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LMKL-Net: A Fast Localized Multiple Kernel Learning Solver via Deep Neural Networks

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In this paper we propose solving localized multiple kernel learning (LMKL) using LMKL-Net, a feedforward deep neural network. In contrast to previous works, as a learning principle we propose *parameterizing* both the gating function for learning kernel combination weights and the multiclass classifier in LMKL using an attentional network (AN) and a multilayer perceptron (MLP), respectively. In this way we can learn the (nonlinear) decision function in LMKL (approximately) by sequential applications of AN and MLP. Empirically on benchmark datasets we demonstrate that overall LMKL-Net can not only outperform the state-of-the-art MKL solvers in terms of accuracy, but also be trained about *two orders of magnitude* faster with much smaller memory footprint for large-scale learning.

1 Introduction

Multiple kernel learning (MKL) is a classic yet powerful machine learning technique for integrating heterogeneous features in the reproducing kernel Hilbert space (RKHS) by linear or nonlinear combination of base kernels. It has been demonstrated successfully in many applications such as object detection in computer vision [46] as one of the joint winners in VOC2009 [8].

In the literature many MKL formulations have been proposed. For instance, [3] proposed a block-ℓ_1 regularized formulation for binary classification, and later [20] proposed using ℓ_p norm as regularization in MKL. As a classic example we show the objective in SimpleMKL [39] as follows:

$$\min_{\mathbf{w}_m, \boldsymbol{\beta}\in\Delta^{M-1}, \boldsymbol{\zeta}, b} \frac{1}{2}\sum_{m=1}^{M}\frac{1}{\beta_m}\|\mathbf{w}_m\|_2^2 + C\sum_{i=1}^{N}\zeta_i, \text{ s.t. } y_i\left(\sum_{m=1}^{M}\mathbf{w}_m^T\Phi_m(x_i)+b\right) \geq 1-\zeta_i, \zeta_i \geq 0, \forall i, \tag{1}$$

where the set $\{(x_i, y_i)\}$ denote training data, $\mathbf{w}_m, \forall m$ and b denote the classifier parameters, $\boldsymbol{\beta} = [\beta_m] \in \Delta^{M-1}$ denote the weights for M kernels lying on the $(M-1)$-simplex space, $\boldsymbol{\zeta} = [\zeta_i]$ denote the slack variables for the hinge loss, $C \geq 0$ is a predefined regularization constant, $(\cdot)^T$ denotes the matrix transpose operator, and $\Phi_m(\cdot)$ denotes the feature map in RKHS for the m-th kernel $\mathbf{K}_m$ so that $\mathbf{K}_m(x_i, x_j) = \Phi_m(x_i)^T\Phi_m(x_i), \forall x_i, \forall x_j$.

Based on the dual we can rewrite Eq. 1 as follows:

$$\min_{\boldsymbol{\beta}} J(\boldsymbol{\beta}) \ s.t. \sum_m \beta_m = 1, \beta_m \geq 0, \forall m \tag{2}$$

$$\text{where } J(\boldsymbol{\beta}) = \begin{cases} \max_{\boldsymbol{\alpha}} -\frac{1}{2}\sum_{i,j}\alpha_i\alpha_j y_i y_j \sum_m \beta_m \mathbf{K}_m(x_i, x_j) + \sum_i \alpha_i \\ s.t. \ \sum_i \alpha_i y_i = 0, 0 \leq \alpha_i \leq C, \forall i. \end{cases} \tag{3}$$

Here $\alpha_i, \forall i$ denote the Lagrangian variables in the dual. To optimize Eq. 2 typically alternating optimization algorithms are developed, that is, learning $\boldsymbol{\beta}$ in Eq. 2 while fixing $\boldsymbol{\alpha}$ and learning $\boldsymbol{\alpha}$

Preprint. Work in progress.

in Eq. 3 by solving a kernel support vector machine (SVM) problem while fixing $\boldsymbol{\beta}$. Such family of algorithms in MKL are called *wrapper methods*. From this perspective Eq. 1 essentially learns a kernel mixture by *linear* combination of base kernels. Accordingly we can write the *decision function*, f, in SimpleMKL for a new sample $z \in \mathcal{X}$ as follows:

$$f_{Simple}(z) = \sum_i \alpha_i y_i \left[\sum_m \beta_m \mathbf{K}_m(z, x_i) \right]. \tag{4}$$

