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Abad, A.; Paskov, M.; Kojima, K.; Parsons, K.; Thomsen, B.C.; Savory, S.J.; Bayvel, P.

TR2016-125 September 2016

### Abstract

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*European Conference on Optical Communication (ECOC)*

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# Experimental Demonstration of Nonbinary LDPC Convolutional Codes for DP-64QAM/256QAM

Toshiaki Koike-Akino<sup>1</sup>, Kenya Sugihara<sup>2</sup>, David S. Millar<sup>1</sup>, Milutin Pajovic<sup>1</sup>, Wataru Matsumoto<sup>2</sup>, Alex Alvarado<sup>3</sup>, Robert Maher<sup>3</sup>, Domaniç Lavery<sup>3</sup>, Milen Paskov<sup>3</sup>, Keisuke Kojima<sup>1</sup>, Kieran Parsons<sup>1</sup>, Benn C. Thomsen<sup>3</sup>, Seb J. Savory<sup>4</sup>, Polina Bayvel<sup>3</sup>

<sup>1</sup> Mitsubishi Electric Research Laboratories (MERL), Cambridge, MA 02139, USA. [koike@merl.com](mailto:koike@merl.com)

<sup>2</sup> Information Technology R&D Center, Mitsubishi Electric Corp., Ofuna, Kanagawa 247-8501, Japan

<sup>3</sup> Optical Networks Group, University College London (UCL), Torrington Place, London, WC1E 7JE, UK

<sup>4</sup> University of Cambridge, Dept. of Engineering, 9 JJ Thomson Avenue, Cambridge, CB3 0FA, UK

**Abstract** We show the great potential of nonbinary LDPC convolutional codes (NB-LDPC-CC) with low-latency windowed decoding. It is experimentally demonstrated that NB-LDPC-CC can offer a performance improvement of up to 5 dB compared with binary coding.

## Introduction

Recent optical communications systems have used soft-decision (SD) decoding with low-density parity-check (LDPC) codes<sup>1–9</sup>. Although modern LDPC codes already achieve near-capacity performance in binary additive white Gaussian noise (BiAWGN) channels, conventional bit-interleaved coded modulation (BICM) based on binary LDPC codes has a fundamental limit compared to the theoretical bound, in particular for high-order modulation. By employing BICM iterative demodulation (BICM-ID), the performance can be significantly improved<sup>10</sup>. However, BICM-ID requires SD feedback from the decoder to demodulator. Hence, BICM-ID can be less practical due to the high complexity and large latency. By contrast, with nonbinary (NB) LDPC codes<sup>11–16</sup>, turbo demodulation is not needed while achieving the theoretical bound. This scheme called nonbinary-input coded modulation (NBICM)<sup>15</sup> offers even better performance than BICM-ID while keeping the total complexity low, especially when combined with high-order and high-dimensional modulation. This is a great advantage of NB-LDPC compared to BICM and BICM-ID. However, the major obstacle has laid in the fact that the decoder complexity increases with the Galois field (GF) size.

Recently, it was suggested<sup>14</sup> that the complexity issue of nonbinary decoding can be mitigated by introducing LDPC convolutional codes (LDPC-CCs)<sup>2–9</sup> with windowed decoding (WD). LDPC-CCs have drawn significant interest in recent years because of their theoretical features such as a saturation property and the practical feasibility of WD, which is capable of low-latency and low-memory decoding. In this pa-

per, we experimentally demonstrate a significant performance gain provided by NB-LDPC-CC in comparison to BICM, for dual-polarization 64-ary quadrature-amplitude modulation (DP-64QAM) and DP-256QAM. As the complexity of WD is roughly proportional to the window size and the maximum column weight, we consider the minimum column weight of 2 and small window size  $W = 6$  for low-power decoding.

