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### Abstract

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# Partition-Based Probabilistic Shaping for Fiber-Optic Communication Systems

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**Abstract:** Various aspects of distribution matchers (DMs) with constant and variable composition are reviewed. LDPC-coded fiber simulations of 64QAM shaped via a multiset-partition DM show significantly increased reach and information rate over a constant-composition DM.

**OCIS codes:** (060.2330) Fiber optics communications, (060.4080) Modulation.

## 1. Introduction

Probabilistic shaping has attracted a lot of attention in the fiber-optic community over the recent years for several reasons. With probabilistic amplitude shaping (PAS) [1], a low-complexity and powerful framework for integrating shaping into existing coded modulation systems has been proposed. Potential shaping gains in the order of 1 dB of signal-to-noise ratio (SNR) are significant especially for highly-optimized long-haul fiber links. Furthermore, the rate adaptivity offered by PAS allows to fill the spectral efficiency gaps between the quadrature amplitude modulation (QAM) formats without having to change the forward error correction (FEC) overhead.

Much effort has been put into demonstrating the benefits of probabilistic shaping in simulations [2], experiments [3], and field trials [4], and to optimize distributions in the presence of fiber nonlinearities [5]. The realization of any shaping gain crucially depends on the distribution matcher (DM)—an integral part of PAS—that has been studied in detail only recently, and several advanced DM techniques have been devised. In this paper, we review general aspects of DMs and carry out a detailed performance comparison between the conventional constant-composition DM (CCDM) [6] and a recently proposed non-constant-composition DM called multiset-partition DM (MPDM) [7].

## 2. Fundamentals of Distribution Matching for Probabilistic Amplitude Shaping

The integration of a DM into a communication system with PAS is shown in Fig. 1. The task of the DM is to transform a sequence of  $k$  uniform data bits into a length- $n$  block of shaped amplitudes  $A$  at the transmitter and to carry out the inverse operation at the receiver. The most common DM system is CCDM for which every output block has the same composition, i.e., the empirical amplitude distribution of each CCDM output is identical. The CCDM mapping and demapping functions are based on arithmetic coding, which is sequential in the CCDM input length. Furthermore, any finite-length DM causes a rate loss that is defined as  $H(A) - \frac{k}{n}$  where  $H(A)$  is the entropy of the shaped amplitudes  $A$  and  $\frac{k}{n}$  the ratio of DM input bits to the output length in symbols. As  $n$  increases, the rate loss generally decreases. A longer DM is, however, not desirable from an implementation point of view as latency and memory requirements become problematic in this case. Hence, fast DM operation with high throughput and low rate loss is highly challenging.

Various DM techniques have been proposed that improve upon the initial CCDM, such as MPDM [7], product bit-level DM [8,9], parallel-amplitude DM with subset ranking [10], streaming DM [11], shell mapping [12], enumerative sphere shaping [13], framing of variable-length DM outputs into fixed-length blocks [14], and DM with mark ratio control [15]. We note that although all of these approaches are conceptually different, they share the common aim of reducing the DM complexity, for instance by reducing the required block length or decreasing the serialism, while maintaining or even decreasing the DM rate loss. In the following, we outline the principle of MPDM, which is one of the aforementioned advanced DMs, and compare its performance in detail to CCDM.

## 3. Multiset-Partition Distribution Matching (MPDM)

The recently proposed MPDM uses different compositions per DM output block which are selected according to a multiset partitioning problem such that the target distribution is achieved not in every single output block but as an average over all blocks. The idea of a pairwise MPDM, which is considered herein, is that for each used composition there must exist a complement such that their combination gives the target composition. Since a CCDM always generates sequences with this target distribution, MPDM can be viewed as a generalization of CCDM to non-constant compositions. By further requiring the number of occurrences of each composition to be a power of two, a binary tree can be constructed that allows to address the individual compositions without any additional rate loss. For a fixed DM output length  $n$ , MPDM increases the number of addressable input bits  $k$  and thus reduces the rate loss, see [7]

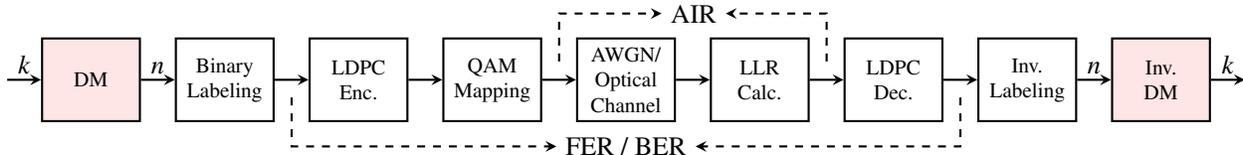


Fig. 1. Block diagram of a communication system with shaped signaling. This paper studies the impact of different distribution matchers (DMs) on information and error rates.

