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TR2018-190 March 06, 2019

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Optical Fiber Communication Conference and Exposition and the National Fiber Optic Engineers Conference (OFC/NFOEC)

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Polar-Coded Modulation for Joint Channel Coding and Probabilistic Shaping

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Abstract: We propose joint channel coding and constellation shaping based on polar-coded modulation. Proposed shaping method offers greater than 0.6 dB gain without the need of external distribution matcher and the increase of decoding complexity.

OCIS codes: (060.4510) Optical communications, (060.1660) Coherent communications, (060.4080) Modulation.

1. Introduction

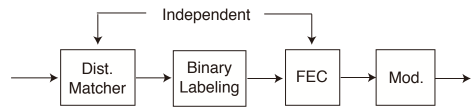
Considering the recent advancement of forward error correction, high-order quadrature-amplitude modulation (QAM) has been widely applied to high data-rate fiber-optic communication systems. For regular uniform QAM modulations, however, it is well known that there is a gap between the Shannon limit and the achievable information rate.

In order to compensate for this gap, various approaches have been extensively studied in the literature [1–8]. These approaches can be largely classified into two types: probabilistic shaping (PS) [1–5] and geometric shaping (GS) [6–8]. The PS generates equispaced constellation points with non-uniform probabilities, whereas the GS uses non-equispaced constellation points with uniform probabilities. Although the performance of the PS is superior to that of the GS in general, the major challenge of the PS lies in low-complexity implementation of an additional distribution matcher, which realizes a desired signal distribution from a given information sequence.

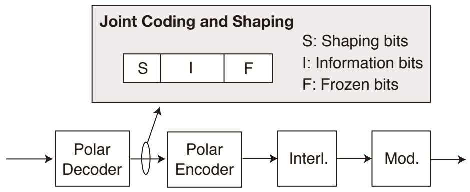
In this paper, we investigate the PS for polar-coded modulation either based on bit-interleaved coded modulation (BICM) or multi-level coding (MLC) schemes, where the distribution matching is realized by the decoding of polar codes. Therefore, our shaping method may be distinguished from other shaping methods in that coding and shaping are jointly performed with a single polar code, i.e., no additional device other than polar encoder and decoder is required in principle. Although the similar shaping method for MLC with the multi-stage decoding (MSD) scheme has been proposed in [9], the main difference of our method is that we introduce the multi-stage shaping, which can get closer to the target distribution, whereas the work [9] is limited to generating non-increasing probabilities of constellation points. Furthermore, we also investigate the shaping for BICM systems, which may be more important in practice.

2. Constellation Shaping for Polar-Coded Modulation

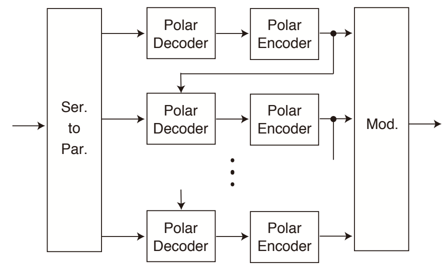
For polar codes, the input sequence to the encoder is divided into two parts: information and frozen bits. Frozen bits of polar codes can be either fixed, i.e., (0 or 1), or information-dependent. Latter approach is often called dynamic frozen bits, and it was originally proposed to improve the minimum distance of polar codes [10]. The main concept behind our shaping method is to employ dynamic frozen bits for constellation shaping. For this purpose, we introduce shaping bit indices in addition to information, frozen bit indices to the polar code input, that are determined by given information bits such that the signal distribution will be closer to the target distribution. We employ the actual decoding operation such as successive cancellation (SC) decoding to find these shaping bits, assuming that every bit locations other than shaping indices are frozen. The bit-wise log-likelihood ratios (LLR) used in the decoding is derived from the target probability mass function (PMF) of the constellation, where we use Maxwell–Boltzmann (MB) distribution. Fig. 1 shows the proposed encoder structures for BICM and MLC systems, which can perform channel coding and distribution matching at the same time unlike the conventional reverse-concatenation shaping method. For a BICM approach, we use a single SC decoder to find shaping bits, and design each bit level from the most significant bits (MSB) to the lowest significant bit (LSB) independently. Although this approach cannot achieve the target PMF exactly, it is possible to achieve good error performance as shown in the subsequent section even with a low computational complexity. On the other hand, for MLC, the shaping operation is successively performed from the MSB to the LSB, and the codeword at the higher level is passed to lower level such that joint distribution can be taken into account.



(a) Conventional probabilistic amplitude shaping.



(b) Proposed BICM polar shaping.



(c) Proposed MLC polar shaping.

Fig. 1: Encoders of conventional reverse-concatenation shaping schemes based on distribution matcher vs. proposed polar-coded BICM/MLC shaping schemes for joint channel coding and probabilistic shaping.

The locations of shaping bits in the input sequence is important to control the signal distribution. In general, the bit index with higher mutual information has a better capability to control the signal distribution, since such a bit can control more code bits than a bit with lower mutual information. Furthermore, if we use good bit indices in terms of the mutual information for shaping, it is easy to find good shaping bits by the simple SC decoding. Therefore, we allocate the shaping bits from the best bit channel to the worst bit channel in this paper. It is important to note that at the receiver side, since shaping bits are allocated after all information bits, we do not have to decode shaping bits, and thus our approach does not increase the decoding complexity.

