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Pajovic, M.; Orlik, P.V.

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Abstract

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Reduced-Dimension Symbol Detection in Random Access Channel

Milutin Pajovic and Philip V. Orlik
Mitsubishi Electric Research Laboratories (MERL)
Cambridge, MA, 02139, USA
Email: pajovic@merl.com

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***Index Terms*—Random access channel, Internet of things (IoT), multi-user detection, reduced-dimension processing.**

I. Introduction

In emerging IoT applications, a number of IoT users communicate with a base station/access point over a shared channel [1]. Due to a large number of users, system designer is prevented from assigning them orthogonal channel resources, such as orthogonal spreading codes. In addition, employing network management and control is impractical because it would incur an excessive communication overhead and latency [2]. Consequently, symbols transmitted by active users collide and need to be separated in the receiver.

Essentially, the described setup resembles that of a multi-user detection (MUD), extensively studied in the literature [3]. However, in contrast to a conventional MUD, only a small number of users in the IoT random access channel setup transmit at the same time, so that only few symbols need to be separated at receiver in a given signaling interval. This fact has been utilized in a number of works to facilitate the formulation of the separation problem as a sparse recovery problem [4], whose solution results in the detection of active users and their transmitted symbols. As such, assuming users are time-synchronized, [5]–[7] separates collided transmissions using element-wise sparse recovery, while [8] recovers transmitted messages using group-wise sparsity. Similarly, separation of time-asynchronous transmissions is considered in [9]. Separation algorithms tailored for networks of smart-meters are proposed in [10], [11]. The case when transmitted symbols are not subject to spreading codes and transmitted packets experience delays and frequency offsets is studied in [12].

Although users in random access channel are assigned non-orthogonal spreading codes, the inherent sparsity in the system, arising from the fact that a user is active with low probability, permits separation of the transmitted messages. However, a closer examination of the proposed separation algorithms reveals that active user detection comprises of cross-correlating the received signal with spreading codes of all users. Equivalently, the number of filters in the receiver filter bank is equal to the overall number of users, which is also the case in a conventional MUD. To overcome this issue, [13] exploits sparsity in the user activity domain and proposes a reduced-dimension processor, with smaller number of filters compared to the number of users.

This paper considers random access channel setup and focuses on reduced-dimension processing of the received signal which facilitates low complexity separation of collided symbols. In particular, we show that the reduced-dimension processor proposed in [13] is essentially based on the zero-forcing (ZF) algorithm and, consequently, propose analogous reduced-dimension processors based on the matched filter (MF) and minimum mean squared error (MMSE) algorithms. As in [13], the pre-whitened versions of the proposed processors are also considered, as well as different ways for the design of a dimensionality reduction transformation matrix. Furthermore, building upon [10], we propose a scheme which aims to remove an impractical requirement that the receiver knows channels from all users, and study it in conjunction with the reduced-dimension processors. Finally, the proposed scheme and processors are validated using simulations.

II. Signal Model

We assume a synchronous multiuser system with N users communicating with the same base station. Each user is equipped with a spreading code $s_n(t)$ acting upon the transmitted symbol, so that $0 \leq t \leq T$, where T is the symbol duration. The signal received at the base station is given by

$$y(t) = \sum_{n=1}^{N} h_n b_n s_n(t) + w(t), 0 \leq t \leq T, \tag{1}$$

where $w(t) \sim \mathcal{CN}(0, \sigma^2)$ is additive white Gaussian noise (AWGN), $h_n \in$ is channel realization between user n and the base station, while b_n is transmitted symbol of user n at the considered signaling interval. Without loss of generality, we assume the users employ binary phase shift keying (BPSK) modulation.

Although the received signal $y(t)$ is conventionally processed through an analog filter bank at the receiver's front end, we stick to discrete-time domain representation for simplicity in exposition. After discretization, where the sampling frequency is assumed equal to the chip rate of $s_n(t)$, signal model (1) is expressed in vector form

$$\mathbf{y} = \mathbf{SHb} + \mathbf{w}, \tag{2}$$

where $\mathbf{S}$ is the matrix of spreading codes whose n-th column is the discretized spreading code $s_n(t)$, denoted $\mathbf{s}_n$, $\mathbf{H} = \text{diag}\,(h_1, \ldots, h_N)$ is the channel matrix, while $\mathbf{b}$ is the vector of transmitted symbols whose n-th entry is the transmitted symbol from user n. Finally, $\mathbf{w}$ is the AWGN vector with $\mathbf{w} \sim \mathcal{CN}(\mathbf{0}, \sigma^2 \mathbf{I}_N)$, where $\mathbf{I}_N$ is order-N identity matrix.

