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# Turbo Product Codes with Irregular Polar Coding for High-Throughput Parallel Decoding in Wireless OFDM Transmission

Toshiaki Koike-Akino†, Congzhe Cao‡, Ye Wang†
† Mitsubishi Electric Research Laboratories (MERL), 201 Broadway, Cambridge, MA 02139, USA
‡ Dept. of Electrical & Computer Engineering, University of Alberta, Edmonton, AB T6G 2V4, Canada
Email: {koike, yewang}@merl.com, congzhe@ualberta.ca

***Abstract*—Ultra-reliable forward error correction (FEC) codes approaching the Shannon limit have played an important role in increasing spectral efficiency of wireless communications. In addition to the error correction performance, both low-power and low-latency decoding are demanded for the fifth generation (5G) wireless applications. In this paper, we introduce turbo product codes (TPC) consisting of multiple polar codes to enable highly parallel decoding for high-throughput and low-latency FEC. With turbo iterative decoding, the proposed polar-TPC can outperform the conventional BCH-based TPC by 0.5 dB, and can approach the performance of the corresponding long polar code within 0.2 dB with a capability of 256-times faster decoding. In addition, we apply irregular polar codes, whose polarization units are pruned, to further reduce the computational complexity by 50% and decoding latency by 80% without sacrificing performance. We analyze the impact of list size, turbo iteration count, and fading channels to demonstrate the potential of the polar-TPC for 5G wireless systems.**

## I. Introduction

Capacity-approaching forward error correction (FEC) based on low-density parity-check (LDPC) codes [1]–[3] has contributed to increasing spectral efficiency and data rates in modern digital communications systems. However, the pursuit of high FEC performance has led to a significant increase in power consumption and circuit size. Hence, a good trade-off between error correction performance and computational complexity as well as decoding latency is of great importance in fifth-generation (5G) wireless applications. Recent research activities on polar codes [4]–[10] have revealed that polar codes can compete against the state-of-the-art LDPC codes, particularly under stringent constraints of decoding complexity and latency [17]–[19]. Remarkably, by using successive cancellation list (SCL) decoding [5], polar codes concatenated with a cyclic redundancy check (CRC) code can nearly approach the Polyanskiy bound at short block lengths [6]. As a result, the polar codes are now considered as the promising FEC candidates in the 5G wireless standards. Nevertheless, one of the unsolved major issues underlying the SCL decoding includes the difficulty of parallel implementation unlike belief propagation (BP) decoding, which has been widely used for LDPC codes. Although BP decoding is also applicable to polar codes, the performance has never been competitive with the SCL decoding. This paper introduces a spatial coupling of parallel short-block polar codes in the form of turbo product codes (TPC) [12]–[14] in order to enable highly parallel/pipeline SCL decoding.

The conventional TPC typically uses algebraic codes such as Bose–Chaudhuri–Hocquenghem (BCH) codes [12]–[14]. The TPC employs an iterative soft-input soft-output (SISO) decoding under the so-called turbo principle, to drastically improve the performance of the classical product codes. In [12], Pyndiah introduced a simplified Chase algorithm generating soft-decision messages given multiple hard-decision candidates, to achieve close to the maximum-likelihood performance.

As for spatially-coupled polar codes, there exist a few studies concatenating multiple codes; e.g., Hamming codes [24], LDPC codes [25]–[27], BCH codes [28], convolutional codes [29], Reed–Solomon (RS) codes [30], as well as CRC codes [5]. However, most work rely on iterative BP decoding and hence they suffer from a huge performance gap against the SCL decoding for polar+CRC codes. In this paper, we study a polar-based TPC as a promising alternative which enables massively parallel SCL decoding without major modifications. Specifically, Pyndiah's Chase algorithm can be readily adopted for turbo iterations across parallel SCL decoding, which produces multiple candidates in the list, compatible for soft-decision calculations. We demonstrate that 256-times higher-throughput SCL decoding is realizable with the proposed polar-TPC while achieving the near optimal performance within a gap of 0.2 dB.

In order to further increase the decoding throughput at a lower decoding complexity, we introduce irregular polar codes [20]–[22] whose polarization units are irregularly pruned. It is verified that the decoding complexity and latency can be significantly reduced by at least 50% and 80%, respectively, without incurring any performance degradation. We analyze the impact of list size, iteration count, and fading channels to show the advantage of the polar-TPC employing the parallel SCL decoding against the conventional BCH-TPC and BP decoding for 5G wireless systems.

The contributions of this paper are summarized as follows.

- **Massively parallel and pipeline polar decoding**: The major drawback of non-parallel SCL polar decoding is dealt with by coupling multiple short-length polar codes in the form of TPC. The polar-TPC of $256^2$-bit length enables 256-times higher-throughput and 128-times lower-latency decoding per turbo iteration in principle.

