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## Abstract

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# An IoT System to Estimate Personal Thermal Comfort

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Abstract—Thermal comfort in office buildings is emerging as an important variable that can be used to maximize employee productivity. In this paper we propose a new Internet of Things (IoT) based system that creates a personalized model of thermal comfort. To create this model, our system collects telemetry via an IoT network of sensors and user inputs. This data is then input into machine learning algorithms that continuously calibrate and update a personalized thermal comfort model for the user. To facilitate the individuality of our models, the system combines personal measurements from the Microsoft Band, such as biometric readings and user feedback, with environmental measurements such as temperature, humidity, and air speed. In this work, we evaluate a broad set of classification and regression algorithms. Our experimental results show that using our IoT based system improves the mean squared error of the thermal prediction by about 50% when compared to the industry standard method developed by P.O. Fanger.

*Keywords*-IoT, Thermal Comfort, Machine Learning, Wearable Devices

#### I. INTRODUCTION AND BACKGROUND

The growing diversification of today's workforce has created new social challenges in the office environment. A particular challenge that has emerged is that of keeping a diverse office population comfortable throughout the workday. Comfort, and particularly thermal comfort, has been shown to improve not only the happiness of workers, but also their productivity and social interactions. For example, one study by Hedge et. al. [1] found that reducing temperatures such that the average female office worker felt chilly increased the typing mistakes by 74% and reduced output (productivity) by 46%. Another study by IJzerman and Semin [2] showed that warmth in the office environment encouraged closeness and friendliness. Thus, creating a comfortable environment by optimally setting the office temperature can be a significant competitive advantage to companies, saving as much as 12.5% in worker wages [1].

However, finding the optimal office temperature is not easy and requires that the preferences of multiple individuals can be accurately modeled. Obtaining such models is not trivial due to the combination of complex thermodynamics of the human body and the non-linear mapping between environmental variables and personal preferences. The topic of thermal comfort modeling has been studied for at least Daniel Nikovski Mitsubishi Electric Research Labs Cambridge, MA USA nikovski@merl.com

four decades. Presently, the dominant model was developed by Dr. Povl Ole Fanger [3], [4]. Dr. Fanger's model did not model the thermal comfort of a single individual, but rather the mean vote of thermal comfort of a group of individuals. Here the thermal vote is defined to be an integer between 1 and 7 on the Bedford Scale or between -3 and 3 on the ASHRAE scale. Fanger's model is calibrated such that at most 5% of respondents are dissatisfied when the predicted mean vote is comfortable. This model was adopted as an international standard in ISO 7730.

Fanger's model is based on heat balance equations that describe the transfer of heat from the body to the environment. The model only requires one input (room temperature) but relies on multiple additional factors such as metabolic rate, effective mechanical power produced by the body, clothing insulation, surface area of the body, mean radiant temperature, relative air velocity, humidity, convective heat transfer, and clothing surface temperature. These factors are assumed or solved iteratively in the model. The assumptions are made based on Fanger's original experimental work, which focused on a small group of north European men. One particularly criticized assumption stemming from this experimental group is the metabolic rate. Recent work has shown that in the mixed gender office environment, Fanger's model assumptions may be overestimating the metabolic rate of females by as much as 35% [5].

In addition to Fanger's work, there are other heat exchange models that have been developed. Examples of this work include modeling human thermoregulation in the 1960s, refined body segmentation with individualized heat exchange models [6], and subdividing the human body into an active controlling system and a passive controlled system [7]. Competing with these models is a class of models called adaptive models that explain thermal comfort as a function of outdoor and indoor temperature. Examples of this literature include the European Committee for Standartizations CEN method [8], and the American Society of Heating, Refrigerating, and Air-Conditioning Engineers (ASHRAE) method [9]. Building on these approaches, Haldi proposes a probabilistic model for thermal comfort in this PhD dissertation [10]. These models are typically calibrated by season (ex. Summer, Winter, etc.) and address the critique of the physiological models that they are incapable of capturing

seasonal variation in individual preferences. Lastly, data driven approaches for comfort modeling have been proposed by Jiang and Yao [11] and Farhan et. al. [12], but these focus on a few machine learning models or the prediction of comfort on a limited scale.