A conventional MUD approach for detecting transmitted symbols is to process the received signal $y(t)$ (after standard pre-processing steps) through a matched filter bank (MFB) where the template of each matched filter is spreading code $s_n(t), n = 1, \ldots, N$. The outputs from the MFB comprise sufficient statistics for the detection of transmitted symbols in the considered signaling interval. Sticking to the discrete-time domain for simplicity, we note that the MFB applies transformation $\mathbf{S}^H$ on the received signal $\mathbf{y}$ so that the vector of the MFB outputs is given by

$$\mathbf{r}_{\text{MFB}} = \mathbf{S}^H \mathbf{SHb} + \mathbf{S}^H \mathbf{w} = \mathbf{GHb} + \mathbf{w}_1, \tag{3}$$

where $\mathbf{G} = \mathbf{S}^H \mathbf{S}$ encodes the cross-correlations of spreading codes, i.e., the (i, j)-th entry in $\mathbf{G}$ is $[\mathbf{G}]_{i,j} = \mathbf{s}_i^H \mathbf{s}_j$. The MFB output noise is $\mathbf{w}_1 \sim \mathcal{CN}(\mathbf{0}, \sigma^2 \mathbf{G})$. The transmitted symbols are the detected from $\mathbf{r}_{\text{MFB}}$.

III. Reduced Dimension Processors

In the problem setting considered here, only a small number of users $K \ll N$ transmit during a signaling period, while other users are inactive and assumed to transmit symbol 0. This is described with models (1) and (2) and the extended symbol alphabet, consisting of the original symbol alphabet and zero, i.e., in case of BPSK modulation, $b_n \in \{+1, -1, 0\}$.

Since only a small fraction of users transmit during a signaling interval, the problem is inherently sparse. In other words, the vector of transmitted symbols $\mathbf{b}$ is sparse. The sparsity can be exploited by reducing the number of filters in the MFB at the expense of possible performance deterioration. Thus, an aim is to design a reduced dimension processor consisting of M filters with templates $f_m, m = 1, \ldots, M$, where $M < N$ and preferably, $M \ll N$.

A. Reduced Dimension Processor from [13]

The correlating signals $\{f_m\}_{m=1}^{M}$ for the reduced-dimension processor (RDP) are designed in [13] using the concept of bi-orthogonality among signals. A set of signals bi-orthogonal to spreading codes $\{s_n(t)\}_{n=1}^{N}$ is defined as

$$\tilde{s}_n(t) = \sum_{l=1}^{N} \left[\mathbf{G}^{-1}\right]_{nl} s_l(t), \; n = 1, \ldots, N. \tag{4}$$

One may note that any signal from $\{s_n(t)\}_{n=1}^{N}$ is orthogonal to any signal from $\{\tilde{s}_n(t)\}_{n=1}^{N}$. Also, when spreading codes are orthogonal, i.e., $\mathbf{G} = \mathbf{I}$, $\tilde{s}_n(t) = s_n(t)$.

Given the set of bi-orthogonal signals $\{\tilde{s}_n(t)\}_{n=1}^{N}$, the correlating signals $\{f_m(t)\}_{m=1}^{M}$ for the RDP are defined as

$$f_m(t) = \sum_{n=1}^{N} a_{mn} \tilde{s}_n(t), \; m = 1, \ldots, M, \tag{5}$$

where a_{mn} are weighting coefficients. The matrix of weighting coefficients $\mathbf{A} \in {}^{M \times N}$ is constrained to have unit norm columns. Essentially, the design of the RDP boils down to designing $\mathbf{A} \in {}^{M \times N}$.

Combining (4) and (5), and filtering the received signal $\mathbf{y}$ from (2) through the resulting processor, yields after some relatively simple algebraic manipulations the output

$$\mathbf{r} = \mathbf{AHb} + \mathbf{w}', \tag{6}$$

where the colored noise $\mathbf{w}' \sim \mathcal{CN}(\mathbf{0}, \sigma^2 \mathbf{A} \mathbf{G}^{-1} \mathbf{A}^H)$.