- **SISO iterative SCL decoding**: We develop a modified Chase algorithm for SISO SCL decoding, where multiple decoder candidates in the list are exploited to compute soft-decision messages for turbo iterations. We evaluate the impact of list size and turbo iteration count to validate its usefulness in comparison to BP decoding.
- **Irregular inactivation of polarization units**: In order to reduce the decoding complexity, we introduce the irregular polar codes [20]–[22] which inactivate carefully-chosen polarization units in an irregular manner.
- **Complexity and latency analysis**: We analyze the complexity and latency reduction for the proposed irregular polar-TPC to verify the advantage in power-constrained communications. We demonstrate that the decoding complexity and latency can be significantly reduced by at least $50\%$ and $80\%$, respectively, with no performance loss.
- **Wireless fading channels**: The polar-TPC is analyzed over various wireless fading conditions, from strong to weak line-of-sight (LOS) environments, for orthogonal frequency-division multiplexing (OFDM) transmission.

## II. Polar Codes with SCL Decoding

Polar codes have drawn much attention in the coding theory community since their ability to achieve capacity via successive cancellation (SC) decoding was proven in 2009 [4] for any arbitrary discrete-input memoryless channels (DMCs). However, in spite of the theoretical strength, polar codes have not been adopted in practical systems until recently due to their poor performance at short block lengths in comparison to LDPC codes. The recent breakthrough development of SCL decoding [5] has shown that polar codes can compete with state-of-the-art LDPC codes, in particular for low-complexity and latency-constrained systems [6], [17]–[19].

### A. Polar Encoding

An $(N, k)$ polar code with $k$ information bits and $N$ encoded bits ($N = 2^n$) uses an $N \times N$ generator matrix $\boldsymbol{F}^{\otimes n}$ for encoding, where

$$\boldsymbol{F} = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}, \tag{1}$$

is a binary kernel matrix and $[\cdot]^{\otimes n}$ denotes the $n$-fold Kronecker power. Let $\boldsymbol{u} = [u_1, u_2, \ldots, u_N]^{\mathrm{T}}$ and $\boldsymbol{x} = [x_1, x_2, \ldots, x_N]^{\mathrm{T}}$ respectively denote the vectors of input bits and encoded bits. The codeword (for non-systematic polar codes) is given by $\boldsymbol{x} = \boldsymbol{F}^{\otimes n}\boldsymbol{B}\boldsymbol{u}$, where the matrix multiplications are carried out over the binary field (i.e., modulo-2 arithmetic), and $\boldsymbol{B}$ denotes an $N \times N$ bit-reversal permutation matrix [4]. Due to the nature of Kronecker product, polar encoding and decoding can be performed at a complexity on the order of $\mathcal{O}[N \log_2 N]$ over the $n$-stage polarization as shown in Fig. 1. The multi-stage operation of the Kronecker products gives rise to the so-called polarization phenomenon to approach capacity in arbitrary DMCs [4].

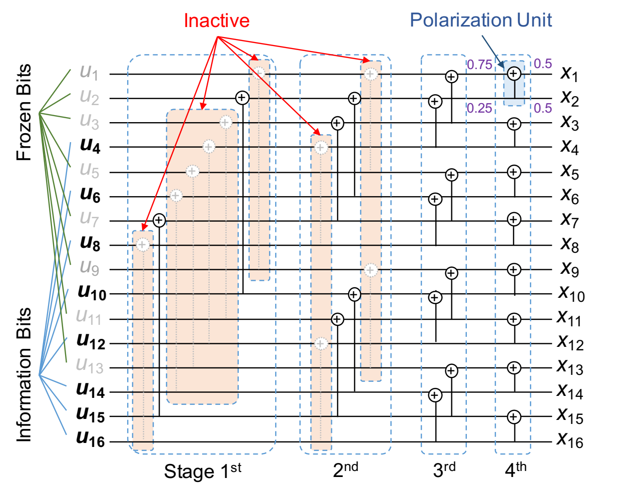



Fig. 1. Irregular polar codes [20]–[22] having 10 inactivations over 32 polarization units ($10/32 = 31.25\%$ complexity reduction).

The polar coding maps the information bits to the $k$ most reliable locations in $\boldsymbol{u}$. The remaining $N - k$ input bits are “frozen” bits, which are typically fixed to zero for symmetric channels. Let $\mathbb{K}$ and $\bar{\mathbb{K}}$ denote the subsets of $\{1, 2, \ldots, N\}$ that correspond to the information bit and frozen bit locations, respectively. By means of various design methods, such as Bhattacharyya parameter [4], density evolution [7], Gaussian approximation [15], and extrinsic information transfer (EXIT) [21], the locations in $\boldsymbol{u}$ with the lowest reliability can be selected as $\bar{\mathbb{K}}$ for frozen bits.

