with a last-mile service system. The mass transit system is described by a set of *trips*, denoted by  $\mathcal{C}$ , with  $n_c := |\mathcal{C}|$ . Each trip originates at a station in set  $\mathcal{S}$  and ends at  $\mathbf{T}_0$ . The trips are regular in the sense that the train stops at all stations in  $\mathcal{S}$  sequentially, with  $\mathbf{T}_0$  as the last stop of each trip. A trip  $c$  leaves station  $s \in \mathcal{S}$  at time  $\tilde{t}(c, s)$  and arrives to  $\mathbf{T}_0$  at  $\tilde{t}(c)$ .

**Destinations:** Let  $\mathcal{D}$  be the set of destinations where the CVs make stops, with  $K := |\mathcal{D}|$ , where we assume  $\mathbf{T}_0 \in \mathcal{D}$ . For each destination  $d \in \mathcal{D}$ , let  $\tau(d)$  be the total time it takes a CV to depart  $\mathbf{T}_0$ , travel to  $d$  (denoted by  $\tau^1(d)$ ), stop at  $d$  for passengers to disembark (denoted by  $\tau^2(d)$ ), and return to  $\mathbf{T}_0$  (denoted by  $\tau^3(d)$ ). Therefore,  $\tau(d) = \tau^1(d) + \tau^2(d) + \tau^3(d)$ . Let  $\mathcal{T} := \{1, \dots, t^{\max}\}$  be an index set of the operation times of both systems. We assume that the time required to board passengers into the CVs is incorporated in  $\tau^1(d)$ . For simplicity, the boarding time is independent of the number of passengers.

**Last-mile system:** Let  $V$  be the set of CVs, with  $m := |V|$ . Denote by  $v^{\text{cap}}$  the number of passengers that can be assigned to a single CV trip. Each CV trip consists of a set of passengers boarding the CV, traveling from  $\mathbf{T}_0$  to a destination  $d \in \mathcal{D}$ , and then returning back to  $\mathbf{T}_0$ . Therefore, passengers sharing a common CV trip must request transportation to a common building. We also assume that each CV must be back at the terminal by time  $t^{\max}$ .

**Known Passengers:** Let  $\mathcal{J}$  be the set of known passengers. Each passenger  $j \in \mathcal{J}$  requests transport from a station  $s(j) \in \mathcal{S}$  to  $\mathbf{T}_0$ , and then by CV to destination  $d(j) \in \mathcal{D}$ , to arrive at time  $t^r(j)$ . The set of passengers that request service to destination  $d$  is denoted by  $\mathcal{J}(d)$ . Let  $n := |\mathcal{J}|$  and  $n_d := |\mathcal{J}(d)|$ . Each passenger  $j \in \mathcal{J}$  must arrive to  $d(j)$  between  $t^r(j) - T_w$  and  $t^r(j) + T_w$ .

**Uncertain Passengers:** We assume a finite set of scenarios, denoted by  $\mathcal{Q}$ , representing different realizations of the uncertain passengers. Let  $\hat{\mathcal{J}}(q)$  be the set of uncertain passengers in scenario  $q \in \mathcal{Q}$ . Each passenger  $j \in \hat{\mathcal{J}}(q)$  requests transport from a station  $s(j) \in \mathcal{S}$  to  $\mathbf{T}_0$ , and then by CV to destination  $d(j) \in \mathcal{D}$ , to arrive at time  $t^r(j)$ . The set of passengers that request service to destination  $d$  is denoted by  $\hat{\mathcal{J}}(q, d)$ . Let  $\hat{n}_q := |\hat{\mathcal{J}}(q)|$  and  $\hat{n}_{q,d} := |\hat{\mathcal{J}}(q, d)|$ .