B. Detection of Active Users and Symbols

The output from the RDP, such as one in (6), is used to detect active users and their transmitted symbols. A variety of algorithms from the literature can be adopted for this task. Two algorithms from [14] and [4], and considered in [13], are briefly described here and used in the rest of the paper. We assume the output $\mathbf{r}$ from a reduced-dimension processor (not necessarily (6)) is given by

$$\mathbf{r} = \mathbf{Cb} + \mathbf{v}, \tag{7}$$

where $\mathbf{C}$ is a mixing (i.e., sensing) matrix, $\mathbf{b}$ is a sparse vector of transmitted symbols, and $\mathbf{v}$ is noise. In case of the RDP described in Section III-A, $\mathbf{C} = \mathbf{AH}$ and $\mathbf{v} = \mathbf{w}'$.

The reduced dimension decorrelating (RDD) algorithm cross-correlates each column in $\mathbf{C}$ with the RDP output $\mathbf{r}$,

$$t_n = \left| \Re \left[\mathbf{c}_n^H \mathbf{r} \right] \right|, \tag{8}$$

where $\mathbf{c}_n$ is the n-th column of $\mathbf{C}$. Assuming the number of active users K is known, the active users are detected from the indices of the K largest statistics $\{t_n\}_{n=1}^{N}$. The BPSK symbols of the detected active users are recovered as

$$\hat{b}_n = \text{sgn} \left(r_n \Re \left[\mathbf{c}_n^H \mathbf{r} \right] \right), \tag{9}$$

where the sign operator, sgn{}, is in accordance with the real-valued symbol alphabet. Overall, the RDD algorithm is a one-shot detector of active users and their symbols.

The reduced dimension decision feedback (RDDF) algorithm is an iterative procedure which follows the principles of the successive interference cancellation (SIC), where one active user and its symbol are recovered in each iteration. Specifically, an active user in the k-th iteration is detected as

$$n_k = \arg\max_n \left|\Re\left[\mathbf{c}_n^H \mathbf{v}^{k-1}\right]\right|, \tag{10}$$

where $\mathbf{v}^{k-1}$ is the residual evaluated in the previous iteration. Note that $\mathbf{v}^{(0)} = \mathbf{r}$. The transmitted symbol of the k-th active user is detected as

$$b_n^{(k)} = \text{sgn}\left(r_{n_k}\Re\left[\mathbf{c}_{n_k}^H \mathbf{r}\right]\right) \tag{11}$$

Finally, the residual is updated as

$$\mathbf{v}^k = \mathbf{y} - \mathbf{C}\mathbf{b}^{(k)}, \tag{12}$$

where $\mathbf{b}^{(k)}$ is the vector whose entries are the estimated symbols of the users recovered up to and including iteration k. The entries corresponding to all other users are 0.

We note that the RDD and RDDF require the number of active users K as an input. In case K is unknown, ad-hoc methods can be employed to estimate K. In the sequel, we assume K is known.

C. Proposed Reduced-Dimension Processors

We discretize the bi-orthogonal signals $\tilde{s}_n(t)$ from (4) and correlating signals $\{f_m(t)\}_{m=1}^M$ from (5), and denote their vector representations with $\tilde{\mathbf{s}}_n$ and $\mathbf{f}_m$, respectively. Denoting with $\mathbf{F}$ the transformation matrix of the RDP whose columns are $\mathbf{f}_m$, $m = 1, \dots, M$, we observe that

$$\mathbf{F} = \mathbf{A}\mathbf{G}^{-1}\mathbf{S}^H \tag{13}$$

As a sanity check, processing the received signal $\mathbf{y}$ from (2) through $\mathbf{F}$ yields (6). A careful examination of (13) reveals that the RDP $\mathbf{F}$ is essentially zero-forcing (ZF) processor, cascaded with the dimension-reduction processor represented with the transformation matrix $\mathbf{A}$. Thus, we refer to this processor as the reduced-dimension zero-forcing (RD-ZF) processor (even though [13] refers to is as the RD-MFB). To distinguish it from other processors proposed in the sequel, we denote $\mathbf{F}_{\text{RD-ZF}} \triangleq \mathbf{F}$, $\mathbf{r}_{\text{RD-ZF}} \triangleq \mathbf{r}$, $\mathbf{C}_{\text{RD-ZF}} = \mathbf{AH}$ and the covariance matrix of the resulting colored noise is $\mathbf{\Sigma}_{\text{RD-ZF}} = \sigma^2\mathbf{A}\mathbf{G}^{-1}\mathbf{A}^H$.