In contrast with prior work, this work captures the emerging capabilities of the Internet of Things and wearable technology to personalize machine learning models to a degree which was not available even five years ago. This work is based on the fact that we can monitor biometric measurements such as heart rate and skin temperature, as well as room measurements such as temperature, humidity, and airspeed, and combine these with un-sensed features such as length of day, day of the year, and time of the day. These unprecedented capabilities allow us to use a new set of features (simulated sensors) that, while not as precisely biologically detailed as in the case of thermodynamic models, allow us to avoid using imprecise estimates of parameters such as clothing insulation or metabolic rate.

The remainder of this paper will be divided into four sections. The first section, section II, will describe the IoT platform used to collect the sensor data for thermal comfort. Section III will discuss the machine learning algorithms and the cloud based implementation that is used to derive the thermal comfort models. Section IV will discuss briefly the user dashboard developed to provide realtime feedback to the user. Finally, the paper will conclude by showing results our work and discussing the implications of this project in section V.

# II. BUILDING THE IOT PLATFORM

The idea in this research is to fuse environmental data with data obtained from a wearable device and to create a personal predictive thermal comfort model. This research differs from prior work describing comfort models from a thermodynamics perspective, or using indoor and outdoor temperature to predict comfort. This work is also different from prior studies that modeled the output of Fanger's equation. Examples of this include Atthajariyakul and Leephakpreeda's [13] Feed Forward Artificial Neural Network model, and Castilla et. al.'s [14] polynomial model. In this work, we directly solicit feedback from the user and use this feedback to calibrate the machine learning model.

#### A. Room Sensor Node

We begin by describing the sensor node that collects room data. We collect four principle features (measurement types) from the room: temperature, humidity, air speed and occupancy. The measurement of temperature is accomplished by placing three sensors in different locations around the room. The placement of sensors in different locations facilitates the creation of new simulated measurements, such as temperature gradients, that allow the machine learning algorithms to infer additional variables, such as the warming effect of solar irradiation in the summer or the cooling effect of windows in the winter. In this project, three temperature sensors were used: the NEST Learning Thermostat accessed via the NEST API through a NEST account, an Omega iSD-TC industrial grade thermocouple, and the inexpensive DHT11 Arduino platform temperature and humidity sensor.

Similar to temperature, humidity is sensed at multiple locations in the room. Two sensors are equipped with both a humidity and temperature sensor: the NEST Learning thermostat and the DHT11 Arduino sensor. Because these are placed in different parts of the room, the sensor readings enable the generation of simulated features, such as a humidity gradient that might be generated by heating from the HVAC system. The air speed around a user is approximated by readings from a Modern Device Wind Speed Sensor. This sensor is currently located near the user but will be replaced in the future with more precise user-centered algorithms that estimate air speed around the user. Such algorithms are currently under development at MERL [15]. Lastly, the system measures room occupancy such that this information can be incorporated in smart HVAC control methods that allow greater energy efficiency. Occupancy is measured by two Parallax PIR sensors that are placed near the entry point and near the user. A picture of the room sensor node can be seen in Fig. 1.



Figure 1. Room Sensor Node

The Arduino sensor readings from the occupancy sensor, the DHT11, and the Windspeed Sensor are collected using an Arduino Mega 2560. This Arduino is connected to one of two gateways in this project, a Raspberry Pi B+. The Raspberry Pi serves as the room sensor node gateway that receives data from the NEST Learning Thermostat, the Omega thermocouple via Ethernet, and Arduino Data via its USB port. This gateway then packages the data from each sensor into a JSON packet and forwards this packet to Microsoft's Azure Cloud using the Advanced Message Queuing Protocol (AMQP). A secondary function of the Raspberry Pi is to run a messaging queue that ensures massage retention when communication failure is observed. This messaging queue is hosted on a local server, on the Raspberry Pi, running RabbitMQ. Lastly, we should note that this system could be further optimized by constructing small voltage division circuits that allow us to connect the Arduino Sensors directly to the Raspberry Pi GPIO ports.