## GMI of BICM and NBICM

Generalized mutual information (GMI)<sup>17</sup> has been recently used to predict SD performance of various modulation formats. The normalized GMI can be extended<sup>14</sup> for any nonbinary coding as

$$I_{\text{GMI}} = 1 - \mathbb{E} \left[ \log_Q \sum_q \exp(-L_q) \middle| B = 0 \right],$$

where  $\mathbb{E}[\cdot]$  denote the expectation,  $\{L_0, \dots, L_{Q-1}\}$  denote the log-likelihood ratio (LLR) vector as  $L_q = \log \Pr(B = 0) / \Pr(B = q)$  for the  $q$ -th element of  $\mathbb{GF}(Q)$ ,  $Q$  is the GF size, and  $B$  is the transmitted element. When  $Q = 2$ , it reduces to the conventional GMI for BICM systems. If the GF size  $Q$  matches the modulation order  $M$ , the above GMI is simply called MI for some literature as a coded modulation bound. Fig. 1 shows the normalized GMI for  $M$ -ary QAM with different GF size. Although binary coding systems (BICM with  $Q = 2$ ) have little degradation from nonbinary coding systems for high rate regimes, BICM can suffer more than 0.5 dB loss in particular for higher-order modulation in mid-/low-rate regimes. In contrast, the GMI of the NBICM systems can closely approach the Shannon limit for low signal-to-noise ratio (SNR). Note that even when  $Q < M$ , NBICM shows

some gain over BICM.

It was experimentally demonstrated<sup>17</sup> that high-order QAM with low-rate code provides higher spectral efficiency; e.g., low-rate 16QAM having an overhead (OH) of 194% can be optimal. It suggests that the performance of mid-/low-rate LDPC codes is also of a great importance.

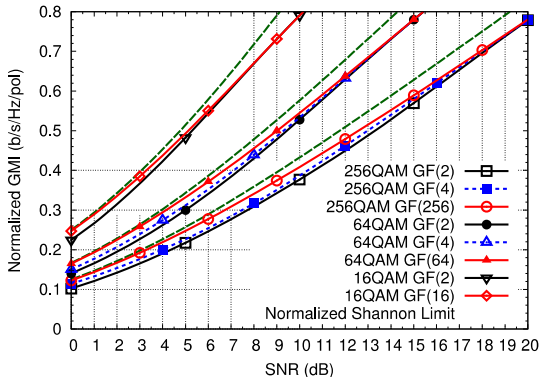


Fig. 1: Normalized GMI for 16/64/256QAMs.

In this paper, we use quasi-cyclic (QC) NB-LDPC-CCs denoted by a protograph of  $(J, K, L, N)_{GF(Q)}$ , where  $J$  is a column weight,  $K$  is a row weight,  $L$  is a termination length, and  $N$  is a QC size. The codeword length is 38,400 bits long, which is identical to a state-of-the-art LDPC code<sup>5</sup>. To keep the same codeword length for various GF size, the QC size is scaled by  $Q$ . More specifically, we consider two protographs  $(2, 20, 20, 384/\log_2 Q)_{GF(Q)}$  and  $(2, 4, 50, 384/\log_2 Q)_{GF(Q)}$  for the code rates of 0.79 (26.6% OH) and 0.49 (104% OH), respectively, for  $Q \in \{2, 4, 8, 16, 64, 256\}$ . We use low-latency WD having a limited window size of  $W = 6$  and adaptive stopping criterion<sup>15</sup>. Such low-weight codes with small window size allows significant reduction in computational complexity and memory requirement for nonbinary decoding.

### Experimental setup

NB-LDPC-CC performance was validated experimentally in a back-to-back configuration for DP-64QAM and DP-256QAM. The experimental setup<sup>18,19</sup> is illustrated in Fig. 2. A pair of digital-to-analog converters (DACs) operating at 20 GSa/s was used to generate 64QAM and 256QAM signals at 10 GBd, including 1% pilot symbols. These signals were filtered with a root-raised-cosine filter with a roll-off factor of 0.1%. After amplification, these signals were applied to an I/Q modulator operating in the linear regime. The optical carrier was generated by an external cavity laser (ECL), with a linewidth of 100 kHz. Polarization-multiplexing was emulated passively

in the optical domain with a delay of 489 symbols. Noise loading was performed by coupling in a variable power source of amplified spontaneous emission (ASE) noise. A discrete component coherent receiver was used with a bandwidth of 70 GHz, while the local oscillator was an ECL with linewidth of 100 kHz. Quantization was performed using an oscilloscope with 63 GHz bandwidth and 160 GSa/s. Offline post-processing was then performed.