Motivated by the above observation, we propose to use other types of linear processors in conjunction with the dimensionality reduction transformation $\mathbf{A}$. As such, the combination of the matched filter with the dimensionality reduction matrix $\mathbf{A}$, referred to as the reduced-dimension matched filter (RD-MF), is defined as

$$\mathbf{F}_{\text{RD-MF}} = \mathbf{A}\mathbf{S}^H. \tag{14}$$

The signal at the RD-MF output is obtained from (2) and (14), and is given by

$$\begin{aligned} \mathbf{r}_{\text{RD-MF}} = \mathbf{F}_{\text{RD-MF}}\mathbf{y} &= \mathbf{AGHb} + \mathbf{w}_{\text{RD-MF}} \\ &= \mathbf{C}_{\text{RD-MF}}\mathbf{b} + \mathbf{w}_{\text{RD-MF}}, \end{aligned} \tag{15}$$

where $\mathbf{C}_{\text{RD-MF}} = \mathbf{AGH}$ and $\mathbf{w}_{\text{MF}} \sim \mathcal{CN}(\mathbf{0}, \mathbf{\Sigma}_{\text{RD-MF}})$, with $\mathbf{\Sigma}_{\text{RD-MF}} = \sigma^2\mathbf{AGA}^H$.

Similarly, the reduced-dimension minimum mean square error (RD-MMSE) processor is obtained by cascading the MMSE filter with the dimensionality reduction transformation $\mathbf{A}$, and is given by

$$\mathbf{F}_{\text{RD-MMSE}} = \mathbf{A}\left(\mathbf{G} + \sigma^2\mathbf{I}\right)^{-1}\mathbf{S}^H \tag{16}$$

The RD-MMSE output signal is using (2) and (16) given by

$$\begin{aligned} \mathbf{r}_{\text{RD-MMSE}} &= \mathbf{A}\left(\mathbf{G} + \sigma^2\mathbf{I}\right)^{-1}\mathbf{GHb} + \mathbf{w}_{\text{RD-MMSE}} \\ &= \mathbf{C}_{\text{RD-MMSE}}\mathbf{b} + \mathbf{w}_{\text{RD-MMSE}} \end{aligned} \tag{17}$$

where $\mathbf{C}_{\text{RD-MMSE}} = \mathbf{A}\left(\mathbf{G} + \sigma^2\mathbf{I}\right)^{-1}\mathbf{GH}$ and $\mathbf{w}_{\text{RD-MMSE}} \sim \mathcal{CN}(\mathbf{0}, \mathbf{\Sigma}_{\text{RD-MMSE}})$ with

$$\mathbf{\Sigma}_{\text{RD-MMSE}} = \sigma^2\mathbf{A}\left(\mathbf{G} + \sigma^2\mathbf{I}\right)^{-1}\mathbf{G}\left(\mathbf{G} + \sigma^2\mathbf{I}\right)^{-1}\mathbf{A}^H. \tag{18}$$

We recall that the goal is to detect K active users and their symbols. To reduce the computational complexity arising from using conventional, fully-fledged processors consisting of N filters (one for each user), the received signal is processed through a processor consisting of M filters. Mathematically, the received signal is projected into a lower, M-dimensional subspace by applying $\mathbf{A}$ to one of the conventional processors. The detection of active users and their symbols in the reduced-dimension space is possible because only a small number of users transmit at a given signaling interval, thereby giving rise to sparsity in the user activity domain.

In our study, we adopt the RDD and RDDF algorithms for detecting active users and their transmitted symbols from the RD-ZF, RD-MF and RD-MMSE outputs.

D. Pre-whitened Reduced Dimension Processors

Notably, the noise processes at the RDP outputs are Gaussian distributed, of zero mean, but non-identity covariance matrix. Since the RDD and RDDF detection algorithms do not take noise statistics into account, thereby inherently assuming white noise, we consider pre-whitening the received signal $\mathbf{y}$ so that the output noise from the RDP is white.