For systematic polar codes, we employ an encoding procedure [10] which is to write the $k$ data bits into a vector $\boldsymbol{u}$ at the bit-reversal permutation of the locations $\mathbb{K}$, with the other locations set to zero, and then apply the polar encoding procedure twice, while setting the frozen bit locations in $\bar{\mathbb{K}}$ to zero on the intermediate result between the encodings. This procedure for systematic coding can be expressed as follows:

$$\boldsymbol{x} = \boldsymbol{F}^{\otimes n}\boldsymbol{B}\phi_{\bar{\mathbb{K}}}(\boldsymbol{F}^{\otimes n}\boldsymbol{B}\boldsymbol{u}), \tag{2}$$

where $\phi_{\bar{\mathbb{K}}}(\cdot)$ denotes setting the frozen bit locations in $\bar{\mathbb{K}}$ to zero. A concatenation of a CRC code is often used for SCL decoding to increase the minimum Hamming distance [11].

### B. Polar Decoding

Let $\boldsymbol{y} = [y_1, y_2, \ldots, y_N]^{\mathrm{T}}$ denote the vector of decoder inputs after wireless communications over fading channels, including modulator and demodulator operations. Due to the assumption of memorylessness, the transition probability $W_N(\boldsymbol{y}|\boldsymbol{x})$ between $\boldsymbol{x}$ and $\boldsymbol{y}$ is defined as $W_N(\boldsymbol{y}|\boldsymbol{x}) = \prod_{i=1}^{N} W(y_i|x_i)$. The original SC decoding proposed in [4] was recently improved by the SCL decoder in [5], which incorporates list decoding and an embedded CRC code to improve the performance for short-block polar codes. The SC decoder proceeds sequentially over the bits, from index 1 to $N$, where for each index $i \in \{1, 2, \ldots, N\}$, an estimate $\hat{u}_i$ for bit $u_i$ is made as follows: if $i \in \bar{\mathbb{K}}$, then $\hat{u}_i$ is set to frozen zero, otherwise, when $i \in \mathbb{K}$, $\hat{u}_i$ is set to the most likely value for $u_i$ given the channel outputs and assuming that the previous estimates $[\hat{u}_1, \hat{u}_2, \dots, \hat{u}_{i-1}]$ are correct.

The SCL+CRC decoder proceeds similarly to the SC decoder, except that for each data bit index $i \in \mathbb{K}$, the decoder retains both possible estimates, $\hat{u}_i = 0$ and $\hat{u}_i = 1$, in the subsequent decoding paths. To avoid handling an exponentially increasing number of paths, the list-decoding approach limits the number of paths to a fixed-size list of the most likely partial paths. The SCL decoder also employs a CRC code embedded in the data bits, which allows it to select the final decoding as the most likely path with a valid CRC. The combination of list-decoding with the embedded CRC code to reject invalid paths yields significantly improved performance [5].

Various other decoding algorithms have been proposed with different complexity and error rate performance, such as BP decoding and soft cancellation (SCAN) decoding. Among those decoding algorithms, BP decoding and SCAN decoding can directly generate soft information of coded bits such that it is straightforward to use them in the TPC scheme that requires turbo iterations. It turns out, however, SCL decoding concatenated with CRC achieves the best performance and makes polar codes very competitive with other state-of-the-art coding schemes. Therefore, in this paper, we consider the modification of SCL decoding to generate soft information of coded bits for polar-TPC decoding.

### C. Irregular Polar Coding

The authors recently introduced irregular degree distribution, as widely used in LDPC codes, for polar codes in [20]–[22]. The irregular polar codes inactivate some polarization units as shown in Fig. 1 to obtain potential error-rate performance improvement in addition to complexity and latency reduction. By pruning polarization units, we can reduce the encoding and decoding complexity without destroying the radix-2 Cooley–Tukey like architecture, thus maintaining the basic SCL decoding structure.

For an $(N, k)$ polar coding, there are $n = \log_2(N)$ polarization stages, each of which contains $N/2$ polarization units (thus $N_\mathrm{U} \triangleq N \log_2(N)/2$ polarization units in total), as shown in Fig. 1 for $N = 16$. Polar codes exploit the so-called polarization phenomenon, where each polarization unit provides degraded and improved reliability. For example, when the coded bits $x_1$ and $x_2$ have uniform reliability having erasure rate (Bhattacharyya parameter) of $0.5$, the upper branch of the polarization unit becomes unreliable with erasure rate of $0.75$, whereas the lower branch improves the reliability to $0.25$. The conventional polar codes have no flexibility in coding architecture except in the location of frozen bits. To increase the degrees of freedom while reducing decoding complexity, the irregular polar coding inactivates carefully chosen polarization units.

The key benefit of such inactivations is three-fold: i) complexity reduction in encoding/decoding computations, ii) decoding latency reduction, and iii) potential performance improvement by adjusting the weight distribution. For the code in Fig. 1, we can achieve a $10/32 \simeq 31\%$ complexity reduction since no computation is required at all for inactivated units in both encoding and decoding. Note that an appropriate choice of polarization units to be inactivated causes no performance penalty (rather, in fact, slightly improved performance can be achieved), as discussed in [20]–[22].