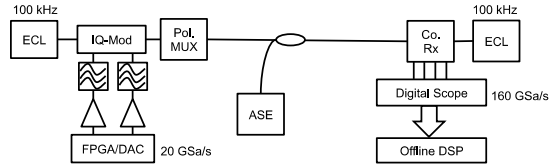


Fig. 2: Experimental setup<sup>18,19</sup>.

Our receiver digital-signal processing consisted of conventional deskey, 4th power intradyne frequency estimation, and matched filtering. A  $2 \times 2$  equalizer was used to compensate for polarization rotation, residual intersymbol interference removal and timing recovery. The equalizer was radially trained for good convergence, before being switched to pilot-aided operation. A radius directed error term was calculated based on the pilot symbols only, with updating performed using the least-mean-square algorithm and an error term averaged over 10 pilot symbols. Recently proposed carrier phase estimation<sup>18</sup> was then performed. We calculated LLR vectors using a clustering algorithm to account for transmitter distortion. The NB-LDPC-CC was then decoded using WD based on fast Fourier transform  $Q$ -ary sum-product algorithm.

### Experimental results

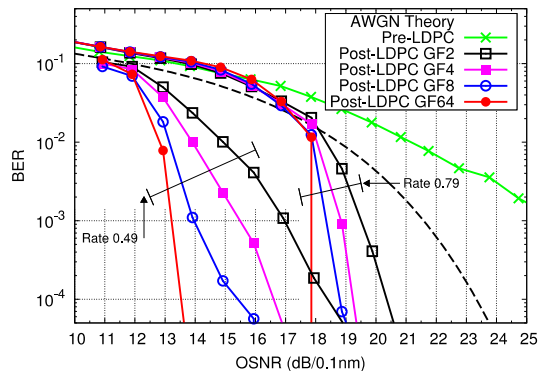


Fig. 3: Experimental results for DP-64QAM

The results of our experiments are presented in Figs. 3 and 4. Although pre-LDPC performance exhibits an error floor and a large penalty from theoretical AWGN performance, LDPC-CCs

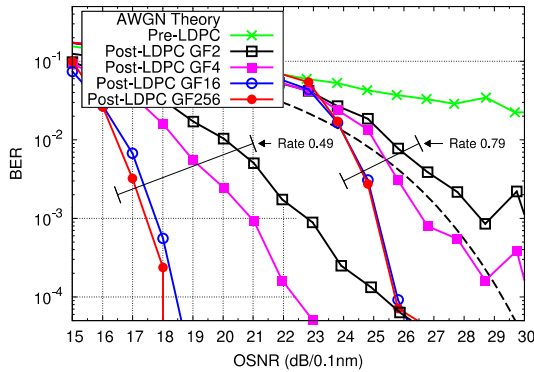


Fig. 4: Experimental results for DP-256QAM

were able to achieve error-free performance over 65,536 symbols for both DP-64QAM and DP-256QAM at high SNRs. More importantly, the bit-error-rate (BER) performance can be significantly improved by increasing the GF size. In particular for 256QAM with low-rate code, the performance improvement by nonbinary coding is more than 5 dB gain at a BER of  $10^{-3}$ . The reason why NB-LDPC-CCs offer more significant gains in comparison to the GMI predictions in Fig. 1 is because we considered practical WD for LDPC-CCs, using a very small window size  $W = 6$  and column weight of 2 for low-power decoding.

## Conclusions

We have experimentally demonstrated NB-LDPC-CC performance in back-to-back configuration using 10 GBd DP-64QAM and 256QAM, with transmitter and receiver laser linewidths of 100 kHz. Significant performance improvement by up to 5 dB gain was confirmed in the experiments. Using low-latency WD with small window size for low-weight NB-LDPC-CCs, the required computational complexity and memory size for nonbinary decoding can be maintained low, while achieving excellent BER performance.

## Acknowledgements

This work was in part funded by the UK EPSRC Programme Grant EP/J017582/1, and the Royal Academy of Engineering/ the Leverhulme Trust Senior Research Fellowship held by SJS.

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