For a general RDP processor $\mathbf{F}$ which gives rise to the output noise correlation matrix $\mathbf{\Sigma}$, the pre-whitened RDP, $\tilde{\mathbf{F}}$, is obtained as

$$\tilde{\mathbf{F}} = \mathbf{\Sigma}^{-\frac{1}{2}}\mathbf{F} \tag{19}$$

Consequently, the resulting mixing matrix $\tilde{\mathbf{C}}$ is using (2) given by

$$\tilde{\mathbf{C}} = \tilde{\mathbf{F}}\mathbf{SH} \tag{20}$$

such that the output from the pre-whitened RDP is

$$\tilde{\mathbf{r}} = \tilde{\mathbf{C}}\mathbf{b} + \tilde{\mathbf{w}}, \tag{21}$$

where $\tilde{\mathbf{w}} \sim \mathcal{CN}(0, \mathbf{I})$.

Using for $\mathbf{F}$ in (19) one of the considered $\mathbf{F}_{\text{RD-ZF}}$, $\mathbf{F}_{\text{RD-MF}}$ or $\mathbf{F}_{\text{RD-MMSE}}$, and consequently for $\mathbf{\Sigma}$, $\mathbf{\Sigma}_{\text{RD-ZF}}$, $\mathbf{\Sigma}_{\text{RD-MF}}$ or $\mathbf{\Sigma}_{\text{RD-MMSE}}$ yields pre-whitened RD-ZF, RD-MF or RD-MMSE, respectively. The RDD and RDDF detection algorithms, as described in Section III-B, are applied to model (21) with the corresponding $\tilde{\mathbf{C}}$, depending on the type of the employed processor.

E. Selection of Dimensionality Reduction Matrix $\mathbf{A}$

Each of the reduced dimension processors from Section III-C relies on the dimensionality reduction transformation $\mathbf{A}$ to effectively project the detection problem into a lower dimensional space. The detection of active users and their symbols is essentially a sparse recovery problem. The separability of active users in the reduced dimension space directly depends on the coherence properties of the sensing matrix $\mathbf{C}$ ($\mathbf{C}_{\text{RD-ZF}}$, $\mathbf{C}_{\text{RD-MF}}$ or $\mathbf{C}_{\text{RD-MMSE}}$, depending on the employed processor). We recall that the coherence of a matrix is defined as the largest cross-correlation between its different columns. Consequently, $\mathbf{A}$ should be selected so as to minimize the coherence of the sensing matrix $\mathbf{C}$. Importantly, the coherence of $\mathbf{C}$ does not depend on diagonal channel matrix $\mathbf{H}$, so that $\mathbf{A}$ can be pre-computed offline without knowing $\mathbf{H}$.

An ad-hoc design method for selecting $\mathbf{A}$, used in [13], is to randomly sample a number of candidates for $\mathbf{A}$ from some pre-fixed domain and choose the one with the smallest coherence. A pre-fixed domain can be a set of random matrices with i.i.d. $\mathcal{CN}(0,1)$ entries, with columns normalized to unit norm. Another choice can be a set of partial discrete Fourier transform (DFT) matrices, generated by selecting uniformly at random M rows from the discrete Fourier transform (DFT) matrix of order N.

Aside from selecting $\mathbf{A}$ which has the smallest coherence among a number of randomly generated candidates, $\mathbf{A}$ can also be selected so that the sensing matrix $\mathbf{C}$ (corresponding to the considered processor) has the smallest coherence. More specifically, in case of the RD-MF processor, we select $\mathbf{A}$ with the objective to minimize the coherence of $\mathbf{C}_{\text{RD-MF}}$; more precisely, of $\mathbf{AG}$ because $\mathbf{H}$ is diagonal and, in general, unknown in advance. Similarly, in case of the RD-MMSE processor, $\mathbf{A}$ can be selected so as to minimize the coherence of $\mathbf{A}\left(\mathbf{G}+\sigma^2\mathbf{I}\right)^{-1}\mathbf{G}$.

In each of the above cases, if pre-whitened processor from Section III-D is used, $\mathbf{A}$ can be selected so that the coherence of the resulting pre-whitened processor $\tilde{\mathbf{C}}$ is minimized.

In the simulation part, we numerically study different approaches in selecting dimensionality reduction transformation $\mathbf{A}$ for all considered RDPs.