### D. Related Work and Motivations

We here note that this paper is distinguished from our previous reports [17]–[23] as follows. We studied interleaver design for polar-coded modulation in [17], achieving a $0.5$ dB gain. This work was extended with constellation shaping for wireless fading channels in [18], [19], achieving a grater-than $2.5$ dB improvement. The concept of irregular polar coding was first proposed for optical communications with experimental validations in [20], [22], and later extended to wireless massive antenna systems in [21]. We then applied irregular polar coding to TPC for Tbps-class high-throughput optical interconnetcs in [23], in which an error floor analysis was provided via importance sampling [14] to validate the feasibility of a bit-error rate (BER) below $10^{-15}$ with the aid of BCH concatenation. In this paper, we investigate the performance of the irregular polar-TPC in wireless fading channels, and with more detailed analysis for list size.

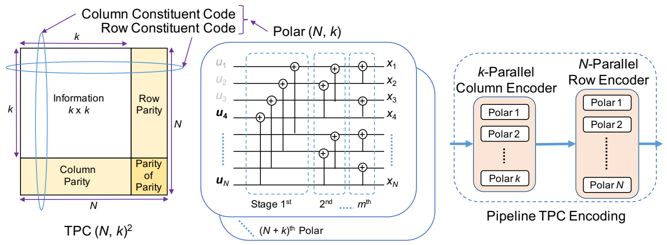
We are specifically motivated to develop a hardware-friendly parallelizable SCL decoding for high-throughput, low-power, and low-latency communications. Rather than decoupling a long polar code [15], we focus on a spatial coupling of parallel short-length polar codes in the framework of TPC to tackle the decoder parallelism. Compared to the concatenation techniques with other coding schemes [24]–[29], the polar-TPC can provide higher parallelism such as 256 parallel decoders with a homogeneous implementation architecture. In addition, the SCL decoder is found to be well-suited for the Pyndiah's Chase algorithm [12] to calculate soft information, without explicitly considering error patterns used in BCH-TPC. To the best of our knowledge, there is no other intensive analyses conducted for polar-based TPC in literature to date.

## III. Polar Turbo Product Coding (Polar-TPC)

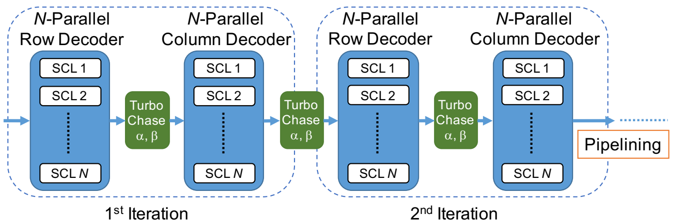
In this section, we describe the encoding and decoding of polar TPCs, where polar codes are spatially coupled in the TPC structure. In order to perform iterative decoding with soft information, we develop a modified SCL decoding algorithm that generates soft information of coded bits.

### A. Massively Parallel/Pipeline Encoding and Decoding

Fig. 2(a) illustrates the proposed $(N, k)^2$ TPC using multiple polar constituent codes, instead of BCH codes. For the two-dimensional spatial coupling architecture, the TPC performs two-stage encoding, i.e. column-encoding and row-encoding over a $k \times k$ information block, to generate an $N \times N$ encoded block. For the two-stage encoding, we can implement a highly parallel encoder, i.e., $k$-parallel column encoding followed by $N$-parallel row encoding. Each polar code $(N, k)$ performs $n$-stage polarization steps ($n = \log_2 N$) with pre-defined




(a) Polar-TPC encoder




(b) Polar-TPC decoder

Fig. 2. Turbo product coding $(N, k)^2$ based on polar codes, capable of highly parallel/pipeline SCL decoding.

$(N-k)$-bit frozen insertion. We use systematic polar coding based on [10] for each column and row constituent codes.

As shown in Fig. 2(b), the TPC decoder performs row-decoding and column-decoding iteratively via turbo processing based on Pyndiah's Chase algorithm (parameterized by $\alpha$ and $\beta$) [12]. Since the decoding can be performed in a fully parallel fashion for $N$ component codes independently (and capable of pipelines across turbo iterations), the TPC enables roughly $N$-times higher decoding throughput compared to a single uncoupled $N^2$-length polar code as the latency of SCL decoding is nearly proportional to the codeword length [16]. Note that there is no benefit in the computational complexity because the SCL decoder has an order of complexity $\mathcal{O}[LN \log_2 N]$ for a list size of $L$; specifically, TPC requires $2N$-times SCL decoding per iteration (thus $\mathcal{O}[2LN^2 \log_2 N]$), whereas the long polar code $(N^2, k^2)$ requires single SCL decoding having an identical complexity order of $\mathcal{O}[LN^2 \log_2 N^2]$. Nonetheless, the capability of hardware-friendly parallel decoding is of a great advantage for high-throughput transmissions.