Localized MKL (LMKL) [10; 22; 30] and its variants such as [24] are another family of MKL algorithms which learn *localized* (*i.e.* data-dependent) kernel weights. For instance, [10] propose replacing $\boldsymbol{\beta}$ in Eq. 1 with a function η, leading to the following objective:

$$\min_{\mathbf{w}_m, \eta_m(x) \geq 0, \boldsymbol{\zeta}, b} \frac{1}{2} \sum_m \|\mathbf{w}_m\|_2^2 + C \sum_i \zeta_i, \text{ s.t. } y_i \left(\sum_m \eta_m(x_i) \mathbf{w}_m^T \Phi_m(x_i) + b \right) \geq 1 - \zeta_i, \zeta_i \geq 0, \forall i, \tag{5}$$

where $\eta_m(x_i)$ denotes the *gating function* that takes data x_i as input and outputs the weight for the m-th kernel. The decision function is as follows:

$$f_{Loc}(z) = \sum_i \alpha_i y_i \left[\sum_m \eta_m(z) \mathbf{K}_m(z, x_i) \eta_m(x_i) \right] + b. \tag{6}$$

Comparing Eq. 5 with Eq. 1, we can see that LMKL essentially *relaxes* the conventional MKL by introducing the data-dependent function η in LMKL that may better capture data properties such as distributions for learning kernel weights. Since the learning capability of η is high, previous works usually prefer regularizing it using explicit expressions such as Gaussian similarity in [10].

Beyond learning linear mixtures of kernels, some works focus on learning nonlinear kernel combination such as involving product between kernels [7; 2; 17; 27] or deep MKL [43; 4] that embeds kernels into kernels. Among these works, the primal formulations may be nontrivial to write down explicitly. Instead some works preferred learning the decision functions directly based on Empirical Risk Minimization (ERM). For instance, in [7; 43] the nonlinear kernel mixtures are fed into the dual of an SVM to learn the parameters for both kernel mixtures and the classifiers. MKL, including multi-class MKL [50] and multi-label MKL [18; 44], can be further generalized to multi-task learning (MTMKL) [16; 12; 32]. However, these research topics are out of scope of this paper.

Motivation: From the dual perspective, solving kernel SVMs such as Eq. 3 involves a constrained quadratic programming (QP) problem for computing the Lagrangian variables that determine *linear* decision boundaries in RKHS. This is the key procedure that consumes most of the computation. Moreover, in the literature of LMKL it lacks of a principle for learning the gating functions rather than manually tuning the functions such as enforcing Gaussian distributions as prior. In addition the optimal decision functions may not be necessarily linear in RKHS for the sake of accuracy. Therefore, it is highly desirable to have principles for learning both gating and decision functions in LMKL.

Deep neural networks (DNNs) have been proven as a universal approximator for an arbitrary function [28]. In fact recently researchers have started to apply DNNs as efficient solvers to some classic numerical problems such partial differential equations (PDEs) and backward stochastic differential equations (BSDEs) in high dimensional spaces [48; 40; 25]. Conventionally solving these numerical problems with large amount of high dimensional data is very time-consuming. In contrast DNNs are able to efficiently output approximate solutions with sufficient precision.

Contributions: Based on the considerations above, we propose a simple neural network, namely LMKL-Net, as an efficient solver for LMKL. As a learning principle we parameterize the gating function as well as the classifier in LMKL using an attentional network (AN) and a multilayer perceptron (MLP), respectively, without enforcing any (strong) prior explicitly. We expect that by fitting the data, the network can approximate both underlying optimal functions properly. The localized weights learned from AN guarantee that the kernel mixtures are indeed valid kernels. Empirically we demonstrate the superiority of LMKL-Net over the state-of-the-art MKL solvers in terms of accuracy, memory, and running speed.

2 Related Work

Localized MKL (LMKL): The success of LMKL highly depends on the gating functions. Due to different design choices, previous works can be categorized into either data/sample related [10; 13; 24] or group/cluster related [49; 31; 22]. For instance, in [10] the gating function is defined explicitly as a Gaussian similarity function, while in [22] the gating function is represented as the likelihoods of data in the clusters that are predefined by the likelihood function. Recently [30] proposed viewing the gating function in LMKL as an explicit feature map that can generate an additional kernel on the data. In all of these works, the classifier parameters are shared among all the data samples.