IV. Overcoming Unknown CSI at Receiver

The implicit assumption in the development so far is that the uplink channels from all users are known at the receiver. This is, in fact, a common assumption in the random access channel setup. However, little or no progress has been made in designing a protocol which would effectively enable receiver to learn channels from all users. The main challenge lies in having a large number of users assigned to the same base station so that estimating all channels with conventional techniques where network control coordinates the orthogonal transmission of pilots from each of the users to the base station is impractical. This problem is even more exacerbated when channels vary in time.

A more practical scheme, proposed in [10], assumes time domain duplex (TDD). The base station broadcasts pilot symbols that facilitate downlink channel estimation at each user in parallel. Each user then precodes its symbols based on the estimated channel by employing the zero-forcing (ZF) precoder. Due to the TDD and ZF precoding, the received signal at the base station does not explicitly depend on the channel coefficients. However, one issue with this approach is that if a certain channel is low in magnitude, the ZF precoded signal has a relatively large power. While this issue does not pose a significant challenge in a network of smart meters [10], this is, in general, undesirable.

To overcome an issue arising from ZF precoding of transmitted symbols, we propose that users precode their symbols using a unit magnitude ZF precoder. That is, the precoder n is given by

$$p_n = \frac{h_n^*}{|h_n|}, \tag{22}$$

where h_n is the channel coefficient of user n. The vector of transmitted symbols from all users is therefore

$$\tilde{\mathbf{b}} = \mathbf{Pb}, \tag{23}$$

where $\mathbf{P} = \text{diag}\{p_1, \ldots, p_N\}$. Substituting $\tilde{\mathbf{b}}$ in place of $\mathbf{b}$ in (2) yields

$$\mathbf{y} = \mathbf{SHPb} + \mathbf{w}. \tag{24}$$

Finally, substituting (22) into (24) yields

$$\mathbf{y} = \mathbf{S}\tilde{\mathbf{H}}\mathbf{b} + \mathbf{w}, \tag{25}$$

where $\tilde{\mathbf{H}} = \text{diag}\{|h_1|, \ldots, |h_N|\}$, and hence the received signal only depends on channel magnitudes.

The received signal (25) can then be processed through any of the reduced dimension processors and all discussion from Section III applies here. In fact, a careful examination reveals that RDD and RDDF detection of active users and their transmitted symbols do not require knowledge of the channel magnitudes. Moreover, this scheme, where users apply unit magnitude ZF precoding on their symbols so that the detection is possible without channel state information at the receiver, is amenable to any constant modulus modulation format.

V. Simulation Results

The proposed reduced-dimension processors are validated with Monte-Carlo simulations. As in [13], the simulation scenario consists of $N = 100$ users, of which $K = 2$ are active at a given signaling interval. Also, the spreading codes assigned to users are not orthogonal and their cross-correlation matrix is given by

$$\mathbf{G} = \mathbf{U}\text{diag}\{1/400, 2/400, \ldots, 100/400\}\mathbf{U}^H, \tag{26}$$

where $\mathbf{U}$ is a randomly generated unitary matrix. An elaboration of this selection for $\mathbf{G}$ can be found in [13].

We perform 500,000 Monte-Carlo runs, randomized over indices of $K = 2$ active users, transmitted BPSK symbols, AWGN realizations and, where applicable, channel coefficients. The received signal is processed with the RD-ZF, RD-MF or RD-MMSE, where the RD-ZF is the benchmark. A successful detection is only declared when all active users and

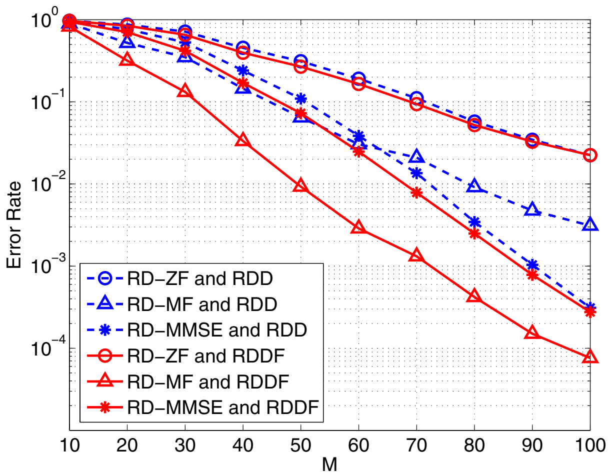

Fig. 1: Error rate versus M for the considered RDP's with RDD and RDDF detections.