**Problem Statement:** The ILMTP-APU is the problem of assigning train trips and CVs to each known passenger so that the uncertain passengers in any of

the scenarios  $\mathcal{Q}$  can be feasibly scheduled and the expected value of a convex combination of the total transit time and the number of CV trips utilized is minimized. A solution therefore consists of:

- A partition  $\mathbf{g} = \{g_1, \dots, g_\gamma\}$  of  $\mathcal{J}$ , where each group  $g_l$  is associated with a departure time  $t_l^{\mathbf{g}}$  indicating the time the CV carrying the passengers in  $g_l$  departs  $\mathbf{T}_0$ , satisfying all request time and operational constraints. For passenger  $j \in \mathcal{J}$ , let  $\mathbf{g}(j)$  be the group in  $\mathbf{g}$  where  $j$  belongs.
- For each scenario  $q \in \mathcal{Q}$ , a partition  $\widehat{\mathbf{g}}(q) = \{\widehat{g}_{q,1}, \dots, \widehat{g}_{q,\widehat{\gamma}(q)}\}$  of  $\widehat{\mathcal{J}}(q)$ , where each group  $\widehat{g}_{q,k}$  is associated with a departure time  $t_k^{\widehat{\mathbf{g}}(q)}$  and an indicator function  $\sigma(q,k) \in \{1, \dots, \gamma\} \cup \{\emptyset\}$ .  $\sigma(q,k) \neq \emptyset$  indicates that uncertain passenger group  $\widehat{g}_{q,k}$  shares the last-mile trip with known passenger group  $g_{\sigma(q,k)}$ . In other words, groups leave from the terminal at the same time (i.e.  $t_{\sigma(q,k)}^{\mathbf{g}} = t_k^{\widehat{\mathbf{g}}(q)}$ ) and the combination of confirmed passenger group and all such shared passenger groups in a scenario does not exceed the CV capacity, i.e.  $|g_l| + \sum_{k \in \{1, \dots, \widehat{\gamma}(q) : \sigma(q,k)=l\}} |\widehat{g}_{q,k}| \leq v^{\text{cap}}$  for each  $l \in \{1, \dots, \gamma\}$  and  $q \in \mathcal{Q}$ .

### 3 Structure of Optimal Solutions

The deterministic version of ILMTP has optimal solutions with a structure that is particularly helpful for defining compact models. For each destination, passengers can be sorted by their desired arrival times and then grouped sequentially. This structure is valid to minimize passenger average waiting time [18] as well as the number of CV trips [17] hence leading to the compact DD-based model in [17]. For ILMTP-APU, however, a more elaborate structure is required.

For example, let us suppose that for a particular time range there is a single CV of capacity 4 available, 4 known passengers, and just 1 unknown passenger in 1 out of 10 scenarios. Furthermore, let us assume that a first trip with the CV incurs no wait time whereas a second trip would impose a wait time of  $w$  on any passenger involved, and that the desired arrival time of the uncertain passenger falls strictly in the middle of those of the known passengers. If we sort and group all passengers regardless of their categories, then at least 2 trips will always be necessary and at least 1 known passenger has to wait  $w$ . But if we define a second trip only for the unknown passenger, then the average cost of the solution is reduced to a tenth because the second trip and the corresponding wait time  $w$  only materialize in 1 out of 10 scenarios. Since uncertain passengers have a smaller impact on the objective function, it is intuitive that they might be delayed with respect to known passengers if the schedule remains feasible. We formalize this two-tier structure using the groups from the previous section.

**Proposition 1.** *When ILMTP-APU is feasible, there is an optimal solution where the groups of passengers for each category – known or uncertain from a scenario  $q$  – are grouped sequentially by their desired arrival times.*

*Proof.* Let us assume, by contradiction, that there is an instance for which the statement does not hold for a group of known passengers involving a passenger

$j$  with destination  $d$  in group  $g_1$  and another group  $g_2 \supseteq \{j-1, j_+\}$  for some  $j_+ > j$ . Furthermore, among the optimal solutions for such an instance, let us choose the optimal solution for which the first index  $d$  of the destination where such a grouping of known passengers does not exist is maximized; and among those solutions the one that maximizes the index  $j$  of the first passenger for which there is a group defined by passengers before and after  $j$  is maximized.

