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Abstract

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***Index Terms*— Source nonlinearity, frequency modulated continuous wave, automotive radar.**

I. Introduction

A frequency modulation continuous wave (FMCW) radar transmits linearly frequency-modulated continuous waves with a time-frequency pattern given by a saw tooth or triangular pattern [1]–[6]. Reflected signals from multiple reflectors of interest are mixed with local oscillator signals, generating digital beat signals with analog-to-digital converters (ADCs) at a low sampling rate. Since the peak frequency of beat signal is proportional to the distance of a reflector, the fast Fourier transform (FFT) of the beat signal can be used to identify the peak and estimate the distance. One issue here is that the range resolution degrades when the FMCW source is not completely linearly modulated. The source nonlinearity can be caused by open-loop voltage controlled oscillators (VCOs) and due to source sensitivity to temperature and vibrations.

The source nonlinearity can be compensated with *hardware* and *software* solutions. Hardware solutions include the use of a predistorted VCO control voltage to have a linear FM output and complex synthesizer concepts with phase-locked loop (PLL). However, this approach may fail when external conditions, e.g., temperature, are changed. The use of direct digital synthesizer offers a cost-effective solution, but the transmitted bandwidth is limited when compared to the one obtained by directly sweeping the VCO. Several local oscillators can be used to stitch a large bandwidth at the cost of increased system complexity. On the other hand, software solutions rely on a reference reflector at a known distance or a dedicated delay line to allow an initial estimation of the source nonlinearity function. With the estimated nonlinearity function, the nonlinearity effect can be compensated to the beat signal and then the range of reflectors can be recovered. Specifically, [3] and [4] proposed the residual-video-phase (RVP) correction algorithm to remove the nonlinearity effect over the entire range profile. However, when the source nonlinearity is time varying, this approach needs to repeat the reference step to update the estimation of source nonlinearity function [7].

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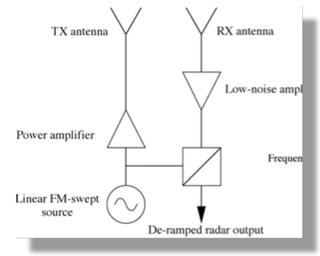


Fig. 1. FMCW-based automotive radar systems.

In this paper, we introduce a computational nonlinearity correction approach to jointly estimate range parameters of multiple reflectors and the nonlinearity function from the beat signal. The key assumption here is that the source nonlinearity function can be described by a transient parametric model over a short time interval. Given the parametric model for the nonlinearity function, the beat signal can be shown to be the sum of K responses characterized, in a nonlinear fashion, by K different range parameters and a shared set of parametric coefficients for the nonlinearity function. Therefore, by jointly estimating the two sets of unknown parameters from the beat signal, we can recover the range of multiple reflectors.

II. Signal Model

Consider an FMCW automotive radar system of Fig. 1 which transmits a unit-magnitude linear FM signal as

$$s_t(t) = e^{j2\pi(f_c t + 0.5\alpha t^2 + \epsilon(t))}, \tag{1}$$

where t is the time variable, f_c is the carrier frequency, α is the chirp rate, and $\epsilon(t)$ is the source nonlinearity function.

With K reflectors at distances of $R_1, \cdots, R_K$, the received signal is the sum of delayed and attenuated copies of the transmitted signal

$$\begin{aligned} s_r(t) &= \sum_{k=1}^{K} A_k s_t(t-\tau_k), \\ &= \sum_{k=1}^{K} A_k e^{j2\pi(f_c(t-\tau_k)+0.5\alpha(t-\tau_k)^2+\epsilon(t-\tau_k))}, \end{aligned} \tag{2}$$

where A_k is proportional to the reflectivity of the k-th reflector, and $\tau_k = 2R_k/c$ is the corresponding round-trip time delay.

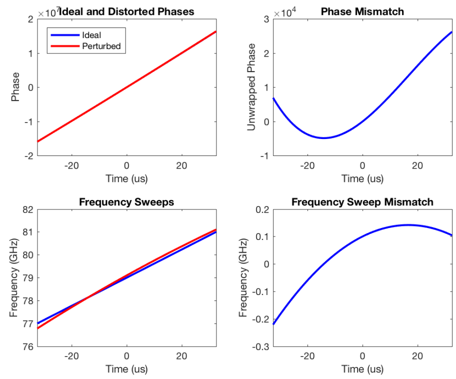


Fig. 2. A third-order polynomial function for source nonlinearity function and its effects on the phase and frequency domains.