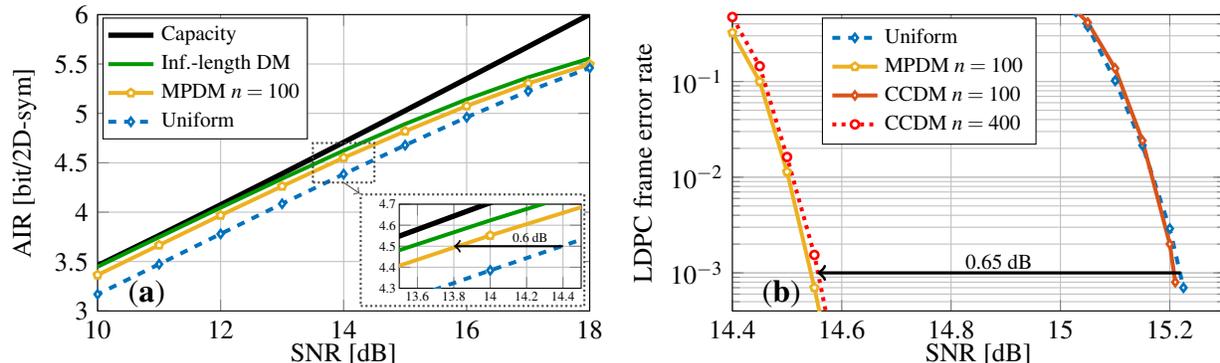


Fig. 2. AWGN simulations of uniform and shaped 64QAM. The gain of MPDM-shaped 64QAM over uniform is 0.6 dB at an AIR of 4.5 bit/2D-sym (a). With LDPC decoding, a similar shaping gain of 0.65 dB is obtained at a net information rate of 4.5 bit/2D-sym (b).

for details. For mapping the uniform input according to the different compositions, regular CCDM operation can be performed. In this paper, we use the parallel-amplitude architecture with subset ranking as DM algorithm [10], which has lower serialism than arithmetic-coding CCDM. In the following, we compare CCDM and MPDM in terms of achievable information rate (AIR) and net information rate after low-density parity-check (LDPC) decoding.

#### 4. Additive White Gaussian Noise (AWGN) Results

The performance of uniform 64QAM and 64QAM shaped via CCDM or MPDM is evaluated over an AWGN channel. The simulation setup is depicted in Fig. 1. All compositions are based on quantized Maxwell-Boltzmann distributions. Figure 2 (a) shows as a function of SNR the bit-wise AIRs, which include the DM rate loss and are computed via numerical integration as outlined in [7, Appendix]. Included for reference is the Shannon capacity  $\log_2(1 + \text{SNR})$  and the AIR limit of a DM with zero rate loss. We observe that MPDM-shaped 64QAM has a significantly higher AIR than uniform 64QAM over the entire relevant SNR range. At an output length of  $n = 100$  symbols and an information rate of 4.5 bit per 2D symbol (bit/2D-sym), 0.6 dB shaping gain over uniform 64QAM is achieved, which is just 0.2 dB below the maximum shaping gain of 0.8 dB that an ideal DM without any rate loss can obtain.

Monte Carlo simulations with DVB-S2 LDPC codes were carried out to corroborate the above AIR results. In Fig. 2 (b), a shaping gain of 0.65 dB of MPDM-shaped 64QAM over uniform signaling is found at a post-LDPC frame error rate (FER) of  $10^{-3}$ . The net information rate is set to 4.5 bit/2D-sym, which means that a rate-3/4 LDPC code is used for uniform 64QAM. Since a non-uniform input distribution introduces extra redundancy, a rate-4/5 LDPC code is used for shaped signaling to achieve the same overall information rate. Further shown in Fig. 2 (b) are decoding results for CCDM. We observe that a fourfold length increase is required for a CCDM to achieve the same performance as MPDM. Reducing the CCDM length to  $n = 100$  symbols gives a FER curve that is similar to uniform 64QAM. This is because the gain from using a shaped distribution and the rate loss of such a short CCDM effectively cancel each other out.