The key to obtaining a contradiction is the fact that the length of the time window for arrival is identical for all passengers each of whom have access to the same public transit service. Let us denote by  $g_A \in \{g_1, g_2\}$  the group with earliest arrival time, say  $t_A$ ; let us denote the other group by  $g_B$ , for which the departure time  $t_B$  is such that  $t_B \geq t_A$ ; and let us denote the indices of passengers on either group as  $\{j_1, j_2, \dots, j_{|g_A|+|g_B|}\}$ , where  $t_{j_i}^r \leq t_{j_{i+1}}^r$  for  $i = 1, \dots, |g_A| + |g_B| - 1$ . Note that all of these passengers have either  $t_A$  or  $t_B$  in their time windows. Since at least  $|g_A|$  passengers can arrive at time  $t_A$ , it follows that the time window of the first  $|g_A|$  passengers includes  $t_A$ . If not, then some of these first passengers would contain  $t_B$  in their time window instead, implying that some among the remaining  $|g_B|$  passengers have  $t_A$  in their time window, and thus that the passengers are not sorted by desired arrival times. Thus,  $t_A \in [t_{j_{|g_A|}}^r - \omega, t_{j_1}^r + \omega]$ . Similarly, at least  $|g_B|$  passengers can arrive at time  $t_B$ , and thus  $t_B \in [t_{j_{|g_A|+|g_B|}}^r - \omega, t_{j_{|g_A|+1}}^r + \omega]$ . Hence, we can replace the passengers of group  $g_A$  with  $\{j_1, \dots, j_{|g_A|}\}$  and those of group  $g_B$  with  $\{j_{|g_A|+1}, \dots, j_{|g_A|+|g_B|}\}$  while preserving their arrival times, size, and consequently with no change to the feasibility or optimality of the solution. However, this exchange implies that up to destination  $d$  and passenger  $j$  all passengers are grouped by sorted arrival times, hence contradicting the choice of  $d$  and  $j$ .

Without loss of generality, we can apply the same argument for uncertain passengers by also choosing the maximum index of a scenario  $q$  for which the groupings are not continuous and finding the same contradiction.  $\square$

The following result, which is independent from Proposition 1, is also helpful to simplify the modeling of ILMTP-APU.

**Proposition 2.** *When ILMTP-APU is feasible, there is an optimal solution where at most one group of uncertain passengers for each scenario is assigned to each group of known passengers.*

*Proof.* If multiple groups are assigned, they have the same arrival times in any optimal solution and thus can be combined without loss of generality.  $\square$

## 4 Decision Diagram for Single Building

We use a DD to represent the groups from Proposition 1 for each destination. A DD is a set of paths between a universal source node (root) and a universal sink node (terminal), each containing a common number of arcs. The  $i$ -th arc corresponds to a decision regarding the  $i$ -th passenger sorted by desired arrival time: either passenger  $i$  is the last in a group and a departure time is chosen,

or subsequent passengers join the group. The endpoints of each arc are nodes representing how many passengers have been accumulated to define a group. Arcs either increment this number by 1 (up to CV capacity) or set it back to 0.

Following similar notation as in [17], we define for each destination  $d \in \mathcal{D}$  a DD  $D^d = (\mathbf{N}^d, \mathbf{A}^d)$  for the known passengers.  $\mathbf{N}^d$  is partitioned into  $n_d + 1$  ordered layers  $\mathbf{L}_1^d, \mathbf{L}_2^d, \dots, \mathbf{L}_{n_d+1}^d$  where  $n_d = |\mathcal{J}(d)|$ . Layer  $\mathbf{L}_1^d = \{\mathbf{r}^d\}$  and layer  $\mathbf{L}_{n_d+1}^d = \{\mathbf{t}^d\}$  consist of one node each; the *root* and *terminal*, respectively. Each arc  $a \in \mathbf{A}^d$  is directed from its *arc-root*  $\psi(a)$  to its *arc-terminal*  $\omega(a)$ . If  $\psi(a) \in \mathbf{L}_i^d$ , then  $\omega(a) \in \mathbf{L}_{i+1}^d$ . It is assumed that every maximal arc-directed path connects  $\mathbf{r}^d$  to  $\mathbf{t}^d$ . Similarly, for each scenario  $q \in \mathcal{Q}$ , we define a DD  $D^{d,q} = (\mathbf{N}^{d,q}, \mathbf{A}^{d,q})$  by using the corresponding upper index  $q$  for disambiguation.