In contrast to previous works, we propose using an attentional network to approximate the unknown optimal gating function. This parameterization can avoid the need of any prior on the function.

Large-Scale MKL: Several MKL solvers have addressed the large-scale learning problem from the perspective of either computational complexity or memory footprint, such as SILP-MKL [42], SimpleMKL [39], ℓ_p-norm-MKL [20], GMKL [45], SPG-GMKL [15], UFO-MKL [37], OBSCURE [36], and MWUMKL [29]. None of them, however, is proposed for LMKL.

Differently, LMKL-Net is able to solve the large-scale LMKL problem efficiently with much smaller memory footprint, thanks to stochastic gradient descent (SGD). Empirically we observe significant speedup using LMKL-Net, compared with the solvers above.

Optimization: Wrapper methods are widely used in MKL that actually alternate the optimization between solving (multi-class) SVM problems and updating the kernel weights. Such methods include semi-infinite linear program (SILP) [42], reduced gradient [39], LPBoost [9], Newton's method [20], mirror descent [14], spectral projected gradient (SPG) [15], and triply stochastic gradients [23]. [11; 6] provided nice reviews on different MKL algorithms. Such methods, however, cannot be scalable well in general without using some clever implementation techniques such as computing kernels on the fly [15; 23].

To overcome this problem, online learning based approaches have been proposed for MKL that have much lower memory requirement. For instance, [26], [37], [36] and [34] proposed utilizing stochastic gradient descent (SGD) to optimize the primal of MKL. [1] proposed a multiple epochs of stochastic variance reduced gradient (SVRG) approach for ℓ_p-norm MKL.

In our work we employ SGD to train LMKL-Net as well that can only have weak convergence in probability [5], due to the non-convexity of our network. We notice that very recently [41] proposed a deep kernel machine optimization (DKMO) framework that embeds kernel matrices using Nyström kernel approximations and learns task-specific representations through the fusion of multiple embeddings using deep learning. As a classifier it has been demonstrated that empirically DKMO can improve performance. It is very unclear, however, whether DKMO indeed solves an MKL problem in terms of optimization. On the contrary our LMKL-Net is a valid solver for LMKL.

3 LMKL-Net

3.1 Mathematical Modeling

Key Notations: We denote $\mathbf{K}(x) \in \mathbb{R}^{N\times M}, \forall x \in \mathcal{X}$ as a matrix for data sample x with N training samples and M kernels, $y \in \mathcal{Y}$ as the class label of x, $h : \mathbb{R}^{N\times M} \times \Omega \to \Delta^{N\times M}$ as a function parameterized by Ω for learning localized kernel weights on a $(NM-1)$-simplex, $g : \mathbb{R}^{N} \times \Pi \to \mathbb{R}^{|\mathcal{Y}|}$ as a classifier parameterized by Π for $|\mathcal{Y}|$ classes, and ℓ as a loss function.

Joint Gating Function: Letting $\mathbf{K}_{\eta_m}(z, x_i) = \eta_m(z)\eta_m(x_i)$ denote the kernel defined by the gating function η_m, we then can rewrite the kernel mixture in Eq. 5 as $\sum_m \eta_m(z)\mathbf{K}_m(z, x_i)\eta_m(x_i) = \sum_m \mathbf{K}_m(z, x_i)\mathbf{K}_{\eta_m}(z, x_i)$. This perspective of LMKL has been explored in [30]. However, there are two major difficulties to compute the gating function η_m. First of all we may require the access to original data, which may not be available. Secondly in each update of function $\eta_m, \forall m \in [M]$ we have to recalculate $\eta_m(x_i), \forall x_i \in \mathcal{X}$, which may be very time consuming and memory inefficient.

To overcome these problems in conventional LMKL we are inspired by the kernel mixture in Eq. 4 and propose a new gating function defined on not only z but also x_i, namely $\eta_m(z, x_i)$, and define our kernel mixture as $\sum_m \eta_m(z, x_i)\mathbf{K}_m(z, x_i)\eta_m(z, x_i) = \sum_m \eta_m(z, x_i)^2\mathbf{K}_m(z, x_i)$. In contrast

