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Irregular Polar Turbo Product Coding for High-Throughput Optical Interface

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Abstract: We propose polar turbo product code (TPC) to enable parallel/pipeline decoding, for high-throughput transmission. With irregular polar codes, the computational complexity and latency can be significantly reduced, yet outperforming BCH-constituent TPC by 0.5 dB.

OCIS codes: (060.4510) Optical communications, (060.1660) Coherent communications, (060.4080) Modulation.

1. Introduction

Capacity-achieving forward error correction (FEC) codes such as low-density parity-check (LDPC) [1, 2] have contributed to increasing data rates of optical transceivers. However, pursuit of high coding gain has led to a significant increase in power consumption. Hence, a good trade-off between performance and complexity is of importance, in particular when considering upgrades of optical interface to beyond 100Gb/s. Staircase codes [3] and turbo product codes (TPC) [4–6] have been considered as lightweight FEC candidates. This paper investigates hardware-friendly polar codes, capable of parallel/pipeline decoding, as a potential alternative for such high-throughput applications.

Recent development of successive cancellation list (SCL) decoding for polar codes [7–10] has shown a highly competitive performance (approaching the finite-length Polyanskiy bound) against state-of-the-art LDPC codes, in particular for low-complexity and latency-constrained systems. One major drawback of polar codes lies in the difficulty of parallel decoding, leading to low throughput and large latency. In order to tackle this issue, we propose a new class of TPC which constitutes spatially-coupled parallel short-length polar codes, rather than uncoupled long polar code.

In this paper, we show that the proposed polar-TPC can outperform a conventional TPC based on Bose–Chaudhuri–Hocquenghem (BCH), and the penalty due to shorter lengths of each constituent polar codes can be effectively compensated by turbo iteration to approach near optimal performance of an uncoupled long polar code. In order to further increase the decoding throughput at a lower decoding complexity, we introduce irregular polar codes [11] whose polarization units are irregularly pruned. It is verified that the decoding complexity and latency can be significantly reduced by up to 72% and 88%, respectively, without incurring any performance degradation.

2. Polar Turbo Product Codes for High-Throughput Parallel/Pipeline Decoding

Fig. 1 illustrates the proposed TPC using multiple polar constituent codes, instead of BCH codes. For a two-dimensional spatial coupling architecture, the TPC performs two-stage encoding, i.e. column-encoding and row-encoding over a k^2 information block, to generate an N^2 encoded block. Each polar code (N,k) performs m-stage polarization steps ($m = \log_2 N$) with pre-defined $(N-k)$-bit frozen insertion (we use systematic polar coding based on [13]). The decoder performs row-decoding and column-decoding iteratively via Pyndiah's turbo Chase processing (parameterized by α and β) [4]. Since the decoding can be performed in parallel for N component codes independently (and capable of pipelines across turbo iterations), the TPC enables roughly N-times higher decoding throughput compared to a single uncoupled N^2-length code as the latency of SCL decoding is nearly proportional to N [12]. Note that there is no benefit in the computational complexity because the SCL decoder has an order of complexity $\mathbb{O}[LN\log_2 N]$ for a list size of L; specifically, TPC requires $2N$-times SCL decoding per iteration (thus $\mathbb{O}[2LN^2\log_2 N]$), whereas the long polar code (N^2,k^2) requires single SCL decoding having an identical complexity order of $\mathbb{O}[LN^2\log_2 N^2]$. Nonetheless, the capability of hardware-friendly parallel and pipeline decoding is of a great advantage for high-throughput optical transmissions. It should be also noted that multiple error patterns required for Chase processing are automatically obtained in SCL decoding without any modifications.

We show the bit-error-rate (BER) performance benefit of the proposed codes in Fig. 2, where we consider polar-TPC $(256,239)^2$ for an overhead of 14.73% and a list size of $L = 16$. For a BCH-TPC $(256,239)^2$, we use Berlekamp–Massey decoding with 32-pattern Chase processing. For comparison, we also present the BER performance of SCL

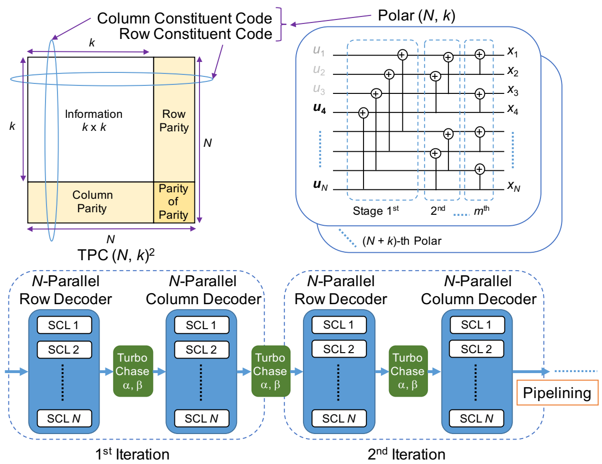

Fig. 1: Turbo product coding $(N,k)^2$ based on polar codes, with parallel/pipeline decoding.