#### B. Wearable Sensor Node

The second sensor node in the personal thermal comfort estimation system is the wearable sensor node. Despite the large number of available wearable sensors, there is a shortage of platforms that have sensors in excess of the basic heart rate sensor and accelerometers. Here the Microsoft Band 2 stands out as the best wearable sensor node. This is because it has more than 11 sensors including heart rate, skin temperature, ambient light, galvanic skin response, barometer, altimeter, pedometer, accelerometer, gyroscope, distance measurement, calories, and UV light exposure. In addition, through its open API, the Band allows access to all of these sensors and internal algorithm outputs at a sampling rate of 1 Hz on a continuous basis. This sample rate is more than sufficient for the purpose of estimating personal thermal comfort. Lastly, one should note that despite being sampled continuously, the Band retains its long battery life, lasting more than 8 hours on average from a full charge. Thus this wearable sensor node is suitable for deployment in an office thermal comfort optimization scenario.

The gateway for the Band sensor node is a Nokia 635 Windows Phone. To enable this phone to act as an IoT gateway, an application was written that subscribes to the Band Sensors and then packages the incoming data into JSON packets that are then forwarded to Azure. Because there are currently no mobile AMQP libraries on the Windows Phone platform, the data is encrypted using 256 bit encryption and sent to Azure via HTTPS. In addition, the phone maintains a list of measurements whose sending operations failed. This list substitutes for mobile deployments of messaging queues such as RabbitMQ that are not available for the Windows Phone.

We enable the application to schedule a background task on the Windows Phone such that the sensor data is transmitted to Azure continuously from behind the phone lock screen. This is implemented to improve the battery life of the application. Here too, we note that the phone is capable of lasting a complete work day on a single charge.

Lastly, the phone application facilitates another important feature of the system: feedback collection. Using speech recognition by Cortana, the phone app is capable of listening to the user and recording his state as: Very Cold, Cold, Chilly, Comfortable, Warm, Hot, or Very Hot. This feedback can also be given via the microphone on the Band, which makes the phone gateway merely a transmission device that can be placed anywhere within Bluetooth signal range.

# III. ADDING MACHINE LEARNING

Having created the telemetry system to collect the data, the next step is to focus on the machine learning component that will enable the generation of personalized comfort models. In general there are two approaches to creating a predictive model. The first approach is to predict discrete classes of thermal comfort. This can be accomplished by training classification algorithms that determine which of the 7 user comfort states correspond to the current set of sensor measurements. The second approach is to predict a continuous value of thermal comfort. That is, we acknowledge that a user is discretizing his comfort state into the 7 point Bedford or ASHRAE scale. Using these discretized samples, a regression function is trained that predicts a continuous value of thermal comfort. The sections to follow briefly describe the machine learning methods used. For a full and detailed description refer to [16].

#### A. Classification Methods

There are five classification methods whose performance was evaluated in this project. These methods are Logistic Regression, Support Vector Machines, Linear Discriminant Analysis, Quadratic Analysis and K Nearest Neighbor. Logistic Regression is a method that models the probability of a particular class (thermal comfort state) given a particular sample of data using the logistic function. Linear and Quadratic Discriminant Analysis are methods of obtaining classification boundaries assuming the conditional probability of a sample of data, given a particular class that can be described by a multivariate Gaussian distribution. Support Vector Machines (SVMs) are a method of finding boundaries between neighboring classes that maximize the distance (margin) between the nearest points of each class and the boundary. SVMs can take advantage of the kernel trick, which means the problem can be solved in an alternative space where boundaries could be easier to find. Lastly, K Nearest Neighbor is a non-parametric method of classification where the mean vote of the K nearest neighbors is used to determine the query class.

# B. Regression Methods

In addition to the classification methods, ten regression methods were used to model the continuous function of thermal comfort. Broadly, these methods can be classified as linear regression methods, probabilistic regression methods and non-linear regression methods.