### *B. Iterative Decoding for TPC*

Let $x_{i,j}$ and $y_{i,j}$ denote the element in the $i$th row and $j$th column of the transmitted and received code block for $i \in \{1, 2, \dots, N\}$ and $j \in \{1, 2, \dots, N\}$. At the receiver, the demodulator calculates the log-likelihood ratio (LLR), $L_{\text{ch}}(i,j)$ of the $(i,j)$th bit, where $L_{\text{ch}}(i,j)$ is given by

$$L_{\text{ch}}(i,j) = \ln \frac{\Pr(y_{i,j}|x_{i,j}=0)}{\Pr(y_{i,j}|x_{i,j}=1)}, \tag{3}$$

and is fed into the polar-TPC decoder as the *a priori* information in the first row-decoding. The TPC decoder first performs $N$-parallel SCL decoding for every row with $L_{\text{apr}}(i,j)$ as prior information. In order to perform iterative decoding over columns and rows, we propagate back and forth the soft-decision decoder output, i.e., the *a posteriori* information for each coded bit $L_{\text{app}}(i,j)$, such that extrinsic information $L_{\text{ext}}(i,j)$ is available for decoding in the next iteration, where

$$L_{\text{ext}}(i,j) = L_{\text{app}}(i,j) - L_{\text{apr}}(i,j). \tag{4}$$

We leave the description of the SISO SCL decoding in the next subsection.

After completion of $N$-parallel row-decoding, the TPC decoder starts to perform the $N$-parallel SCL decoding for every column, given the *a priori* information updated as follows:

$$L_{\text{apr}}(i,j) = L_{\text{ch}}(i,j) + L_{\text{ext}}(i,j). \tag{5}$$

After obtaining the soft-decision decoder output $L_{\text{app}}(i,j)$ from the column SCL decoding, we again obtain the extrinsic information for each coded bit according to (4). At this point, we complete a full iteration of TPC decoding. Further turbo iterations can be performed in pipeline by the TPC decoder to achieve better performance.

### *C. SISO SCL Decoding*

In [12], the Chase algorithm is applied to BCH-TPC decoding to generate soft information based on hard-decision decoders. The method uses multiple BCH decoded output candidates for hypothetically flipped error patterns across several unreliable received bits. Since the original SCL decoding does not provide soft information, we integrate the analogous Chase algorithm with SCL decoding of polar codes such that soft information $L_{\text{app}}(i,j)$ can be obtained efficiently. In fact, the SCL decoding is compatible with the Chase algorithm since $L$ candidates of the decoded sequence $\hat{\boldsymbol{u}}$ are already considered in the list. Hence, there is no need to explicitly generate multiple error patterns as used for BCH-TPC.

Let $D_\ell$ denote the squared distance between the $\ell$th candidate in the list (for $\ell \in \{1, 2, \dots, L\}$) and the received sequence $\boldsymbol{y}$ as follows:

$$D_\ell = \sum_{j=1}^{N} \left(y_j - \hat{u}_j\right)^2. \tag{6}$$

For each decoded bit $\hat{u}_i$ for $i \in \{1, 2, \dots, N\}$, we let $S_{\text{one}}(i)$ and $S_{\text{zero}}(i)$ be the distances between the best candidate of $\hat{u}_i$ among all $L$ paths and the decision $\hat{u}_i = 1$ and $\hat{u}_i = 0$, respectively, where the best candidate corresponds to the shortest distance $D_\ell$ of the path that $u_i$ belongs to. Using Pyndiah's approximation [12], the soft output information of the SCL decoder is then given as

$$L_{\text{app}}(i) = \frac{S_{\text{zero}}(i) - S_{\text{one}}(i)}{4}, \tag{7}$$

if $S_{\text{zero}}(i)$ and $S_{\text{one}}(i)$ are both available. Otherwise when all candidate paths cannot result in $\hat{u}_h = 1$ (or $\hat{u}_h = 0$), we simply use

$$L_{\text{app}}(i) = \beta \cdot (2\hat{u}_i - 1), \tag{8}$$

where $\beta$ is a predefined constant. Following the method in [12], we use another constant $\alpha$ to scale the final soft output as $\alpha \cdot L_{\text{app}}$. When a CRC code is used, the CRC-valid candidates may be adjusted by yet another constant $\gamma$.