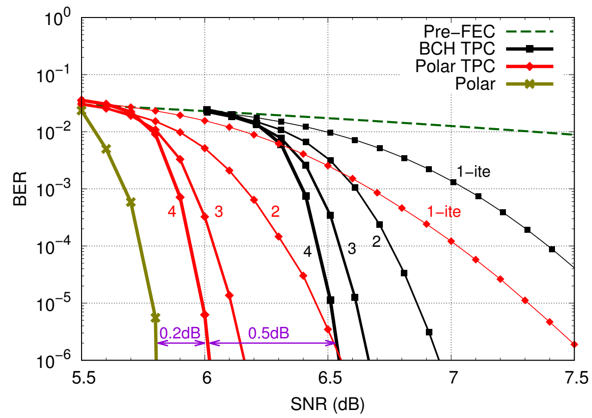

Fig. 2: Polar-TPC vs. BCH-TPC for $(256,239)^2$ with 1–4 turbo iterations.

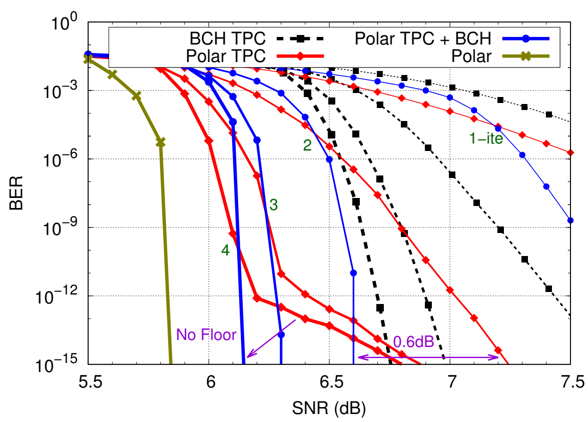

Fig. 3: Error floor analysis with IS [15] for polar-TPC with/without outer BCH.

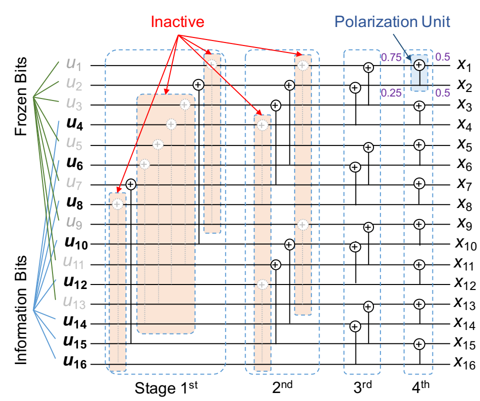

Fig. 4: Irregular polar code [11] with $10/32 = 31\%$ inactivated polarization units for reduced complexity.

decoding ($L = 32$) for a long polar code $(256^2, 239^2)$, concatenated with cyclic redundancy check (CRC) 16 bits whose polynomial is designed via [14]. We observe in Fig. 2 that turbo iterations offer significant improvement in BER performance to compensate for the penalty of short polar constituent codes. Remarkably, after four iterations, the BER of the proposed polar-TPC approaches that of the long polar code within 0.2 dB, even though the decoding throughput can be 256-times faster in principle. Moreover, it is verified that the polar-TPC can outperform the conventional BCH-TPC by greater than 0.5 dB.

Although it is known that the polar codes have no error floor [7], the TPC usually exhibits error floor due to turbo iterations [15]. Since most optical systems require very low BER down below 10^{-15}, we shall make an analysis of the error floor. However, classical Monte-Carlo (MC) simulations have a practical limitation to observe sufficiently large number of errors in confidence for such low BER regimes. To analyze the error floor of polar-TPC, we use the importance sampling (IS) approach with mean translation according to [15], which discusses the error floor of BCH-TPC. The IS, which is a generalized version of the MS, uses a weighted sample mean to obtain a higher-confident estimate of BER given a lower number of simulation samples. The technique enforces more frequent errors by generating noise near the boundary of neighboring codewords. Using the Hamming weight analysis for polar codes [14], we simulated the IS for polar-TPC in Fig. 3. Unfortunately, error floors above a BER of 10^{-15} were found for 3 and 4 turbo iterations. This high error floor compared to BCH-TPC is due to the shorter minimum Hamming distance; the minimum Hamming distance of polar code $(256,239)$ is $d = 4$ (the second minimum is 6), while that of extended BCH code $(256,239)$ is $d = 6$. In order to mitigate the error floor, we consider an outer BCH code for hard-decision cleaning after polar-TPC decoding. For a fair comparison with an identical overhead, we present the BER performance of polar-TPC $(256,240)^2$ with a concatenation of an outer BCH code $(240^2, 239^2)$ in Fig. 3. It is