The arcs between layers of the diagram correspond to the passengers that request transportation to the destination. Each node  $u$  is associated with a *state*  $\mathbf{s}(u)$  corresponding to the number of immediately preceding passengers that is grouped with the next passenger. Each arc  $a$  is associated with a label  $\phi(a) \in \{0, 1\}$  on whether passenger  $\psi(a)$  is not the last one in the group, in which case  $\phi(a) = 0$  and  $\mathbf{s}(\omega(a)) = \mathbf{s}(\psi(a)) + 1 \leq v^{\text{cap}}$ , or else  $\phi(a) = 1$ . There can be multiple arcs between the same nodes in the latter case, each arc  $a$  corresponding to a different CV start time  $t^0(a)$ . Accordingly, each arc  $a$  such that  $\phi(a) = 1$  has a corresponding wait time  $W(a)$  for all passengers in the group and a label  $\chi(a, t) \in \{0, 1\}$  to indicate that a CV would be active at time  $t$  (i.e.  $t^0(a) \leq t \leq t^0(a) + \tau(d)$ ) if arc  $a$  is chosen. Hence,  $\phi(a) = 0$  implies  $\chi(a, t) = 0$ .

## 5 IP Formulation

We define a formulation by which we group the passengers using a path from each DD. Some of the groups of known and uncertain passengers are combined, and we aim for a feasible schedule of the resulting groups using the CV fleet.

We introduce binary variable  $x_a \in \{0, 1\} \forall a \in \mathbf{A}^d, d \in \mathcal{D}$ , to denote the choice of the particular arc in the DD for known passengers. Similarly, we introduce binary variable  $y_a^q \in \{0, 1\} \forall a \in \mathbf{A}^{d,q}, d \in \mathcal{D}$  to denote the choice of the particular arc in the DD for uncertain passengers in scenario  $q \in \mathcal{Q}$ . Let  $\tilde{\mathbf{A}}^{d,q} = \{(a_1, a_2) \in \mathbf{A}^d \times \mathbf{A}^{d,q} \mid \phi(a_1) = 1, \phi(a_2) = 1, t^0(a_1) = t^0(a_2) \text{ and } \mathbf{s}(\psi(a_1)) + \mathbf{s}(\psi(a_2)) + 2 \leq v^{\text{cap}}\}$ . The set  $\tilde{\mathbf{A}}^{d,q}$  denotes the set of feasible pairs of known passenger group and uncertain passenger group of scenario  $q$ , i.e. identical start time on the CV and the capacity constraint is satisfied. Let  $z_{a_1, a_2}^q \in \{0, 1\} \forall (a_1, a_2) \in \tilde{\mathbf{A}}^{d,q}$  denote the decision of pairing the group of known passengers represented by arc  $a_1$  and group of uncertain passengers represented by arc  $a_2$ .

The objective function can be expressed as

$$\begin{aligned} f(\alpha) = & \sum_{d \in \mathcal{D}} \sum_{a \in \mathbf{A}^d} [\alpha W(a) + (1 - \alpha)] x_a \\ & + \frac{1}{|\mathcal{Q}|} \sum_{d \in \mathcal{D}} \sum_{q \in \mathcal{Q}} \left( \sum_{a \in \mathbf{A}^{d,q}} [\alpha W(a) + (1 - \alpha)] y_a^q - (1 - \alpha) \sum_{(a_1, a_2) \in \tilde{\mathbf{A}}^{d,q}} z_{a_1, a_2}^q \right) \end{aligned}$$



The following constraint imposes that only one group of uncertain passengers is paired with a group of known passengers if the latter is selected:

$$\sum_{a_2: (a_1, a_2) \in \tilde{A}^{d,q}} z_{a_1, a_2}^q \leq y_{a_1} \forall d \in \mathcal{D}, q \in \mathcal{Q}, a_1 \in A^d : \phi(a_1) = 1. \quad (1a)$$