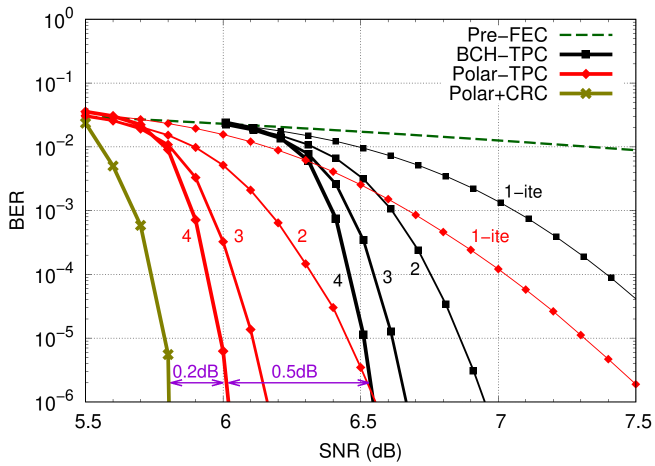



Fig. 3. Polar-TPC $(256, 239)^2$ vs. polar codes $(256^2, 239^2)$ with $I = 1, 2, 3, 4$ turbo iterations and a list size of $L = 16$.

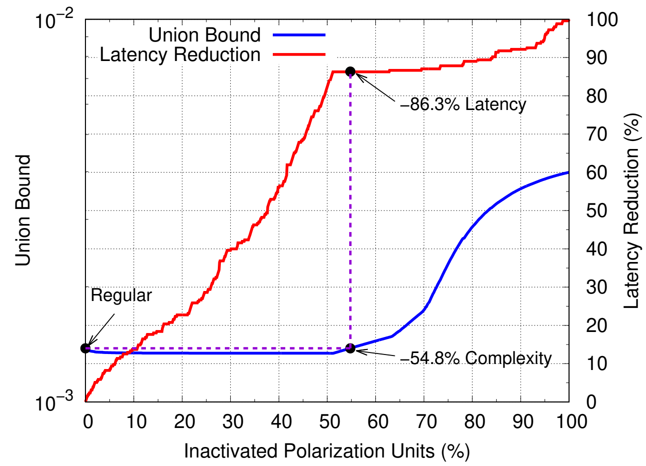



Fig. 5. Union bound and latency reduction of irregular polar code $(256, 239)$ at an SNR of 7 dB: up to 54.8% complexity reduction and 86.3% latency reduction without performance loss.

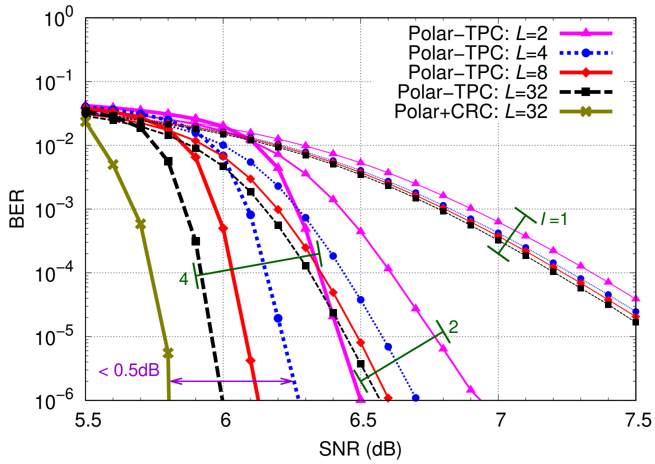



Fig. 4. List size impact: Polar-TPC $(256, 239)^2$ vs. polar codes $(256^2, 239^2)$ with $I = 1, 2, 4$ turbo iterations and a list size of $L = 2, 4, 8, 32$.

## IV. Performance Results

### A. Polar-TPC vs. BCH-TPC in AWGN Channels

In [23], we revealed that the polar-TPC can outperform the conventional BCH-TPC in additive white Gaussian noise (AWGN) channels. Here, we re-evaluate the BER performance of the polar-TPC versus the BCH-TPC and the corresponding long uncoupled polar code in Fig. 3 for quadrature phase-shift keying (QPSK) modulation. We consider polar-TPC $(256, 239)^2$ for an overhead of $14.73\%$ and a list size of $L = 16$. Throughout the paper, we use the Chase algorithm parameters across iterations as $\alpha = [0.3, 0.4, 0.5, 0.6, 0.7, 1, 1, 1]$ and $\beta = [1.8, 1.5, 1.3, 1.9, 2, 4, 8, 16]$ irrespective of conditions. The polar codes were designed by the EXIT method [21] at an SNR of 7 dB. For the BCH-TPC $(256, 239)^2$, we use Berlekamp–Massey decoding with 32 error patterns for the Chase algorithm. For benchmark, we also present the BER performance of the standalone SCL decoding ($L = 32$) for a long polar code $(256^2, 239^2 + 16)$ concatenated with CRC-16 whose polynomial is 0x8005 (CRC is not used for Polar-TPC). We observe in Fig. 3 that turbo iterations offer significant improvement in BER performance to compensate for the penalty of short polar constituent codes. Remarkably, after four iterations, the BER of the proposed polar-TPC approaches that of the long polar code within 0.2 dB, even though the decoding throughput can be 256-times faster in principle. Moreover, it is verified that the polar-TPC can outperform the conventional BCH-TPC by greater than 0.5 dB.