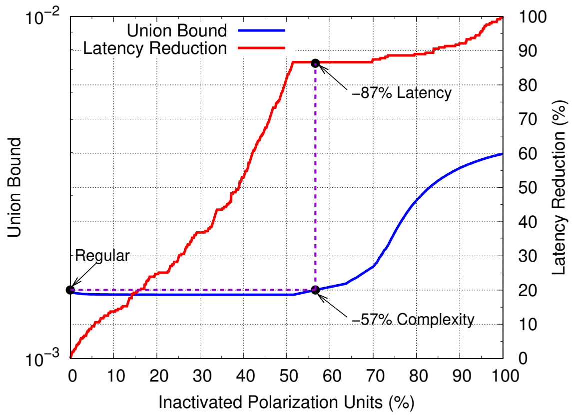

Fig. 5: Union bound and latency reduction of irregular polar code (256, 240) at SNR of 7 dB.

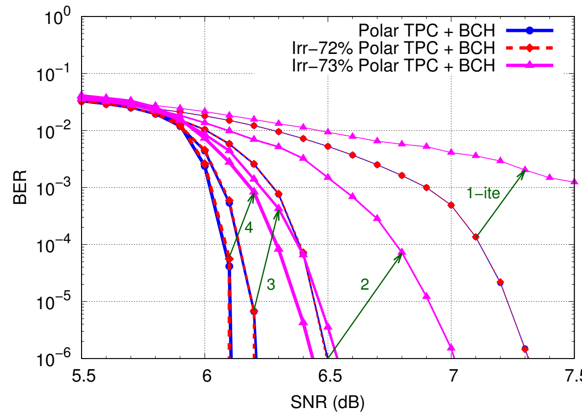

Fig. 6: Irregular polar-TPC: no degradation up to 72% inactivations.

verified that no error floor appears for the polar-TPC with BCH. In addition to the error floor mitigation effect, we can see that the BCH concatenation can offer an additional gain of 0.6 dB for the 2-iteration decoding case. This is because the BCH code can correct up to 29-bit errors, which include the dominant error patterns due to the minimum Hamming distance of $d^2 = 16$ and also the second minimum Hamming distance of $4 \cdot 6 = 24$ for the polar-TPC.

3. Irregular Polar Turbo Product Codes for Low-Power and Low-Latency Decoding

For the proposed polar-TPC, we can consider an adaptive early termination across pipelines to adjust the power consumption as an average number of iterations is typically small. In order to further reduce the power consumption, we apply irregular polar coding [11] to TPC. Irregular codes are constructed with several inactivations of polarization units as shown in Fig. 4, which can potentially reduce the computational complexity, decoding latency, and BER at the same time. Fig. 5 shows the union bound via density evolution (DE) and latency reduction of a polar code $(256, 240)$ at a signal-to-noise ratio (SNR) of 7 dB. It is seen that the union bound can be better than the regular polar code by deactivating no more than 57% polarization units, and the corresponding latency reduction will be 87%. Note that the decoding complexity can be linearly decreased by pruning polarization units, whereas the latency is not always immediately reduced in particular for the locations at later polarization stages (see Fig. 4).

From Fig. 5, the DE analysis suggests that the TPC with irregular polar codes can reduce at least 57% power consumption without any penalty of BER performance. We show the simulated performance of irregular polar-TPC in Fig. 6, where we found that the performance can be kept almost identical to the regular polar-TPC, even with 72% inactivations. Hence, the irregular polar-TPC can realize nearly $1/4$ power consumption and about $1/10$ latency.

4. Conclusion

We introduced a new polar-TPC which is capable of parallel and pipeline decoding for high-speed optical interface. It was shown that polar-TPC can achieve significant performance gain grater than 0.5 dB compared with BCH-TPC, and perform closely to a single uncoupled polar code while 256-times higher throughput can be realized. Moreover, we applied irregular polar codes to TPC so that 72% complexity and 88% latency can be removed with no BER loss.

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