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201 Broadway, Cambridge, Massachusetts 02139

Lattice Precoding for Multi-Span Constellation Shaping

Toshiaki Koike-Akino, David S. Millar, Keisuke Kojima, Kieran Parsons

Mitsubishi Electric Research Laboratories (MERL), Cambridge, MA 02139, USA. koike@merl.com

Abstract *We introduce lattice precoding (LP) for constellation shaping, which takes kurtosis into account at multiple spans to mitigate nonlinear distortion. The proposed method achieves 0.35 b/s/Hz gain for 64QAM shaping in long-haul nonlinear fiber-optic communications.*

Introduction

To compensate for a fundamental loss (up to 1.53 dB) underlying regular quadrature-amplitude modulation (QAM), various constellation shaping methods have been investigated in optical communications community[1–14]. With Gaussian-like constellations, the achievable information rate is improved in an additive white Gaussian noise (AWGN) channel. There are two major approaches in the literature: probabilistic shaping (PS)[1–9] and geometric shaping (GS)[10–14].

The PS[1–9] modifies the occurrence probabilities of the constellation points to manipulate signal distribution, e.g., via Maxwell–Boltzmann (MB) distribution. Although conventional equalization algorithms can be used with minimal modification, an external entropy coding is required, for example, Huffman coding, trellis shaping[2], shell mapping[3], many-to-one mapping[4], and constant composition distribution matching[8]. With a shaped 64QAM, 15% throughput and 43% reach increases were experimentally verified[5]. Nevertheless, a real-time implementation of low-loss entropy coding has been a highly challenging task for ultra-high-speed optical transmissions.

The GS[10–14] directly modifies the location of the constellation points to approach Gaussian. While each constellation point is equally likely, the demodulation complexity can be increased in general due to the irregular constellation. Some efficient GS optimizations include multi-ring construction[10], Arimoto–Blahut algorithm (ABA)[11–13], and projection mapping[14]. For a particular condition, the GS is reported to outperform the PS[12].

In this paper, we propose a new GS method based on lattice precoding (LP)[17,18]. The LP is a generalized version of Tomlinson–Harashima precoding (THP)[15], and used for wireless communications as a technique called vector perturbation (VP). We have first applied the LP to short-reach fiber-optic communications[17,18], achieving 21% reach extension. In fact, this benefit partly came from the shaping gain achieved by the non-

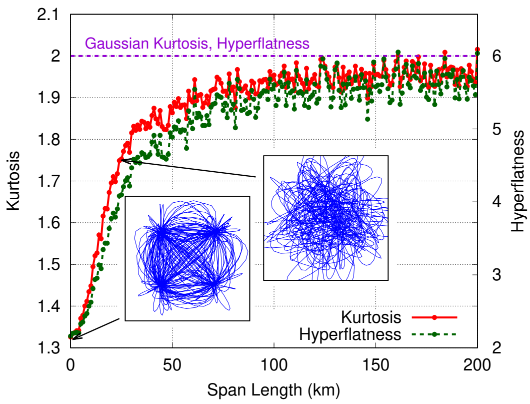

Fig. 1: 4QAM signal constellation statistics over fiber propagation. Signal approaches Gaussian due to CD.

linear lattice operation of the LP, which tries to minimize ℓ_2 (for energy efficiency[18]) and/or ℓ_∞ norms (for peak limitation[17]). Motivated by the fact that nonlinear interference (NLI) depends on kurtosis[6,9], we use the LP shaping to minimize ℓ_4 norm (as well as ℓ_2) so that the achievable rate is maximized at nonlinear fiber channels. Although kurtosis-aware shaping was already discussed[9], the achievable gain was marginal because the signal constellation will be distorted after fiber propagation over multiple spans due to chromatic dispersion (CD). To resolve this issue, we optimize the kurtosis of constellation not only at the initial span input but also at multiple intermediate spans in advance. We show that the LP-based multi-span shaping achieves a significant gain for long-haul optical communications.

Constellation shaping and kurtosis

The effective noise variance due to amplified spontaneous emission (ASE), self-phase modulation (SPM) and cross-phase modulation (XPM) over nonlinear fiber channels is well modeled as[6]:

$$\sigma_{\text{eff}}^2 = \sigma_{\text{ASE}}^2 + \kappa_0 P_{\text{tx}}^3 + P_{\text{tx}}^3\big(\kappa_4(\mu_4 - 2) + \kappa_4'(\mu_4 - 2)^2 + \kappa_6\mu_6\big), \quad (1)$$

where P_{tx} is a signal power, κ_i's are system-dependent coefficients, σ_{ASE}^2 is ASE noise variance, $\mu_k = \mathbb{E}[|X|^k]$ denotes the kth moment of