We then evaluate the impact of the list size $L$ in Fig. 4, where we consider list sizes $L = 2, 4, 8, 32$ and turbo iterations $I = 1, 2, 4$. Although the BER performance degrades as the list size is reduced, a small list size of $L = 4$ still maintains a small gap from the long polar SCL decoding within a 0.5 dB loss. In addition, we can observe a great benefit of turbo iterations even with a very small list size of $L = 2$, showing competitive performance to the BCH-TPC when compared with Fig. 3.

### B. Irregular Polar for Low-Power and Low-Latency Decoding

We next evaluate the complexity and latency reduction when we apply the irregular inactivations of polarization units. Fig. 5 shows the union bound via EXIT analysis [21] and latency reduction of an irregular polar code $(256, 239)$ at a signal-to-noise ratio (SNR) of 7 dB. It is seen that the union bound can be better than the regular polar code (i.e., $0\%$ inactivation) by inactivating no more than $54.8\%$ polarization units, and the corresponding latency reduction will be $86.3\%$. Note that the decoding complexity can be linearly decreased by pruning polarization units, whereas the latency is not always immediately reduced in particular for the locations at later polarization stages (see Fig. 1). Our latency analysis takes a great care of such constraints.

From Fig. 5, the EXIT analysis suggests that the TPC with irregular polar codes may be able to reduce power consumption by $55\%$ without any penalty of BER performance. We show the simulated performance of the irregular polar-

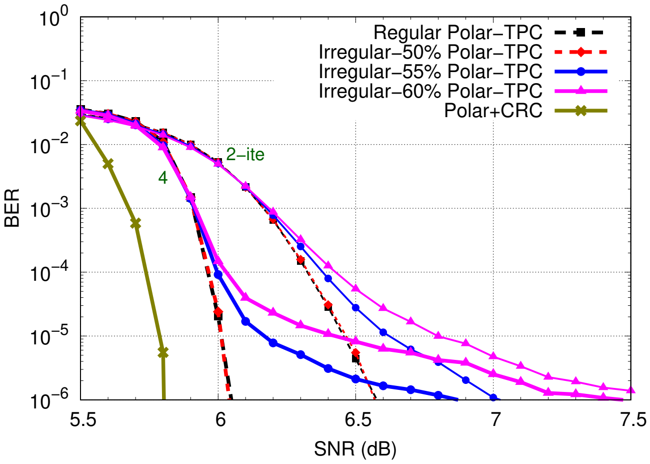



Fig. 6. Irregular polar-TPC $(256, 239)^2$ in AWGN channels (list size $L = 16$, turbo iteration $I = 2, 4$): no performance degradation up to 50% inactivations.

TPC in Fig. 6, where we found that the performance can be kept almost identical to the regular polar-TPC, when 50% of polarization units are inactivated. Hence, the irregular polar-TPC can realize at least $1/2$ power consumption and about $1/5$ decoding latency. However, Fig. 5 reveals that more-than 50% inactivations cause an error floor. This is because the minimum Hamming distance is shortened by pruning many polarization units. Nonetheless, this error floor issue can be readily solved by an outer BCH code, as studied in [23], where the authors found via the importance sampling analysis [14] that an outer BCH concatenation can overcome the error floor and tolerate up to 72% inactivations for realizing an overall BER of $10^{-15}$.

### C. BER Performance in Wireless Fading Channels

The great potential of the polar-TPC was demonstrated in the discussion so far, considering the simple AWGN channels. We finally evaluate the irregular polar-TPC in OFDM transmission over wireless fading channels. Fig. 7 shows the BER performance in Nakagami–Rice fading channels with a Rician factor of $K \in \{10, 5, 0, -10\}$ dB, correspondingly, from stronger to weaker LOS wireless environments. From the figure, we observe the following:

- The wireless fading channel with smaller Rician factor can severely degrade the BER performance for all methods by about 6 dB compared to the AWGN performance.
- The performance degradation of the polar-TPC due to deeper fading with smaller Rician factor can be even larger. The gap of 0.2 dB at AWGN channels will become 1.3 dB at fading channels with $K = -10$ dB. This is simply because the fading flattens the pre-FEC BER curve.
- The irregular polar-TPC with SISO SCL decoding can perform better than the BP decoding (with 100 iterations), and achieve a closer performance to the standalone SCL decoding of the long polar+CRC code.

As studied in [8], [17]–[19], the interleaver design can greatly impact the polar codes in particular for high-order modulation and fading channels. The interleaver design suited for the polar-TPC remains as a future work.