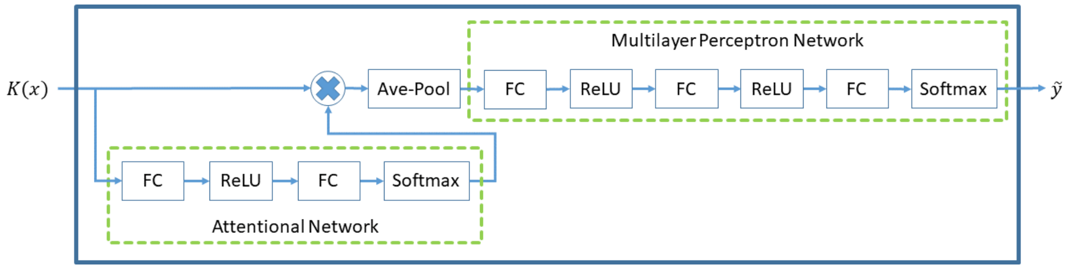


Figure 1: LMKL-Net architecture.

to Eq. 4 where the kernel weights in $\boldsymbol{\beta}$ are constant for all the data, now we would like to learn localized weights based on our joint gating function. In this way we can view $\eta_m(z, x_i)$ as a joint function over variables z and x_i. This shares some similarities with the input kernel matrices that inspire us to learn (approximately) function $\eta_m, \forall m$ based on the kernels.

Decision Function: To further generalize the classifier from linear to nonlinear for an arbitrary data sample z, we propose our decision function as follows:

$$f(z; \omega, \pi) = g\left(\sum_{m=1}^{M} h\Big(\mathbf{K}(z); \omega\Big) \otimes \mathbf{K}(z); \pi\right), \tag{7}$$

where function h is used to approximate the *normalized square* of our joint gating function, that is,

$$\left[h\Big(\mathbf{K}(z); \omega\Big)\right]_{i,m} \simeq \frac{\eta_m(z, x_i)^2}{\sum_{m,i} \eta_m(z, x_i)^2}, \forall m, \forall i, \tag{8}$$

for the entry at (i, m) in the matrix, $\omega \in \Omega, \pi \in \Pi$ are the parameters of h, g, respectively, and $\otimes$ denotes the entry-wise product operator.

Problem Definition: In this paper we only focus on the MKL problems where only kernels and data labels are provided *with no access* to the original data. This setup is heavily used and studied in the literature [11], and thus we only compare our approach with MKL algorithms (see our experimental section in Sec. 4) to emphasize the effectiveness of LMKL-Net in solving MKL problems.

Similar to [7; 43], we would like to learn our decision function based on ERM as follows:

$$\min_{\omega \in \Omega, \pi \in \Pi} \sum_i \ell\Big(y_i, f(x_i; \omega, \pi)\Big), \tag{9}$$

where Ω, Π denote the feasible parameter spaces that may be restricted by some constraints such as regularization. For simplification in expression we assume that all the parameter constraints have been embedded in the feasible spaces implicitly.

3.2 Network Architecture

Now we would like to parameterize the functions h, g in Eq. 7 using DNNs. Fig. 1 illustrates the architecture of our LMKL-Net, where "FC" denotes a fully connected layer, "ReLU" denotes rectified linear units [33] as the activation function, "Ave-Pool" denotes the average pooling operation over the kernels, and "Softmax" denotes a softmax function to normalize the inputs. Accordingly the parameter ω in h is determined by the two FC layers in the attentional network (AN), and the parameter π in g is determined by the three FC layers in the multilayer perceptron network (MLP), respectively. By sequentially applying AN, average pooling, and MLP we indeed compute the decision function in Eq. 7. Empirically we find that ReLU leads to better performance with faster convergence than other activation functions, and adding more "ReLU$\rightarrow$FC" layers into the architecture, however, does not increase accuracy necessarily or significantly, but leads to more computational burden.

We train the network using ADAM [19], a variant of SGD with adaptive learning rates, with mini-batches to learn both kernel weights and classifier parameters *simultaneously*, in contrast to the conventional MKL algorithms such as wrapper methods. LMKL-Net can achieve *weak* convergence