The first method of linear regression used was Ordinary Least Squares (OLS). This method serves as a baseline test to compare the performance of other methods. The OLS method is mathematically described as finding the regression coefficients that minimize the sum of the squared errors:

$$error = \sum_{i=1}^{n} (y_i - \sum_{j=1}^{k} (\beta_j x_{i,j})),$$
(1)

where y are the reported user comfort measurements,  $x_{i,j}$  are the observed sensor readings (features),  $\beta_j$  are the regression coefficients, n are the total number of data samples, and kare the number of coefficients in the regression. In addition to OLS, three modified linear regression methods are tested in this paper, LASSO, Ridge Regression and Elastic Net. These methods add a penalty term to the Least Squares loss function that penalizes large, or in some cases, non-zero coefficients:

LASSO: 
$$\lambda \sum_{j=1}^{k} |\beta_j|$$
 Ridge:  $\lambda \sum_{j=1}^{k} (\beta_j)^2$  (2)

Here  $\lambda$  is a meta parameter of the algorithms that must be tuned for each data set on which LASSO and Ridge Regression are trained. As shown above, the LASSO penalty is the  $L_1$  norm of the regression coefficients, which forces the identified model to be sparse with few non-zero coefficients. This is particularly helpful in revealing which of the inputs are the most important predictors. In the case of thermal comfort, this helps to determine which room and personal measurements most directly determine the state of the human comfort. In contrast, Ridge Regression uses a penalty that is the  $L_2$  norm of the regression coefficients, which penalizes large model coefficients. Ridge Regression does not result in a sparse model but reduces the variance in regression coefficients when some sensor measurements are correlated. Combining the two types of penalties, Elastic Regression seeks to minimize model variance while reducing the effect of inputs that are not relevant:

$$\lambda_1 \sum_{j=1}^k |\beta_j| + \lambda_2 \sum_{j=1}^k (\beta_j)^2$$
 (3)

Lastly, Least Angle Regression builds a regression model by sequentially adding predicting inputs.

To improve on linear regression methods, we incorporate probabilistic regression methods that add information in the form of probabilistic descriptions of noise in the data. In particular, the methods here assume that the labeled data is generated by recording the true data plus Gaussian noise,  $N(0, \sigma^2)$ . Then the labeled data has the form:

$$y(n) = x(n) + \epsilon(n), \tag{4}$$

where  $\epsilon(n) N(0, \sigma^2)$ . Using this model, Bayesian Ridge Regression, assumes the distribution of a probability of a thermal comfort state given the model coefficients and a

sample of data that can be described by a Gaussian distribution. Moreover, Bayesian Ridge Regression assumes the distributions for all thermal comfort states can be described by identically distributed Gaussian functions. The advantage of this approach is that Bayesian Ridge Regression directly estimates the output without the need for algorithm training, or calibration of meta parameters. If the assumption of identically distributed Gaussian distributions is relaxed to independent distributions, then the method is called Automatic Relevance Determination. Lastly, the thermal comfort can also be directly estimated using the covariance matrices and assuming the form of the covariance function. This method is called the Gaussian Process Regression.

Finally, two non-linear methods of regression are used: Support Vector Regression and Kernel Ridge Regression. Kernel Ridge Regression is the determination of a regression model using the ridge regression loss function after the kernel trick has been applied to the data. Support vector regression is a method of finding a regression function that deviates at most  $\gamma$  from the training data. Here  $\gamma$  is minimized while training the support vector regression.

#### C. Model Training and Cross Validation

For the experiments in this paper, the machine learning models described in the previous section are trained on 530 labeled data points. However, in order to eliminate bias in the model fit, the data set is partitioned to train model meta parameters and to perform cross validation to find the average root mean square error (RMSE) of the prediction. To train model parameters, we partition the original data set using a 20/80 split. Here 20% of the data is used to train the model meta parameters and choose the best data kernel, while 80% of the data is used in cross validation to obtain the average RMSE. We chose to use leave-one-out cross validation method because it has the lowest error bias of all the cross validation methods. Details regarding cross-validation methods can be found in [16].