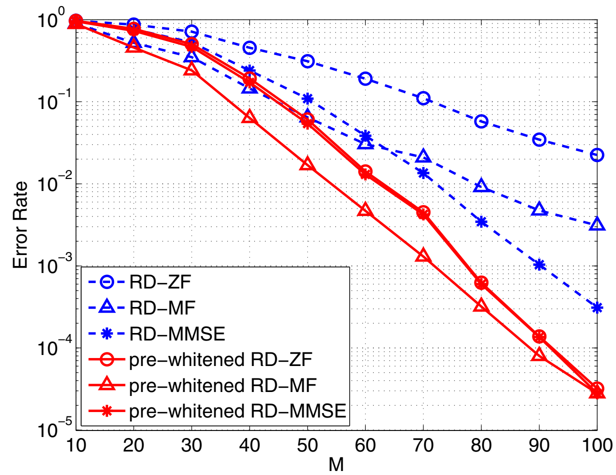

Fig. 2: Error rate versus M for the considered RDP's with and without pre-whitening and RDD detection.

their symbols are correctly detected. An error rate, defined as the ratio of unsuccessful runs to the overall number of runs is used. Given the number of runs, the smallest detectable error rate is 2×10^{-6}. The dimensionality M, i.e., the number of filters in the RDP is varied between 10 and 100, where $M=100$ corresponds to the conventional processor (ZF, MF or MMSE).

The dimensionality reduction matrix $\mathbf{A}$ is obtained by generating 100,000 realizations of partial DFT matrices, where M rows from the DFT matrix of size N are uniformly at random selected, and choosing the one with the most desirable coherence. Unless otherwise stated, the objective in the search is to minimize the coherence of $\mathbf{A}$.

A. Users' Channels Known at Receiver

In this scenario, all channels $h_n = 1, n = 1, \ldots, N$, and are perfectly known at the receiver side. The transmitted symbols have unit power and the received SNR is 23 dB. In all cases, RDD and RDDF algorithms, described in Section III-B, are used for detecting active users and their symbols.

1) Comparison of RDPs: The simulated error rates versus M for RD-ZF (benchmark), RD-MF and RD-MMSE with RDD and RDDF detection algorithms are shown in Fig. 1. As can be seen, both proposed processors outperform the RD-ZF. Also, the RDDF detection, resembling the SIC detection, tends to outperform the RDD detection, which is a one shot detection and has lower computational complexity. While the performance improvement of RDDF over RDD is insignificant for the RD-ZF and RD-MF, it is considerable for the RD-MMSE processor.

2) Pre-whitened Processors: The comparison between error rates attained using the considered processors with and without pre-whitening, in conjunction with the RDD detection is shown in Fig 2. A similar set of plots corresponding to the RDDF detection is shown in Fig. 3. As can be seen, the pre-whitening improves the performance of all considered processors uniformly over the simulated range of M.

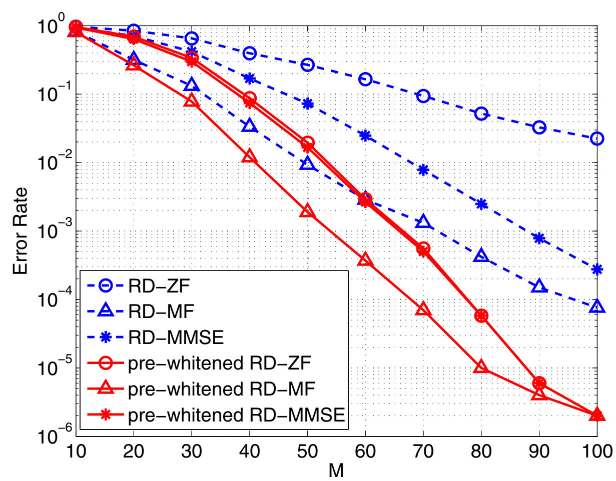

Fig. 3: Error rate versus M for the considered RDP's with and without pre-whitening and RDDF detection.

3) Optimization of $\mathbf{A}$*:* In all previous results, the selected $\mathbf{A}$ has the smallest coherence among 100,000 randomly generated partial DFT matrices. Here we investigate how the error rate performance changes if $\mathbf{A}$ is chosen so that the coherence of the overall mixing matrix, $\mathbf{C}$ or $\tilde{\mathbf{C}}$, is minimized.