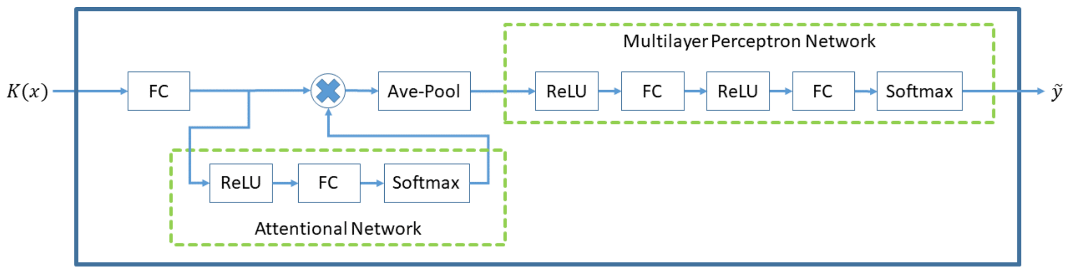


Figure 2: An equivalent architecture for accelerating training and inference of LMKL-Net by sharing weights.

in probability based on the analysis in [5] for nonconvex optimization using SGD. Generally speaking, the computational complexity of LMKL-Net is *linear* to the number of its parameters, the size of mini-batches, and the number of iterations.

Training & Inference Acceleration: In large-scale problems where the training samples are dominant, most of the computation for optimizing LMKL-Net in Fig. 1 are spent on the two FC layers in AN and the first FC layer in MLP. To accelerate both training and inference, we enforce that the first FC layers in both AN and MLP share the same weights. This transfers Fig. 1 to a new network architecture as illustrated in Fig. 2. Empirically we observe almost identical accuracy on different datasets using both architectures, while the new architecture is trained significantly faster. Therefore in our experiments we report our performance based on the architecture in Fig. 2.

Discussion: The idea of neural network based LMKL solver can be utilized for solving some other MKL learning problems. For instance, in order to solve SimpleMKL in Eq. 4 we can feed a constant vector into AN to learn β and use an FC layer with non-negative weights as a constraint instead of MLP to learn α. We observe that in terms of accuracy in practice this network is always worse than LMKL-Net. Sparse learning in kernel SVMs are useful in practice to reduce the size of kernel matrix per data. To mimic this nice property, we can add the group sparsity into the learning of the first FC layer in Fig. 2. However, the investigation on these specific network design choices is out of scope of this paper, and we will consider them in our future work.

4 Experiments

Implementation: In our experiments, we utilize the cross entropy loss because it performs superiorly than other losses and can handle multiclass classification problems inherently. We set the output dimension in the first FC layer in Fig. 2 to 256D, and keep using it in MLP. The output dimension for the FC layer in AN is N so that the entry-wise product operator can work. In our experiments using cross-validation we observe that higher dimensions than 256D do not necessarily lead to better accuracy but with remarkably longer running time, and the accuracy using lower dimensions starts to decrease. We initialize all FC layers randomly based on Gaussian distributions. We also observe that in our experiments having smaller weight decay [21] has little impact on the accuracy while larger weight decay worsens the performance significantly. Therefore we decide not to include weight decay in our experiments. We train the network for 200 epochs with batch size 256 and learning rate 0.001 for ADAM optimizer in DNN training. All these numbers are determined by cross-validation. We also utilize batch-normalization to accelerate the training. The results are reported based on 3 runs.

Benchmarks: For binary problems we test on 6 datasets, *i.e.* adult-8, news20, phishing, rcv1, real-sim, and w7a. For multiclass problems we test on 8 datasets, *i.e.* aloi, covtype, letter, protein, sensit (combined), sensorless, shuttle, and SVHN. We download all the datasets (scaled versions if available) from `https://www.csie.ntu.edu.tw/~cjlin/libsvmtools/datasets/` which also lists the statistical information of the datasets such as numbers of training/validation/test data, classes and dimension of features. Please refer to the website for more dataset details.

Considering the memory limit we intentionally control the number of data samples for generating kernels. Specifically in each dataset we randomly select 20K samples from training/validation data if the samples are more than 20K, otherwise we use all of them. Similarly for test data, we randomly select 10K samples if there are more, otherwise use all. Then by referring to VGG MKL dataset at `http://www.robots.ox.ac.uk/~vgg/software/MKL/` which consists of 10 Gaussian RBF kernels, we create 10 RBF kernels as well using the selected samples. We determine the window

	adult-8	news20	phishing	rcv1	real-sim	w7a	average
UNIFORM	81.94	93.33	46.16	96.37	96.56	90.37	84.12
SPG-GMKL	84.13	90.27	95.26	95.57	92.21	97.05	92.42
LMKL	78.09	**95.52**	52.04	**96.76**	**97.09**	97.75	86.21
Lp-MKL	76.33	-	-	-	-	97.05	-
OBSCURE	84.22±0.09	94.68±0.03	97.25±0.00	96.55±0.05	96.94±0.00	98.50±0.02	94.69
Ours	**84.62±0.15**	93.53±0.03	**98.17±0.25**	96.71±0.07	96.42±0.09	**98.74±0.03**	**94.70**

Table 1: Comparison on binary classification accuracy (%). "-" indicates that we cannot achieve reasonable performance on the dataset.

sizes in RBF-kernels to be proportional to the maximum Euclidean distance among the training features from 0.1 to 1 step by 0.1. We precompute all the kernels as the inputs for all the solvers.