# IV. DISPLAYING THE RESULTS

Having constructed the IoT network that collected telemetry data and trained machine learning models on this data, the final step is to combine all elements into a dashboard where the user can see real-time sensor measurements, the current algorithm estimate of thermal comfort, and a display of the change of model parameters after new user feedback is provided. This dashboard is constructed using the Bokeh package created by Continuum Analytics. A screenshot of the dashboard can be seen in Fig. 2. The dashboard is divided into three columns. The leftmost column displays some of the current measurements observed by the sensors. The center column displays the current model parameters and the changed model parameters after each user feedback recording. The rightmost column displays a plot similar to Fig. 3, which will be discussed in the section to follow. This



Figure 2. Bokeh Dashboard for User Feedback

| Method                          | RMSE  |
|---------------------------------|-------|
| SVM                             | 0.560 |
| Kernel Ridge Regression         | 0.574 |
| Logistic Regression             | 0.575 |
| Support Vector Regression       | 0.585 |
| Bayesian Ridge Regression       | 0.589 |
| Ridge Regression                | 0.597 |
| LASSO                           | 0.601 |
| ARD                             | 0.608 |
| Linear Discriminant Analysis    | 0.621 |
| Ordinary Least Squares          | 0.624 |
| Elastic Net                     | 0.622 |
| K Nearest Neighbor              | 0.634 |
| Gaussian Process Regression     | 0.701 |
| Least Angle Regression          | 0.710 |
| Quadratic Discriminant Analysis | 0.885 |
| Fanger's Method                 | 1.15  |
| Table I                         |       |

RMSE OF THERMAL COMFORT ESTIMATION METHODS

plot shows the current estimate of thermal comfort plotted onto the curve that describes the number of dissatisfied individuals if the mean group thermal comfort vote is as predicted.

#### V. RESULTS AND DISCUSSION

When supplied with telemetry data from the IoT network, the machine learning methods discussed in the previous sections are capable of producing a personalized thermal comfort model for the individual wearer. In this study the ability of the models to predict thermal comfort was evaluated using a set of 530 data points. This data was collected both passively, on days when the ordinary thermostat controlled the room temperature, and actively, by regularly sampling user feedback while varying room temperature and humidity. The results of all learning methods and Fanger's method are shown in Table I. Here the results are shown as Root Mean Squared Error (RMSE) per method and are sorted such that the method with the lowest RMSE is at the top of the table. All machine methods outperform Fanger's method. In fact, the RMSE of Fanger's Method is 1.15, which is about 50% higher than the best machine learning method, the SVM. This is true despite the fact that Fanger's formula is specifically designed as a heat transfer model that describes the transfer of heat from the body to the environment and vice versa.

This stark comparison between Fanger's formula and the machine learning methods is due to the fact that Fanger's formula assumes static values for many of its parameters. As previously mentioned, these values include a fixed metabolic rate, clothing surface temperature, and activity level. We should note that in this study, the inputs to Fanger's formula were slightly increased to include humidity and air velocity. Thus the calculation of Fanger's formula does benefit from the IoT framework.

In contrast, the machine learning methods take into account measurements for many of the variables in Fanger's equation. In addition, the non-linear relationships employed in the SVM and Kernel Ridge Regression were closest to approximating the thermodynamic relationships and physiological relationships. As an example, heart rate is closely linked to metabolic rate and its coefficient in the learned models reflects this.

Another way to evaluate the effectiveness of the thermal comfort prediction method is to compare the predicted percentage of dissatisfied (PPD). The PPD was calibrated by Fanger such that when the average comfort vote was



Figure 3. RMSE of Thermal Comfort Prediction vs. Percent of Dissatisfied Occupants

"comfortable," the percent dissatisfied would be 5%. The plot of percent dissatisfied vs the predicted thermal comfort is shown in Fig. 3. Note that the two quantities have a parabolic relationship, which means that as the mean vote moves away from 0 (comfortable) the percentage of dissatisfied people quickly increases. The background of the plot is also color coded to show the comfort scale: green stands for the comfortable zone, light blue stands for the chilly zone, light red stands for warm zone, blue stands for cold, and red stands for hot.