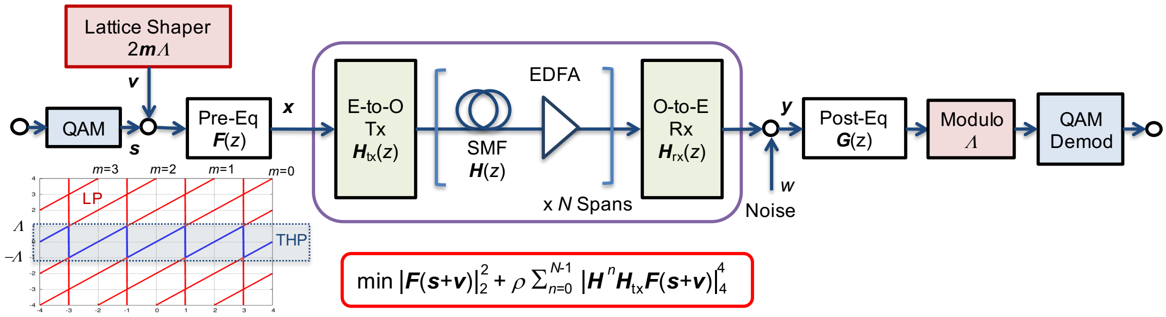

Fig. 2: Lattice precoding (LP) for multi-span constellation shaping, minimizing ℓ_2 and ℓ_4 norms at multiple span inputs.

constellation. Particularly, the kurtosis μ_4 plays an important roll in determining the strength of NLI. An analogous theory was also discussed in enhanced Gaussian noise (EGN) model.

Taking the kurtosis into consideration, the widely used MB distribution can be generalized[9]:

$$P_X(x_i) \propto \exp\left(-\nu|x_i|^2 - \nu'|x_i|^4\right), \tag{2}$$

where $P_X(x_i)$ denotes the probability mass function of constellation point x_i, ν and ν' are shaping parameters to be optimized. When $\nu' = 0$, it reduces to the standard MB distribution. This kurtosis-specific generalization offers additional 0.1 b/s/Hz and 0.2 dB gain over the standard MB for a single-span single mode fiber (SMF)[9]. However, reduced kurtosis at the fiber input will vanish after fiber propagation due to CD, and thus such shaping may not be effective for multi-span long-haul fiber transmissions unless an inline CD management is taken place.

Fig. 1 illustrates the impact of CD across SMF propagation for 4QAM signals whose kurtosis is minimum of $\mu_4 = 1$ at sample timing. We assume a baud rate of 34GBd, root-raised-cosine (RRC) filter with a roll-off of 0.01, and standard SMF whose dispersion parameter is $D = 17$ps/nm/km. Because of the RRC filter, the signal kurtosis is slightly larger than $\mu_4 = 1$ even at zero span. It is observed that the kurtosis rapidly increases over fiber propagation due to CD; specifically, the kurtosis becomes greater than 1.7 after 25 km span. After 80 km distance, kurtosis gain from Gaussian signal ($\mu_4 = 2$) is almost negligible. Note that the hyperflatness (μ_6) also behaves similarly. In this paper, we propose to use the LP so that the signal kurtosis at multiple spans is maintained small.

Lattice precoding for constellation shaping

In the presence of CD, the linear transfer function of fiber channels can be expressed as $H(f) = \exp(-\jmath L(2\pi f)^2\beta_2/2)$, where L is a fiber length and $\beta_2 = -D\lambda^2/2\pi c_0$ is a CD coefficient (c_0 is the speed of light). The total transfer function of the overall systems includes all linear impacts such as electrical-to-optical modulator $H_{\rm tx}(z)$, SMF channel $H(z)$, and optical-to-electrical modulator $H_{\rm rx}(z)$ as well as RRC filters $H_{\rm rrc}(z)$. The fiber channel $H(f)$ causes severe intersymbol interference (ISI) at longer distances L, resulting into Gaussian kurtosis as discussed in Fig. 1. We may use pre-CD compensation filter $F(z)$ and post-CD compensation filter $G(z)$, respectively, at transmitter (Tx) and receiver (Rx).

Fig. 2 illustrates the LP[17,18]. At the Tx, QAM-modulated symbols s are pre-equalized by pre-CD filter of $F(z)$. The pre-equalized signal x and channel output y are expressed as $x(z) = F(z)s(z)$ and $y(z) = H_{\rm rx}(z)H^N(z)H_{\rm tx}(z)x(z) + w(z)$ in z-transform, where w is an effective noise. Here, N denotes the number of fiber spans.