To measure the classification performance, we utilize accuracy that is defined as the number of correctly classified samples divided by the total number of testing samples. The training and testing data samples have been balanced roughly.

Competitors: We compare LMKL-Net with some state-of-the-art MKL solvers with public code that can directly take kernels as input. We tune each solver so that we can report the best performance that we can achieve.

For binary classification, we test GMKL [45][1], SMO-MKL [47][1], SPG-GMKL [15][1], LMKL [10][2], Lp-MKL [20][3], UFO-MKL [37][4], OBSCURE [35][4], and UNIFORM [38] (*i.e.* average kernels with SVMs). We observe that GMKL, SMO-MKL, and SPG-GMKL always return identical results with different running speed, and thus report their results under the name of SPG-GMKL since it is fastest. Similarly for the two online learning methods, in our experiments OBSCURE outperforms UFO-MKL significantly in terms of both accuracy and running speed. Therefore, we only report the results of OBSCURE. We also observe that too much effort is needed such as heavily tuning parameters to make Lp-MKL work on our data and the results are often worse than others. For multiclass classification, we compare LMKL-Net with OBSCURE and UNIFORM because we find that most of existing code that we use cannot handle multiclass classification properly.

Note that among all the competitors OBSCURE is the most related to our LMKL-Net in terms of linear complexity in learning as well as its classification performance. OBSCURE utilizes group sparsity on classifier parameters (*i.e.* $\|\mathbf{w}\|_{2,p}, 1 < p \leq 2$) as regularization and proposes a two-stage online learning algorithm (first online then batch) to optimize the primal formulation with complicate calculation. In contrast, LMKL-Net is a network based solver that can be efficiently trained using SGD. This remarkable difference in optimization leads to the fact that empirically LMKL-Net can be trained significantly faster with much smaller memory demand.

4.1 Binary Classification

We first summarize our comparison results in Table 1. Overall our LMKL-Net outperforms all the other competitors, achieving the best in 3 out of 6 classes, while the other 3 best accuracies is obtained by LMKL. Compared with LMKL, however, LMKL-Net achieves **8.49%** improvement on average in terms of accuracy. OBSCURE performs very closely to LMKL-Net.

In order to understand LMKL-Net, we illustrate the loss and accuracy on training and test data in Fig. 3. We take for example adult-8, news20, and w7a. From the perspective of loss, we can observe clear overfitting in training on the first two datasets, as the testing loss increases with more epochs while the training loss decreases continuously. Surprisingly, we also observe that the quick and serious overfitting on adult-8 actually leads to very slow decrease in test accuracy, while on news20 such behavior is even hardly noticeable. This may explain why our solver performs worst on adult-8 among all the datasets. On the other hand, these observations also indicate the robustness of LMKL-Net in training and inference. In contrast, on w7a LMKL-Net is trained well, leading to the best performance among all the datasets.

[1]`http://www.cs.cornell.edu/~ashesh/pubs/code/SPG-GMKL/download.html`
[2]`http://users.ics.aalto.fi/gonen/icml08.php`
[3]`http://doc.ml.tu-berlin.de/nonsparse_mkl`
[4]`https://github.com/denizyuret/dogma/tree/master/demos`