At the receiver side, the received signal is then mixed with the transmitted signal to generate the beat signal,

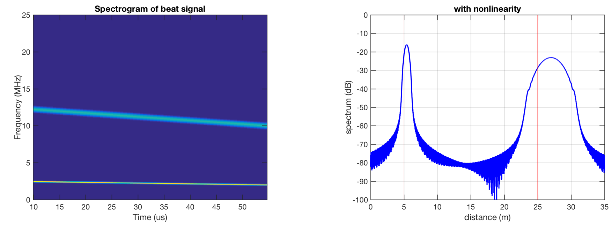
$$\begin{aligned} s_b(t) &= s_r(t)s_t^*(t) \\ &= \sum_{k=1}^{K} A_k e^{j2\pi(f_c\tau_k+\alpha\tau_k t-0.5\alpha\tau_k^2+(\epsilon(t)-\epsilon(t-\tau_k))}. \end{aligned} \quad (3)$$

With a perfect linear FM source, $\epsilon(t)-\epsilon(t-\tau)=0$ in (3) and the beat signal consists of multiple sinusoidal signals in t at frequencies $f_{b_k}=\alpha\tau_k$. On the other hand, when the source nonlinearity is present, the beat signal in (3) is no longer a sum of sinusoidal signals due to $\epsilon(t)-\epsilon(t-\tau)\neq 0$. The resulting phase distortion is *range dependent* as $\epsilon(t)-\epsilon(t-\tau_k)$ is now a function of the delay τ_k. As a result, the spectrum peak of beat signal can be shifted and spread, resulting in degradations in terms of the range resolution and signal-to-noise ratio (SNR). Fig.2 shows a third-order polynomial nonlinearity function and its impact in the phase and frequency domains. The problem of interest here is to estimate the delay parameters τ_k and hence the range parameters R_k when the source nonlinearity function $\epsilon(t)$ is present but unknown.

III. Proposed Nonlinearity Correction Algorithm for Range Estimation

As pointed out earlier, the reference-based approach needs to first estimate the source nonlinearity function $\epsilon(t)$ from the beat signal corresponding to a given reference, e.g., a delay line or a reflector at a known distance. [7] has used a parametric model, i.e., polynomial phase signal (PPS) model, to recover the nonlinearity source function from the response of a known reference.

In this section, we propose a computational nonlinearity correction algorithm without the need of the reference step. Still relying on a parametric modeling, i.e., the PPS model, the proposed algorithm aims to *jointly* estimate 1) the source nonlinearity function $\epsilon(t)$ and 2) range of multiple reflectors from the beat signal, as opposed to the two-step approach which *separately* estimates the two sets of unknown parameters using a reference [7]. The proposed algorithm exploits the fact that, given a parametric model for the nonlinearity function $\epsilon(t)$, the beat signal is the sum of K responses characterized by K



(a) Spectrogram of beat signal (b) Range estimation without nonlinearity correction

Fig. 3. The case of 2 targets at 5 and, respectively, 25 meters with the source nonlinearity given by the third-order polynomial phase function in Fig. 2. (a) the spectrogram of the beat signal shows two chirp signals described in (5); (b) the spectrum distortions for the two reflectors are different.

delay parameters $\{\tau_k\}_{k=1}^M$ and the *common* source nonlinearity function $\epsilon(t)$ parameterized by PPS coefficients.

To illustrate the proposed algorithm, we assume a simple third-order polynomial function for the source nonlinearity function[1]

$$\epsilon(t) = 2\pi(\beta_0+\beta_1 t+\beta_2 t^2/2+\beta_3 t^3/3!), \quad (4)$$

where $\{\beta_p\}_{p=0}^3$ are unknown model coefficients. Replacing $\epsilon(t)$ in (3) with the above parametric model, we have

$$s_b(t) = \sum_{k=1}^{K} \tilde{A}_k e^{j2\pi\left[(\alpha\tau_k+\beta_2\tau_k-0.5\beta_3\tau_k^2)t+0.5\beta_3\tau_k t^2\right]} \quad (5)$$