3. Performance Analysis of Shaped Polar-Coded Modulation

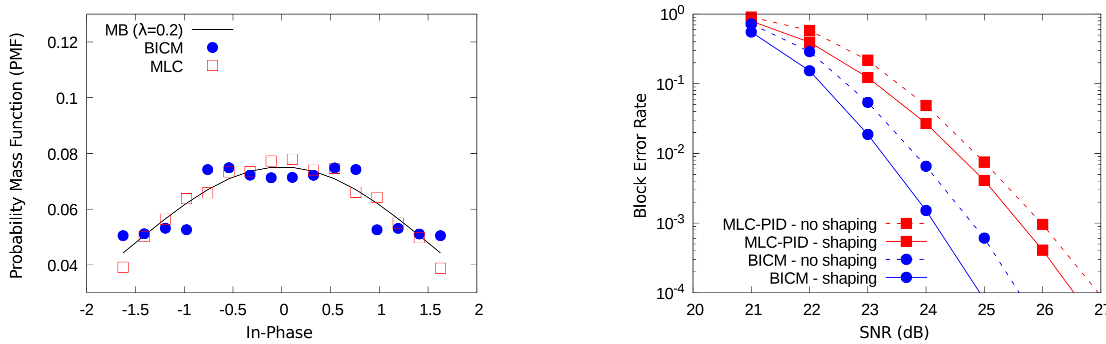
In this section, we evaluate the performance of the proposed shaping method based on both BICM and MLC systems employing 256QAM constellation (16PAM per dimension). We assume that the number of shaping bits is optimized by the exhaustive search in terms of the error rate performance and SC decoding is used as a decoding at the receiver. For BICM, we use quadratic permutation polynomial (QPP) interleavers, which are adopted as an interleaver for turbo codes in wireless communication standards.

Fig. 2a, shows the PMF realized by the proposed polar shaping for BICM and MLC, respectively. From this figure, we observe that the PMF of the MLC scheme agrees with the target MB distribution well, whereas the PMF of the BICM system has large disagreement. This is because BICM approach does not take the joint distribution into account.

In Fig. 2b, we make a performance comparison of BICM and MLC with parallel independent decoding (PID) of each layer, that are resulting in the similar capacity. For a fair comparison, we set the spectral efficiency as 3.2 bits per dimension and block length is chosen as 128 dimensions for both schemes. We use 5 shaping bits for BICM, and for MLC, (0, 5, 2, 1) bits are used at each bit level from MSB to LSB, respectively. We can see from this figure that BICM outperforms MLC-PID scheme in terms of the block error rate performance. This is because MLC has to split the codewords into multiple short codes, whereas BICM can use longer codes. Also, the performance gain by shaping of MLC is slightly smaller than that of BICM, since MLC uses more shaping bits than BICM, which may degrade the error rate performance of component polar codes.

Fig. 3 shows the performance comparison of proposed BICM systems with SC decoding-based shaping and the exhaustive search over all possible combination of shaping bits that achieve the best shaping gain. In this simulation, we make a comparison with the spectral efficiency of 2.0 bits per dimension, block length 64 dimensions, and 5 shaping bits. From Fig. 3, it is observed that the performance with the proposed SC decoding-based approach is close to that with the exhaustive search and their difference is only 0.1 dB. As demonstrated from this result, since we use bit indices with high mutual information for shaping, which means that the low-complexity SC decoding is sufficient for shaping, our method is easy to implement.

Finally, we investigate the performance of proposed system with relatively long code length in Fig. 4, where half-



(a) The PMF of the constellation.

(b) Block error rate (BLER) performances.

Fig. 2: The comparison of MLC-PID and BICM schemes (3.2 bits/dimension, block length: 128 dimensions).

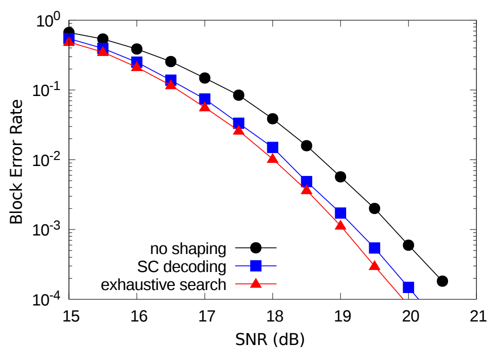


Fig. 3: The effect of decoding schemes used for shaping (2.0 bits/dimension, block length: 64 dimensions).

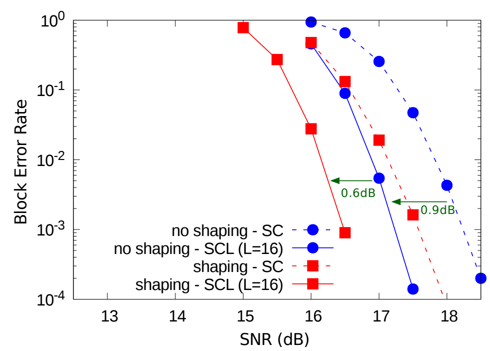


Fig. 4: BICM shaping at medium block length (2.0 bits/dimension, block length: 1024 dimensions).

rate polar codes with code length 4096 bits are used. In this figure, we also plot the performances with SC list (SCL) decoding with a list size of 16. It is observed that SCL decoding improves the performance of SC decoding by approximately 0.9 dB and proposed shaping method further offers the performance gain greater than 0.6 dB.

4. Conclusion

We proposed a novel constellation shaping based on polar-coded modulation. Performance results demonstrated that proposed shaping method achieves significant performance gain greater than 0.6 dB over the system without shaping. Note that the proposed scheme does not require an additional operation at the receiver, such as distribution dematcher.

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