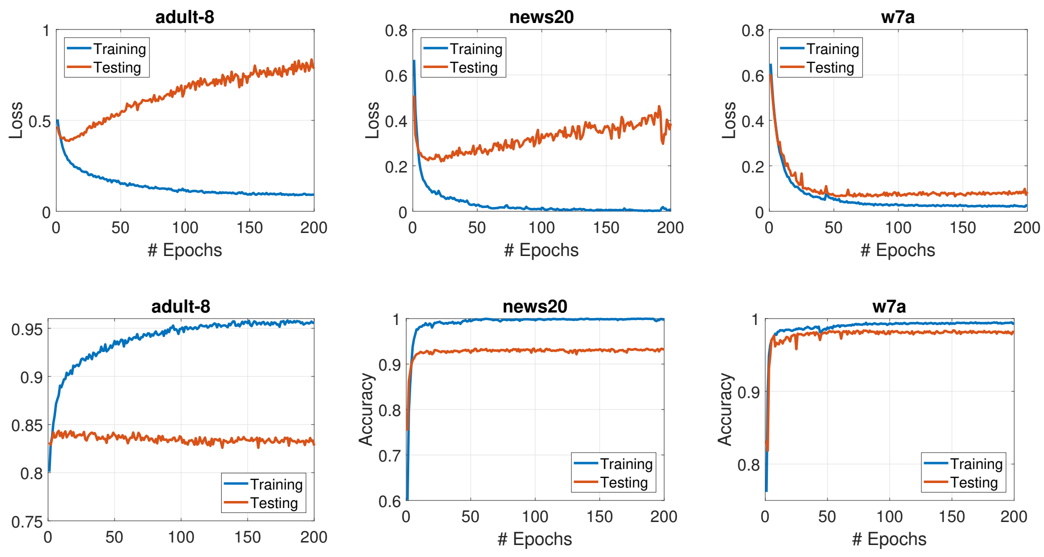
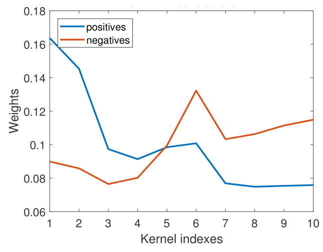
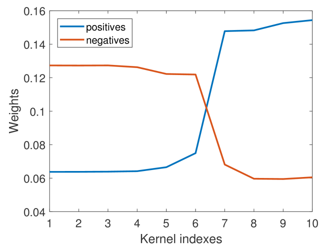


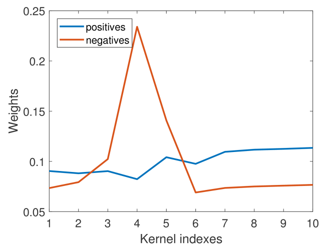
Figure 3: Illustration of training and testing **(Top)** loss and **(Bottom)** accuracy over epochs on different datasets.



(a) adult-8



(b) news20



(c) w7a

Figure 4: Illustration of learned mean kernel weights by averaging over the test data in each binary dataset.

We illustrate the kernel weights learned by LMKL-Net in Fig. 4 as well. To do so, we first marginalize $h(\mathbf{K}(z);\omega)$ in Eq. 8 for each sample z over the training samples $x_i, \forall i$ to compute an M-dim vector, and then compute the mean vectors over positive and negative samples in each test dataset, respectively. As we expect from the nature of data dependency, the patterns on each dataset are quite distinct. For instance, on news20 positives prefer the last four kernels while negatives prefer the rest. The variance in kernel weights is relatively small compared with the mean. Therefore for clarification we only show the mean values.

4.2 Multiclass Classification

We summarize the comparison results in Table 2. Again LMKL-Net performs the best overall with **1.11%** improvement over OBSCURE.

We also illustrate the training and testing behavior of our solver in Fig. 5. The observations here on loss and accuracy are quite similar to those in binary classification. Moreover it seems that the gaps between the training and testing curves for both loss and accuracy are consistent with each other. Namely, it is very likely that smaller gaps in loss will result in better training and testing accuracy, and vice versa.

	UNIFORM	OBSCURE	**Ours**
aloi	86.74	**89.60±0.14**	89.37±0.16
covtype	82.12	**83.56±0.07**	82.48±0.09
letter	97.68	97.70±0.02	**98.35±0.06**
protein	**70.24**	69.95±0.11	69.92±0.27
sensit	**85.20**	83.76±0.05	84.93±0.15
sensorless	98.77	99.23±0.02	**99.68±0.06**
shuttle	99.19	99.84±0.01	**99.87±0.01**
SVHN	59.73	55.73±0.10	**63.62±0.28**
average	84.96	84.92	**86.03**

Table 2: Comparison on multiclass classification (%).

We also test LMKL-Net on VGG MKL dataset as an example of using small-scale datasets. This dataset is created based on Caltech-101 where there are 101 object classes plus a background class.