where $\tilde{A}_k = A_k e^{j2\pi\left[f_c\tau_k-0.5\alpha\tau_k^2+\beta_1\tau_k-0.5\beta_2\tau_k^2+\beta_3\tau_k^3/3!\right]}$ absorbs time-independent terms. It is seen from (5) that the resulting beat signal $s_b(t)$ from K reflectors is a K-component chirp (or linear FM) signal with each component characterized by a weighted complex amplitude $\tilde{A}_k$, a center frequency at $(\alpha\tau_k+\beta_2\tau_k-0.5\beta_3\tau_k^2)$, and a chirp rate of $\beta_3\tau_k$. Fig. 3 (a) shows the spectrogram of the beat signal corresponding to two reflectors at 5 and 25 meters away from the source with the source nonlinearity given by Fig. 2. As shown in (5), the two reflectors result in a beat signal consisting of two chirp signals with the chirp rate determined by the highest polynomial coefficient β_3 and the two delays $\{\tau_1,\tau_2\}$. This is seen in Fig. 3 (a) that the two chirp components in the beat signal lead to two straightline ridges in the time-frequency domain. Fig.3 (b) shows the conventional FFT-based range estimation without any nonlinearity correction. It is seen that the two spectrum peaks are shifted and spread around the true distances. More importantly, the spectral distortion is more significant to the reflector at 25 meters than the other one at 5 meters due to the range-dependent phase distortion.

Based on the parametric model of (5), robust multi-component chirp parameter estimation algorithms, e.g., [8], can be applied to the beat signal $s_b(t)$ to estimate unknown parameters of these K chirp components. Denote the following chirp parameter estimates from a multi-component chirp parameter estimation algorithm

$$\begin{aligned} \hat{a}_k &= (\alpha+\beta_2)\tau_k - 0.5\beta_3\tau_k^2, \\ \hat{b}_k &= 0.5\beta_3\tau_k, \quad k=1,\cdots,K \end{aligned} \quad (6)$$

[1]Due to the space limitation, the generalization to an arbitrary order will be included in a future submission.

Given these K pairs of chirp parameters $\{\hat{a}_k, \hat{b}_k\}$, we can recover K range parameters $\boldsymbol{\tau} = [\tau_1, \cdots, \tau_K]^T$ and the nonlinearity model coefficients $\{\beta_p\}_{p=1}^{3}$ as follows. First, group all K pairs of chirp parameter estimates as $\hat{\mathbf{a}} = [\hat{a}_1, \cdots, \hat{a}_K]^T$ and $\hat{\mathbf{b}} = [\hat{b}_1, \cdots, \hat{b}_K]^T$. Then (6) can be rewritten as

$$\hat{\mathbf{a}} = \frac{2(\alpha + \beta_2)}{\beta_3}\hat{\mathbf{b}} - \frac{2}{\beta_3}(\hat{\mathbf{b}} \odot \hat{\mathbf{b}}) \triangleq \gamma_1 \hat{\mathbf{b}} + \gamma_2 (\hat{\mathbf{b}} \odot \hat{\mathbf{b}}) \quad (7)$$

where $\odot$ denotes the element-wise Hadamard product. Then $\boldsymbol{\gamma} = [\gamma_1, \gamma_2]^T$ is estimated as

$$\hat{\boldsymbol{\gamma}} = [\hat{\gamma}_1, \hat{\gamma}_2]^T = (\mathbf{B}^T\mathbf{B})^{-1}\mathbf{B}^T\hat{\mathbf{a}} \quad (8)$$

with $\mathbf{B} = [\hat{\mathbf{b}}, \hat{\mathbf{b}} \odot \hat{\mathbf{b}}]$. Therefore, we can estimate $(\alpha + \beta_2)$ and β_3 as

$$\alpha + \hat{\beta}_2 = -\hat{\gamma}_1 \hat{\gamma}_2^{-1}, \quad \hat{\beta}_3 = -2\hat{\gamma}_2^{-1} \quad (9)$$

and the range parameter $\boldsymbol{\tau}$ can be estimated as

$$\hat{\boldsymbol{\tau}} = 2\hat{\beta}_3^{-1}\hat{\mathbf{b}} = -4\hat{\gamma}_2\hat{\mathbf{b}}. \quad (10)$$