The fleet size constraint can be modeled for all  $t \in \mathcal{T}, q \in \mathcal{Q}$  as

$$\sum_{d \in \mathcal{D}} \left( \sum_{a \in A^d: \chi(a,t)=1} x_a + \sum_{a \in A^d, q: \chi(a,t)=1} y_a^q \right) - \sum_{d \in \mathcal{D}} \sum_{(a_1, a_2) \in \tilde{A}^{d,q}} z_{a_1, a_2}^q \leq m. \quad (1b)$$

The IP model for the ILMTP-APU is

$$\begin{aligned} \min \quad & f(\alpha) \\ \text{s.t.} \quad & \text{Network flow constraints for } D^d \quad \forall d \in \mathcal{D} \end{aligned} \quad (2a)$$

$$\text{Network flow constraints for } D^{d,q} \quad \forall d \in \mathcal{D}, q \in \mathcal{Q} \quad (2b)$$

Eq. (1a), (1b)

$$x_a \in \{0, 1\} \quad \forall d \in \mathcal{D}, a \in A^d \quad (2c)$$

$$y_a^q \in \{0, 1\} \quad \forall d \in \mathcal{D}, q \in \mathcal{Q}, a \in A^d \quad (2d)$$

$$z_{a_1, a_2}^q \in \{0, 1\} \quad \forall d \in \mathcal{D}, q \in \mathcal{Q}, (a_1, a_2) \in \tilde{A}^{d,q}. \quad (2e)$$

The network flow constraints in (2a)-(2b) guarantee that a path is taken on each decision diagram, which corresponds to the groupings of known passengers and uncertain passengers for each scenario.

## 6 Experimental Results

We ran experiments to test our approach on a machine with an Intel(R) Core(TM) i7-4770 CPU @ 3.40GHz and 16 GB RAM. The code is in `Python 2.7.6` and the ILPs are solved using `Gurobi 7.5.1`. We generated instances with 1000 passengers, 60 CVs with capacity 5, 10 destinations, time windows of 10 minutes, time units of 30 seconds, and 10 scenarios each containing 50 or 100 passengers. The instances are similar to those in [17], but smaller due to problem complexity.

For benchmarking, we also tested the following heuristic **H**: (1) solve the problem optimally for known passengers using the algorithm in [17]; and (2) for each scenario, solve the resulting MIP formulation to maximize the number of uncertain passengers that can be scheduled with the remaining capacity of the trips already scheduled and the remaining availability of the CVs for more trips.

Table 1 summarizes results for  $\alpha = 0.5$ . Each instance  $P_{k,u,i}$  corresponds to the  $i$ -th instance with  $k$  known passengers and  $u$  unknown passengers on each of the 10 scenarios. We report the values for the first stage (known passengers) and for the second stage (uncertain passengers) as well as runtimes. The first- and second-stage values correspond to the first and second terms of  $f(\alpha)$ . If the second stage is infeasible, we report the percentage of scheduled passengers.