The red bars across the plot show the RMSE level for the given method. For example the top bar shows Fanger's method has an RMSE of 1.15, which corresponds to 33% dissatisfaction. The best machine learning method, the SVM, has an RMSE of 0.56 which corresponds to 11.5% dissatisfaction. This means that reducing the RMSE by 50% has yielded a 21.5% reduction in the percent of dissatisfied office occupants.

#### VI. CONCLUSION

This paper presented an IoT based system that uses machine learning to find a personal thermal comfort model. Using low-cost room sensors, a wearable fitness band, a smart phone and a Raspberry Pi, we assembled an IoT system that provides a sufficient level of resolution to improve the accuracy of thermal comfort prediction by about 50%. By improving the accuracy of thermal comfort prediction in a personalized fashion, this system enables the creation of office-wide thermal comfort estimation systems that are capable of solving thermal comfort problems.

#### REFERENCES

 A. Hedge, S. Wafa, and A. Anshu, "Thermal effects on office productivity," in *Proceedings of the Human Factors and Ergonomics Society Annual Meeting*, vol. 49, no. 8, 2005, pp. 823–827.

- [2] H. IJzerman and G. R. Semin, "The thermometer of social relations mapping social proximity on temperature," *Psychological Science*, vol. 20, no. 10, pp. 1214–1220, 2009.
- [3] P. O. Fanger, "Calculation of thermal comfort: introduction of a basic comfort equation," ASHRAE Transactions, vol. 73, 1967.
- [4] Ergonomics of the thermal environment Analytical determination and interpretation of thermal comfort using calculation of the PMV and PPD indices and local thermal comfort criteria, ISO, 2005.
- [5] B. Kingma and W. van Marken Lichtenbelt, "Energy consumption in buildings and female thermal demand," *Nature Climate Change*, vol. 5, pp. 1054–1056, 1967.
- [6] H. Zhang, "Human thermal sensation and comfort in transient non-uniform thermal environments," Ph.D. dissertation, University of California, Berkeley, 2003.
- [7] D. Fiala, "Dynamic simulation of human heat transfer and thermal comfort," Ph.D. dissertation, De Montfort University, 1998.
- [8] Indoor environmental input parameters for design and assessment of energy performance of buildings-addressing indoor air quality, thermal environment, lighting and acoustics, European Standards Commission, 2006.
- [9] Thermal Environmental Conditions for Human Occupancy, ASHRAE, 2013.
- [10] F. Haldi, "Towards a unified model of occupants' behaviour and comfort for building energy simulation," Ph.D. dissertation, EPFL, 2010.
- [11] L. Jiang and R. Yao, "Modelling personal thermal sensations using c-support vector classification (c-svc) algorithm," *Building and Environment*, vol. 99, pp. 98 – 106, 2016.
- [12] A. A. Farhan, K. Pattipati, B. Wang, and P. Luh, "Predicting individual thermal comfort using machine learning algorithms," in 2015 IEEE International Conference on Automation Science and Engineering (CASE), Aug 2015, pp. 708–713.
- [13] S. Atthajariyakul and T. Leephakpreeda, "Neural computing thermal comfort index for HVAC systems," *Energy Conversion and Management*, vol. 46, no. 1516, pp. 2553 – 2565, 2005.
- [14] M. Castilla, J. Ivarez, M. Ortega, and M. Arahal, "Neural network and polynomial approximated thermal comfort models for {HVAC} systems," *Building and Environment*, vol. 59, pp. 107 – 115, 2013.
- [15] B. Kramer, P. Grover, P. Boufounos, M. Benosman, and S. Nabi, "Sparse sensing and dmd based identification of flow regimes and bifurcations in complex flows," *ARXIV*, 2015.
- [16] T. Hastie, R. Tibshirani, and J. Friedman, *The Elements of Statistical Learning: Data Mining, Inference, and Prediction*, ser. Springer Series in Statistics. Springer New York, 2013.