The comparison between error rates when $\mathbf{A}$ is chosen so that its coherence is minimized (referred to as the partial optimization) the the coherence of the overall processor is minimized (referred to as the full optimization) is shown in Fig. 4 for RD-MMSE processor in conjunction with the RDD detection algorithm. As can be seen, minimizing the coherence of $\mathbf{C}$ instead of just $\mathbf{A}$ does not yield any error rate benefit. Similarly, when pre-whitened processor is used, minimizing the coherence of $\tilde{\mathbf{C}}$ instead of just $\mathbf{A}$ has no benefit. The same

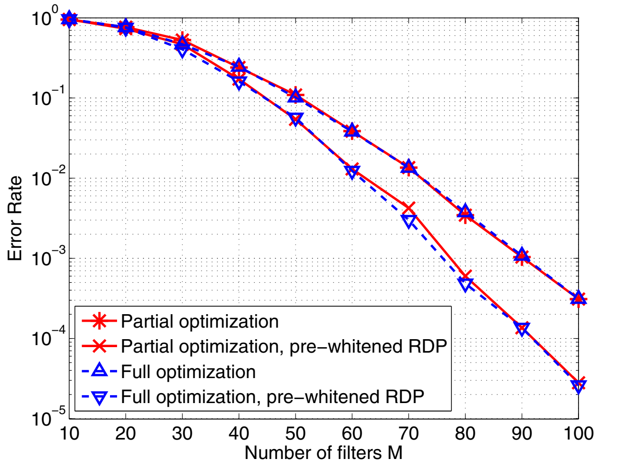

Fig. 4: Error rate versus M for RD-MMSE for partial and full optimizations of $\mathbf{A}$.

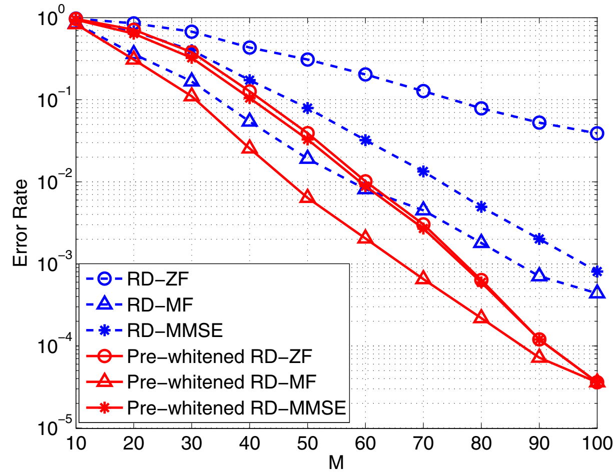

Fig. 5: Error rate versus M for unknown channels at receiver with RDDF detection.

qualitative results have been obtained for other two processors, with and without pre-whitening. We skip the presentation of the obtained plots.

B. Users' Channels Unknown at Receiver

In this simulation scenario, all channels are independent with uniformly distributed magnitudes, $|h_n| \sim \mathcal{U}[1, 1.5]$, and phases, $\arg\{h_n\} \sim \mathcal{U}[0, 2\pi]$. The channels are unknown at the receiver side. The transmitted symbols have unit power and the received SNR is 25 dB. We assume each user perfectly estimates its channel and applies the normalized ZF precoding, as described in Section IV. Upon filtering the received signal through one of the considered RDPs, the active users and their symbols are detected using the RDDF algorithm.

The error rate performance of the proposed scheme in conjunction with the considered RDPs, with and without pre-whitening is shown in Fig. 5. As before, the proposed RD-MF and RD-MMSE outperform the RD-ZF processor. Also, each pre-whitened processor outperforms its non-pre-whitened counterpart.

VI. Conclusion

This paper proposes two novel reduced-dimension processors to facilitate low complexity separation of collided symbols transmitted by active users in an IoT random access channel setting. The pre-whitened versions of the proposed processors which yield white noise output are also studied. In addition, a variety of approaches in the design of dimensionality reduction transformation matrix are considered. Furthermore, we propose a scheme aimed to overcome a common assumption used in the area of IoT random access channel that the receiver knows channels from all users it serves. The tests show that the proposed processors considerably outperform the benchmark.

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