IV. Numerical Results

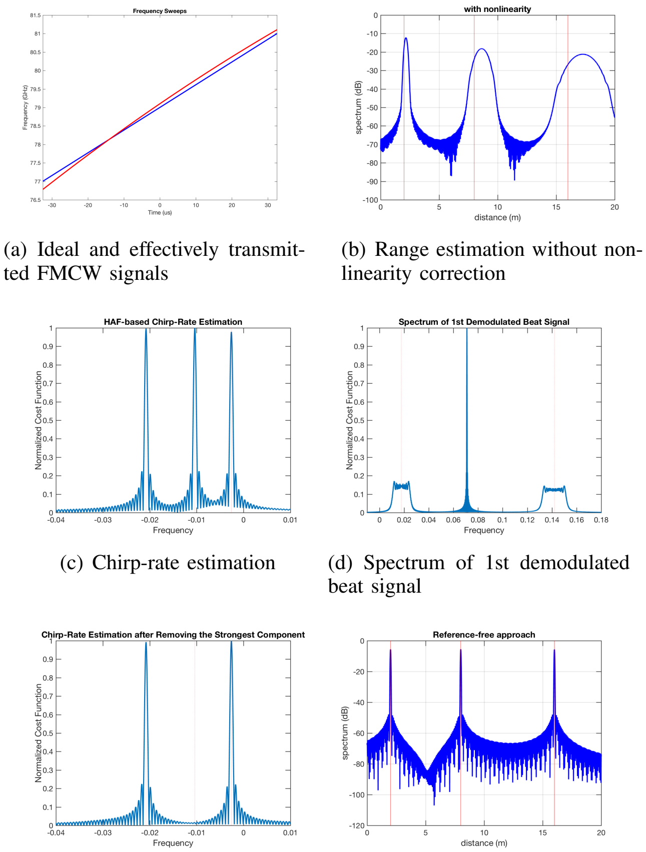
In this section, we provide a preliminary numerical validation of the proposed nonlinearity correction algorithm. Specifically, an FMCW automotive radar system is simulated with a center frequency at 79 GHz, a bandwidth of 4 GHz, a pulse duration of 65 μs, and a sampling frequency of 50 MHz. Three reflectors are present at 2, 8 and 16 m aways from the source. Fig.4 (a) shows the time-frequency spectrum of ideal and effectively transmitted FMCW signal. It is seen that the simulated source nonlinearity causes larger frequency deviations at the beginning and end of the chirp duration. The source nonlinearity function is generated using a third-order ($P = 3$) polynomial function. Fig.4 (b) shows the range estimation result without any source nonlinearity correction.

Fig. 4 (c) shows the high-order ambiguity function (HAF)-based chirp-rate estimation spectrum of the beat signal. Three dominant peaks are seen with locations given by the chirp rate parameters $\{0.5\beta_3\tau_k\}_{k=1}^{3}$ of the three components. Fig. 4 (d) shows the spectrum for estimating the center frequency $(\alpha\tau_k + \beta_2\tau_k - 0.5\beta_3\tau_k^2)$ of the strongest component after the demodulation using the estimated chirp rate parameter.

Fig. 4 (e) shows the HAF-based chirp-rate estimation spectrum after removing the strongest component. It is seen that the strongest component is mostly removed and the chirp-rate parameter of the second strongest component is estimated. Finally, Fig. 4 (f) shows the recovered range estimation spectrum by using the proposed approach which uses the first-order and second-order phase parameters of the chirp components to recover the three delay parameters $\{\tau_k\}_{k=1}^{3}$.

V. Conclusion

In this paper, we introduced a reference-free computational nonlinearity correction algorithm for the FMCW-based ranging system. It deals with the range-dependent nonlinearity-induced distortion by using a joint estimation algorithm of the range parameter and source nonlinearity coefficients. The proposed algorithm imposes a parametric model to describe the source nonlinearity function and recovers the range and parametric coefficients directly from the beat signal.



(a) Ideal and effectively transmitted FMCW signals

(b) Range estimation without nonlinearity correction

(c) Chirp-rate estimation

(d) Spectrum of 1st demodulated beat signal

(e) Chirp-rate estimation after removing the strongest component

(f) Reference-free range estimation

Fig. 4. The idea and effective source sweep spectra and its impact on range estimation.

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