**Table 1.** Comparison of solution obtained with DDO and with heuristic H.

Instance	Heuristic H			DDO Approach		
	1 <sup>st</sup> Stage	2 <sup>nd</sup> Stage	Time (s)	1 <sup>st</sup> Stage	2 <sup>nd</sup> Stage	Time (s)
P <sub>1000,50,1</sub>	21,940.0	(41.6%)	23.8	22,991.5	1,254.2	630.4
P <sub>1000,50,2</sub>	22,434.0	(33.4%)	25.2	23,410.5	1,257.9	535.1
P <sub>1000,50,3</sub>	22,409.5	(35.8%)	25.2	23,403.5	1,271.1	532.2
P <sub>1000,50,4</sub>	22,410.0	(38.2%)	25.6	23,397.0	1,302.2	643.5
P <sub>1000,50,5</sub>	22,099.0	(37.4%)	25.4	23,152.5	1,282.0	1019.9
P <sub>1000,50,6</sub>	22,552.0	(37.6%)	28.7	23,826.0	1,239.8	539.8
P <sub>1000,50,7</sub>	22,397.0	(34.8%)	24.3	23,621.5	1,293.3	646.1
P <sub>1000,50,8</sub>	22,002.0	(42.2%)	25.9	23,003.0	1,264.2	564.6
P <sub>1000,50,9</sub>	22,745.5	(34.8%)	26.0	23,855.0	1,276.0	704.5
P <sub>1000,50,10</sub>	22,206.0	(40.0%)	25.4	23,186.0	1,262.3	526.8
P <sub>1000,100,1</sub>	22,389.5	(31.6%)	24.8	24,078.5	2,503.4	1,171.3
P <sub>1000,100,2</sub>	21,992.5	(40.4%)	25.5	23,718.5	2,510.7	2,142.1
P <sub>1000,100,3</sub>	22,181.0	(31.6%)	39.4	23,814.5	2,523.1	1,330.6
P <sub>1000,100,4</sub>	22,409.5	(31.3%)	23.8	23,923.5	2,577.0	1,535.2
P <sub>1000,100,5</sub>	22,447.5	(40.3%)	29.5	24,162.0	2,496.3	896.0
P <sub>1000,100,6</sub>	22,725.5	(31.4%)	25.2	24,277.0	2,530.4	1,112.4
P <sub>1000,100,7</sub>	22,236.5	(36.9%)	26.5	24,286.0	2,537.8	2,776.1
P <sub>1000,100,8</sub>	22,584.0	(33.0%)	27.3	24,177.5	2,560.4	1,544.1
P <sub>1000,100,9</sub>	22,541.0	(35.6%)	24.9	24,106.5	2,477.9	1,982.5
P <sub>1000,100,10</sub>	22,552.0	(37.6%)	29.0	24,274.0	2,525.3	1,965.6

We note that ignoring the second stage leads to infeasible problems in all cases for heuristic H, and only a portion of the uncertain passengers could be scheduled. Interestingly, the proportion of uncertain passengers that are scheduled when the optimal solution of the known passengers alone is fixed remains approximately the same for 50 and 100 uncertain passengers per scenario. Hence, reducing the number of uncertain passengers does not make heuristic H more suitable.

For  $\alpha = 0$ , we found that the optimal solution of the deterministic case, which is used by H, has the same value as the first stage of DDO. In that case, DDO found solutions that are robust for the scenarios considered while also optimal for the known passengers. In contrast, for  $\alpha > 0$  there is a difference between first stage values for both approaches, which is due to minimizing travel times.

## 7 Conclusion

We considered a two-stage optimization problem of last-mile passenger scheduling subject to a finite set of scenarios representing uncertain additional demand. Our approach based on decision diagram optimization produces solutions that, despite an increase in runtimes, are feasibly robust with respect to all scenarios while minimizing the expected number of last-mile trips necessary to satisfy the demand across all scenarios. The results show the potential of using decision diagrams to solve such challenging problems of scheduling under uncertainty.