To restrict the amplitude of pre-equalized symbols x, THP uses modulo operators at both Tx and Rx. The Tx modulo operator limits symbol amplitudes as $|x| \leq \Lambda$ before the channel input. The modulo operator at the Tx is equivalent to the addition of lattice symbols $v \in 2m\Lambda$ (m is an integer) into the QAM symbols s, as shown in Fig. 2. At the Rx, the channel output y is fed into post-CD filter $G(z)$ followed by the Rx modulo operator, which can auto-cancel any lattice points v.

For THP, the lattice point (or, its integer m) is uniquely determined such that the pre-equalized symbols x are Λ-bounded: $|x| \leq \Lambda$. However, any other lattice points are invariant after the Rx modulo operator. In other words, there are infinite degrees of freedom to choose the lattice perturbation vector v in the LP, in comparison to the conventional THP. This additional flexibility for LP can give us a great opportunity refining the channel input x to be in favor of the system, for example, minimizing peak power[17] or maximizing the energy efficiency[18] to achieve high shaping gain. We optimize the signal x so that the kurtosis at multiple spans input is reduced by considering ℓ_4 norm along with ℓ_2 norm. Specifically, we use sphere detection with 32 survivors to search for the best lattice points $\boldsymbol{m}$ as follows:

$$\min_{\boldsymbol{m}\in\mathbb{Z}^B} \left\|\boldsymbol{F}(\boldsymbol{s}+2\Lambda\boldsymbol{m})\right\|_2^2 + \rho\sum_{n=0}^{N-1}\left\|\boldsymbol{H}^n\boldsymbol{H}_{\mathrm{tx}}\boldsymbol{F}(\boldsymbol{s}+2\Lambda\boldsymbol{m})\right\|_4^4, \quad (3)$$

where $\boldsymbol{s}$, $\boldsymbol{m}$, $\boldsymbol{F}$, $\boldsymbol{H}$, and $\boldsymbol{H}_{\mathrm{tx}}$ are vector/matrix representations of the QAM sequence s, lattice integers m, pre-CD $F(z)$, fiber $H(z)$, and modulator $H_{\mathrm{tx}}(z)$, respectively, for a block length of B symbols. We denote $\|\boldsymbol{x}\|_k = (\sum|x_i|^k)^{1/k}$ as an ℓ_k norm. The regularization factor ρ is adjusted to balance between the shaping gain and NLI mitigation at every span input. Note that the objective function can also take ℓ_6 norm.

Simulation results

We assume $N = 50$ spans of 80 km SMF ($\alpha = 0.2$ dB/km attenuation and $\gamma = 1.3$/W/km nonlinearity), without inline CD management. Erbium-doped fiber amplifier with a noise figure of 5 dB is assumed to compensate span loss. For simplicity, we do not consider hardware impairments such as linewidth, jitters, nonlinearity, and quantization noise for optimal modulator and demodulator, whose transfer functions $H_{\mathrm{tx}}(z)$ and $H_{\mathrm{rx}}(z)$ represent simply RRC lowpass filter $H_{\mathrm{rrc}}(z)$. We use $\Lambda = 1.2$ for lattice points.

Fig. 3 shows the achievable information rate in terms of generalized mutual information (GMI) for the proposed LP multi-span 64QAM shaping. The LP tries to minimize the kurtosis for the first several spans up to the whole 50 spans. It is confirmed that it is beneficial to increase the span of interest to design the signal constellation to improve the mutual information. Specifically, whole span LP shaping achieves 0.35 b/s/Hz improvement of GMI over the single-span LP shaping.

Conclusions

We proposed to use an extended version of THP, called LP, for constellation shaping. The nonlinear lattice shaper is applied such that the signal kurtosis at multiple spans is kept small in order to reduce the total NLI. It was verified that a significant improvement of 0.35 b/s/Hz is achievable over single-span kurtosis minimization.

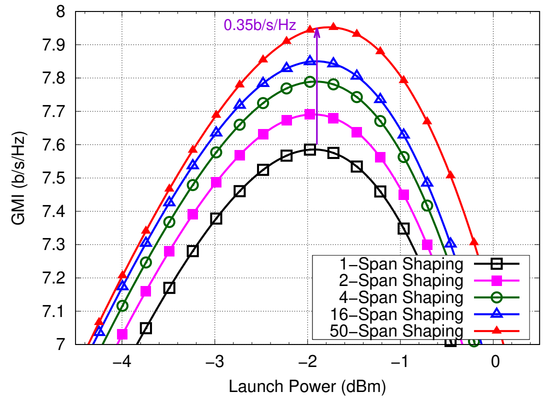

Fig. 3: Achievable information rate in GMI with LP multi-span 64QAM shaping for $N = 50$ spans of SMF $L = 80$ km.

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