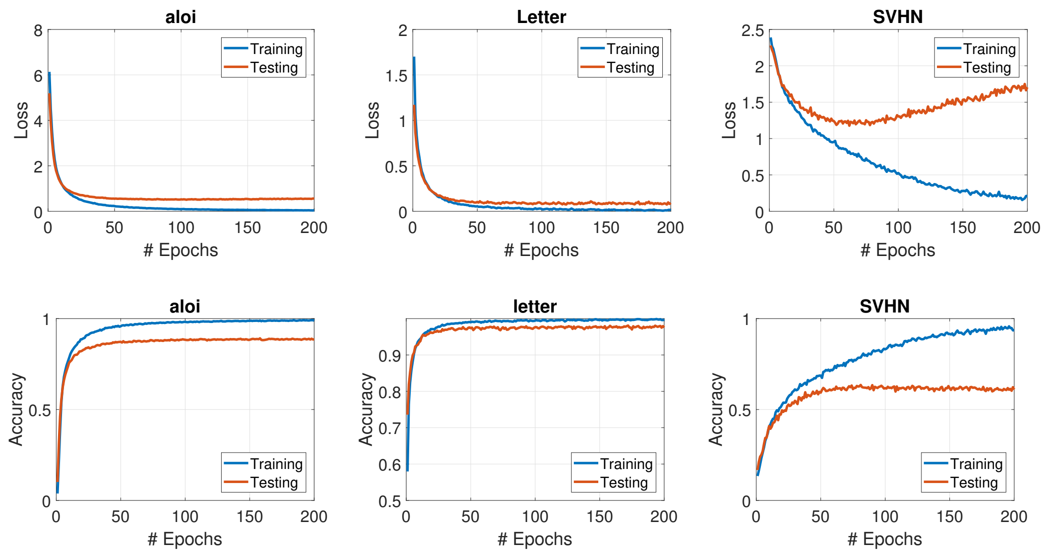


Figure 5: Illustration of **(top)** loss and **(bottom)** accuracy in multiclass classification.

In our experiments we utilize the setup where for each class 15 images are sampled as training data and another 15 images as test data. 10 kernels are created as input. On this dataset our solver achieves 75.2% with **0.6%** improvement over the best number shown on the web page.

4.3 Memory Footprint & Computational Time

Typically it needs about 20GB hard disk to store the precomputed kernels per dataset. Due to the batch size of 256, we need about $256 * 20/20000 = 0.26$GB memory to store the data for SGD training. Compared with the memory for loading all the precomputed kernels, the ratio will be, roughly speaking, $0.26/20 = \mathbf{0.013}$. Empirically Since we observe that even we project the input kernels to a lower dimensional space than 256D, our solver can still achieve similar performance. This indicates that the active memory or ratio can be much smaller dependent on the network architectures.

For computational time we take aloi dataset consisting of 1K classes as an example for large-scale learning. Our test machine has a GTX 1080 graphics card (GPU) and an i7-6850K@3.6GHz CPU. It takes more than 10 hours to learn OBSCURE with multi-threads on CPU. Our LMKL-Net is trained on GPU, and both need about 6s to traverse the dataset once (*i.e.* one epoch with about 20K samples). As we show in Fig. 3 and Fig. 5, our solver usually can converge empirically within 50 epochs. Based on these statistics, we can roughly compute the running time ratio as $6 * 50/(3600 * 10) = \mathbf{0.0083}$. On the other datasets we do observe that the training time of OBSCURE seems scalable with the number of classes given similar sizes of training data. For instance OBSCURE needs less than 5 minutes to be trained for binary classification (but still the fastest among the other solvers). In contrast, LMKL-Net has similar training speed across different datasets as long as the sizes of training data are roughly the same.

5 Conclusion

In this paper we propose a deep neural network, LMKL-Net, as an efficient solver for localized multiple kernel learning (LMKL) problems. LMKL-Net consists of two major components, *i.e.* an attentional network (AN) for learning the gating function and a multilayer perceptron (MLP) for learning a multiclass classifier in LMKL. We expect that the network can approximate the optimal functions in terms of accuracy given the input kernels. Empirically we demonstrate the performance of LMKL-Net on several benchmark datasets and compare it with some state-of-the-art MKL solvers. Overall LMKL-Net outperforms its competitors for both binary and multiclass classification. The robustness in training speed and the characteristic of SGD differentiate LMKL-Net from the other solvers, leading to about two orders of magnitude faster with much smaller memory footprint for large-scale learning.

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