## References

1. Andersen, H.R., Hadzic, T., Hooker, J.N., Tiedemann, P.: A constraint store based on multivalued decision diagrams. In: Proceedings of the 13th International Conference on Principles and Practice of Constraint Programming. pp. 118–132. CP’07, Springer-Verlag, Berlin, Heidelberg (2007), <http://dl.acm.org/citation.cfm?id=1771668.1771682>
2. Ben-Tal, A., Nemirovski, A.: Robust optimization – methodology and applications. *Mathematical Programming* **92**(3), 453–480 (May 2002). <https://doi.org/10.1007/s101070100286>, <https://doi.org/10.1007/s101070100286>
3. Bergman, D., Cire, A., van Hoeve, W., Hooker, J.: Decision diagrams for optimization. Springer (2016)
4. Bergman, D., Cire, A.A.: Decomposition based on decision diagrams. In: Quimper, C.G. (ed.) *Integration of AI and OR Techniques in Constraint Programming*. pp. 45–54. Springer International Publishing, Cham (2016)
5. Bergman, D., Cire, A.A.: Discrete nonlinear optimization by state-space decompositions. *Management Science* **0**(0), null (2018). <https://doi.org/10.1287/mnsc.2017.2849>, <https://doi.org/10.1287/mnsc.2017.2849>
6. Bergman, D., van Hoeve, W.J., Hooker, J.N.: Manipulating mdd relaxations for combinatorial optimization. In: Achterberg, T., Beck, J.C. (eds.) *Integration of AI and OR Techniques in Constraint Programming for Combinatorial Optimization Problems*. pp. 20–35. Springer Berlin Heidelberg, Berlin, Heidelberg (2011)
7. Bergman, D., Lozano, L.: Decision Diagram Decomposition for Quadratically Constrained Binary Optimization. *Optimization Online e-prints* (Oct 2018)
8. Birge, J.R., Louveaux, F.: *Introduction to Stochastic Programming*. Springer (2011)
9. Grosse-Ophoff, A., Hausler, S., Heineke, K., Möller, T.: How shared mobility will change the automotive industry (2017), <https://www.mckinsey.com/industries/automotive-and-assembly/our-insights/how-shared-mobility-will-change-the-automotive-industry>
10. Haus, U.U., Michini, C., Laumanns, M.: Scenario aggregation using binary decision diagrams for stochastic programs with endogenous uncertainty (2017)
11. Kleywegt, A., Shapiro, A., Homem-de Mello, T.: The sample average approximation method for stochastic discrete optimization. *SIAM Journal on Optimization* **12**(2), 479–502 (2002). <https://doi.org/10.1137/S1052623499363220>, <https://doi.org/10.1137/S1052623499363220>
12. Liu, Z., Jiang, X., Cheng, W.: Solving in the last mile problem: Ensure the success of public bicycle system in beijing. *Procedia Soc. Behav. Sci.* **43**, 73–78 (2012)
13. Lozano, L., Bergman, D., Smith, J.C.: On the Consistent Path Problem. *Optimization Online e-prints* (Apr 2018)
14. Lozano, L., Smith, J.C.: A binary decision diagram based algorithm for solving a class of binary two-stage stochastic programs. *Mathematical Programming* (Aug 2018). <https://doi.org/10.1007/s10107-018-1315-z>, <https://doi.org/10.1007/s10107-018-1315-z>
15. Mahéo, A., Kilby, P., Hentenryck, P.V.: Benders decomposition for the design of a hub and shuttle public transit system. *Transportation Science* (2018)
16. McCoy, K., Andrew, J., Glynn, R., Lyons, W.: Integrating shared mobility into multimodal transportation planning: Improving regional performance to meet public goals. Tech. rep., Office of the Assistant Secretary of Transportation for Research and Technology, U.S. Department of Transportation (2018)

17. Raghunathan, A.U., Bergman, D., Hooker, J., Serra, T., Kobori, S.: Seamless multimodal transportation scheduling (2018)
18. Raghunathan, A.U., Bergman, D., Hooker, J.N., Serra, T., Kobori, S.: The integrated last-mile transportation problem (ILMTP). In: Proceedings of the Twenty-Eighth International Conference on Automated Planning and Scheduling, ICAPS 2018, Delft, The Netherlands, June 24-29, 2018. pp. 388–398 (2018), <https://aaai.org/ocs/index.php/ICAPS/ICAPS18/paper/view/17720>
19. Shaheen, S.: U.S. carsharing & station car policy considerations: Monitoring growth, trends & overall impacts. Tech. rep., Institute of transportation studies, Working paper series, Institute of Transportation Studies, UC Davis (2004)
20. Shen, Y., Zhang, H., Zhao, J.: Simulating the First Mile Service to Access Train Stations by Shared Autonomous Vehicle. In: Transportation Research Board 96th Annual Meeting (2017)
21. Thien, N.D.: Fair cost sharing auction mechanisms in last mile ridesharing. Ph.D. thesis, Singapore Management University (2013)
22. Wang, H.: Routing and Scheduling for a Last-Mile Transportation Problem. *Transportation Science* pp. 1–17 (forthcoming) (2017). <https://doi.org/https://doi.org/10.1287/trsc.2017.0753>, <http://pubsonline.informs.org/doi/abs/10.1287/trsc.2017.0753>