## V. Conclusion

We introduced a new polar-TPC which is capable of highly parallel and pipeline decoding for high-speed wireless communications. It was shown that the polar-TPC can achieve significant performance gain greater than 0.5 dB compared with the conventional BCH-TPC, and perform closely (within 0.2 dB) to a single uncoupled polar code, while potentially enabling a 256-times higher throughput. We investigated the impact of list size and turbo iterations as well as polarization unit inactivations. It was verified that at least 50% complexity and 80% latency can be further reduced by introducing irregular polar codes for TPC without any penalty in BER performance. We then evaluated the irregular polar-TPC in wireless fading channels. It was found that the performance gap between the single polar code and polar-TPC will increase up to 1.3 dB as the Rician factor decreases. Nevertheless, the proposed TPC achieves a significant coding gain compared to uncoded systems in fading channels, with a capability of high-throughput parallel decoding.

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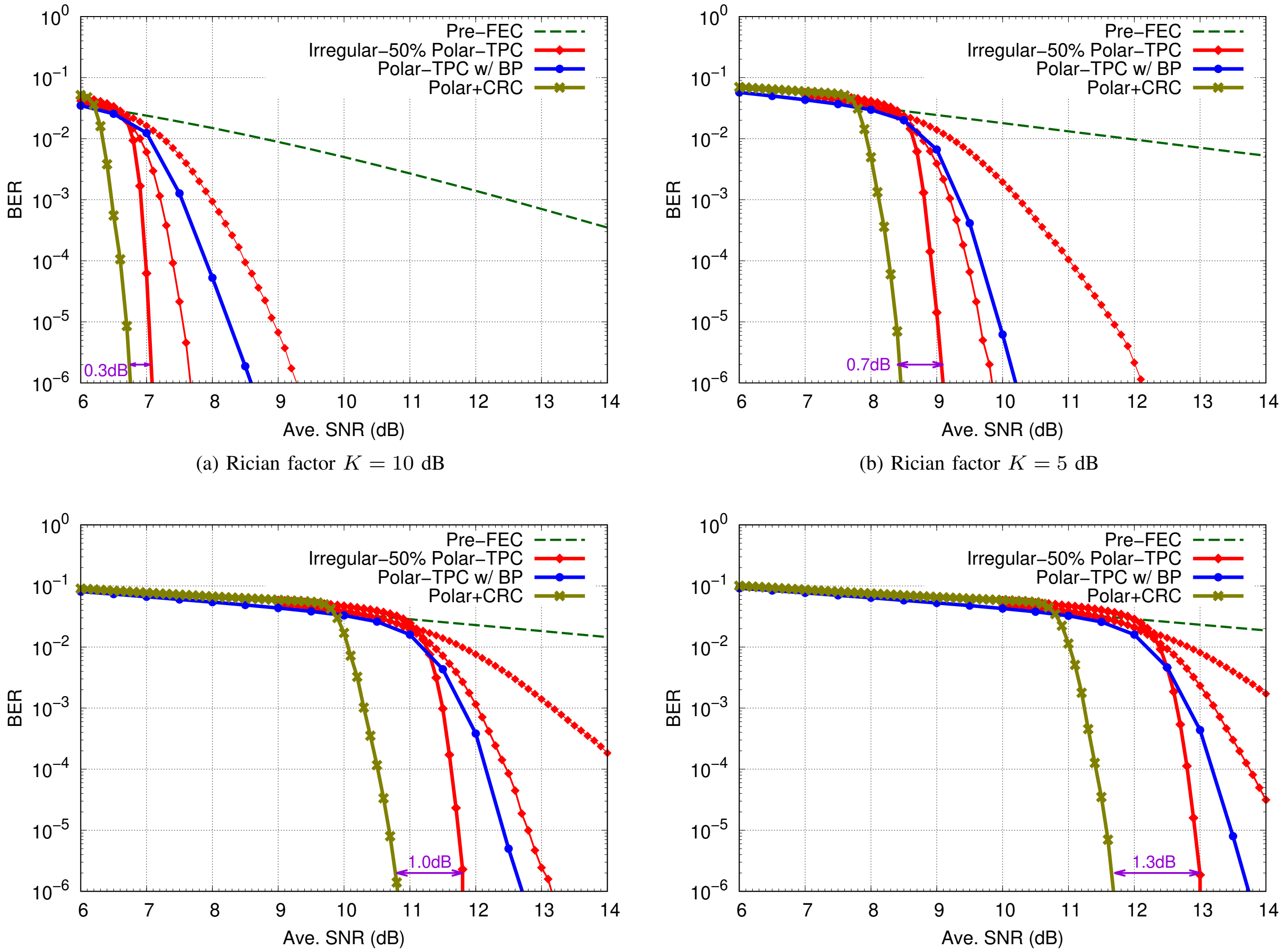


(a) Rician factor $K = 10$ dB

(b) Rician factor $K = 5$ dB

(c) Rician factor $K = 0$ dB

(d) Rician factor $K = -10$ dB

Fig. 7. Irregular polar-TPC in Nakagami–Rice fading channels (a list size of $L = 16$, turbo iterations $I = 1, 2, 4$, BP iterations: 100).


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