#### 5. Fiber Simulations

A dual-polarization optical fiber system is simulated to analyze reach improvements of MPDM over CCDM. Five wavelength division multiplexing (WDM) channels, each modulated with 64QAM at a symbol rate of 34 GBaud and spaced at 37.5 GHz, are transmitted over 80 km spans of single-mode fiber ( $\alpha = 0.2$  dB/km,  $\gamma = 1.3$  1/W/km,  $D = 17$  ps/nm/km), with amplifier noise added at the end of the link. The launch power per channel is set to the optimum of  $-1$  dBm. At the receiver, the center WDM channel is ideally detected. The accumulated dispersion is compensated,

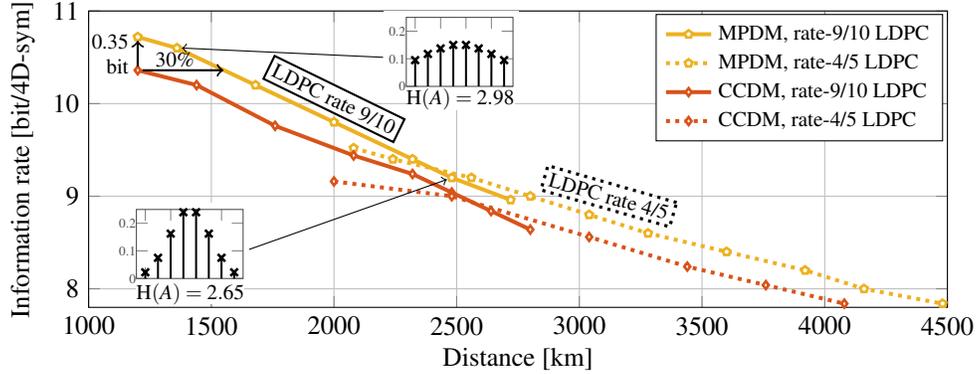


Fig. 3. Net information rate after LDPC decoding versus transmission distance of dual-polarization 64QAM shaped with MPDM and CCDM, each of length  $n = 100$ . The used LDPC codes have rate 9/10 (solid) and 4/5 (dotted). All markers correspond to a BER of less than  $5 \times 10^{-5}$  after decoding. Insets: Amplitude distributions of one 64QAM quadrature, with different shaping overheads.

log-likelihood ratios (LLRs) are computed with circularly symmetric Gaussian noise statistics and decoding of the DVB-S2 LDPC codes is performed. For each considered distance, several million data bits were simulated.

Figure 3 shows the net information rate after LDPC decoding versus reach. We consider CCDM and MPDM, each of length  $n = 100$ , LDPC codes of rates 9/10 and 4/5, and 64QAM. The measured post-LDPC bit error rate (BER) is in all cases smaller than  $5 \times 10^{-5}$  such that an outer FEC of rate 0.9922 can clean up the remaining bit errors [16]. Figure 3 shows that MPDM-shaped 64QAM has up to 0.35 bit/4D-sym higher net information rate than CCDM, corresponding to a reach increase of 30%. For longer distances, the gain in reach is about 15%. We further note that with just two LDPC codes and one QAM format, the feasible reach range is more than 3000 km. This rate adaptivity is enabled by varying the shaped input distribution and thus the entropy  $H(A)$  in bits, see the insets of Fig. 3. Note that at around 2400 km (or 9.2 bit/4D-sym), the MPDM curves of the two FEC rates cross, which means that the total redundancy added by both the LDPC codes and the shaping must be identical. However, the splitting between FEC and DM redundancy, i.e., how much overhead is added by either subsystem, is different. For distances beyond 2500 km, it is beneficial to use more FEC and less shaping overhead, whereas for shorter distances the opposite is the case.

## 6. Conclusion

We have reviewed distribution matchers (DMs), which constitute a key block of modern communication systems with probabilistic shaping. Advanced DMs can improve upon the conventional CCDM by generating output sequences of varying composition. Multiset-partition distribution matching (MPDM) is such a novel DM scheme, and its principle is briefly outlined. Numerical simulations of 64QAM over the AWGN channel show that a short MPDM with an output length of just 100 symbols achieves most of the available shaping gain, whereas a CCDM of this size has performance similar to uniform signaling since its shaping gain and rate loss approximately cancel out. We have further shown reach increases of up to 30% by using MPDM instead of CCDM and demonstrated rate adaptivity over several thousand kilometers with constant QAM order and just two different LDPC code rates.

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