

## A Humidity Integrated Building Thermal Model

Xu, J.; Nikovski, D.N.

TR2016-053 July 2016

### Abstract

A building thermal simulation model is proposed in this paper to predict temperature and humidity for short term response under varying environment inputs. This model is composite of a thermal circuit model to predict zone temperature and an enhanced BRE admittance model to predict zone humidity. Various environment factors such as weather, human activity, radiation, moisture absorption/desorption, ventilation, and condensation are considered. The training and prediction procedures are accelerated with approximations. In both laboratory and field tests, the model shows good performance on temperature and humidity estimation. To the best of our knowledge, it is the first time that a grey-box building thermal model can provide accurate humidity prediction with time step as short as 5 minutes.

*2016 American Control Conference (ACC)*

This work may not be copied or reproduced in whole or in part for any commercial purpose. Permission to copy in whole or in part without payment of fee is granted for nonprofit educational and research purposes provided that all such whole or partial copies include the following: a notice that such copying is by permission of Mitsubishi Electric Research Laboratories, Inc.; an acknowledgment of the authors and individual contributions to the work; and all applicable portions of the copyright notice. Copying, reproduction, or republishing for any other purpose shall require a license with payment of fee to Mitsubishi Electric Research Laboratories, Inc. All rights reserved.



# A Humidity Integrated Building Thermal Model

Jingyang Xu<sup>1</sup> and Daniel Nikovski<sup>2</sup>

**Abstract**—A building thermal simulation model is proposed in this paper to predict temperature and humidity for short term response under varying environment inputs. This model is composite of a thermal circuit model to predict zone temperature and an enhanced BRE admittance model to predict zone humidity. Various environment factors such as weather, human activity, radiation, moisture absorption/desorption, ventilation, and condensation are considered. The training and prediction procedures are accelerated with approximations. In both laboratory and field tests, the model shows good performance on temperature and humidity estimation. To the best of our knowledge, it is the first time that a grey-box building thermal model can provide accurate humidity prediction with time step as short as 5 minutes.

## I. INTRODUCTION

Building thermal simulation models are important since they can be used to evaluate building thermal status in different environment with a relatively low cost. In this paper, we consider a building thermal model for evaluating comfort-ability under the control of a modern heating, ventilation and air conditioning (HVAC) system, where air temperature and humidity for the designated zones are estimated and predicted.

Previous researchers have devoted tremendous amount of efforts on building thermal modeling techniques. As a result, various types of building thermal models are available for applications with different purposes. However, most of the available building thermal models are focusing on either temperature or humidity.

To the aspect of thermal model that predicts temperature. The most complicated type of models are built using thermal simulation software such as Energy-plus [1], TRANSYS, and etc. These software are able to construct building thermal environment with detailed construction information and usually give good prediction performance after tuning using data from real building. The physical models that support the execution of this type of models are complicated. Thus, it is able to have potentially better performance. At the same time, parameter tuning process is necessary for this type of models due to the complex condition of real systems. With these merits, however, this type of building thermal models is expensive to use for general building thermal modeling, not only because each building has to go through a model construction and tuning processes, but also because the difficulty to integrate these software with other logic unit in micro systems.

Another type of building temperature models are built using thermal factors that are generally shared by all buildings. A general structure, usually a grey-box or black-box model, is given based on basic thermal dynamic equations. After the grey-box or black-box models are constructed, the parameter values for the models are obtained by training corresponding models using historical data. Thermal circuit model is one kind of such model utilizing the similarity between electric circuit and the thermal dynamics in building thermal transition process. There are successful practices using thermal circuit model, such as [2], [3]. At the same time, models using neural network models [4] and simulink models [5] are also proposed. This type of models have the advantage of adaptivity to all kinds of buildings and lower cost on model construction. This type of model has already been used to give optimal control of building temperature with minimum energy consumption [6]. However, due to lack of accurate humidity prediction models at time steps as short as 5 minutes, human comfort control are not applicable.

To the aspect of thermal model for humidity prediction, there are models built on complicated thermal dynamic and fluid dynamic equations. Recently, there are also humidity integrated thermal models available as Modelica [8] and TRANSYS [9] libraries, where detailed equations are encapsulated. These models potentially provide accurate simulation and easy implementation for a specific building thermal environment. However, they also require detailed building construction information and also parameter calibration effort, which makes it difficult for massive implementation.

At the same time, there are grey-box models based on analytical equations of moisture transportation. As mentioned in [10], the climate and zone temperature should also be involved in moisture control. BRE admittance model, which is proposed by [11], has been widely used for short term and long term humidity predictions. [12] uses BRE model to predict and prevent mold growth in building structures. There are comparison for BRE admittance model, European indoor class model, and ASHRAE 160P model. In the comparison made by [13] on these three models, ASHRAE intermediate model has the most robust performance. At the same time, BRE admittance model is problematic in the test results. In the comparison by [14], ASHRAE 160P model and BRE admittance model are studied under uncontrolled building environment. However, it is shown in [12] that BRE model has the best performance on short term prediction, which is actually the target of our paper.

Most existing grey-box humidity models focus on humidity only. At the same time, there are studies on simultaneous heat and moisture transfer, however, mostly for porous

<sup>1</sup>Corresponding author. Innovative Decisions and Analytics LLC. Email: jingyang.xu@ida-system.com, jxu7@buffalo.edu

<sup>2</sup>Mitsubishi Electric Research Labs

media such as multi-layered porous media [15], building components [16], [17], [18], soil ground [19], and etc. Even though these studies would help understand the interaction between building thermal system and environment, they do not directly predict building air temperature and humidity when other factors get involved.

However, building thermal models that predict both temperature and humidity with short term are not well studied. The stand-alone humidity models are mostly used in long term humidity prediction with time steps range from one hour to one day. For example, [20] uses dynamic discrete autoregressive moving average (ARMA) model for temperature and humidity prediction in a naturally ventilated pig building. Thus, a humidity integrated building thermal model that works for general buildings and is able to predict temperature and humidity in short time steps would significantly accelerate the massive application of building thermal models.

In this paper, we propose a building thermal model that predicts temperature and humidity within the same framework. The building thermal model is constructed based on combining a thermal circuit model and a BRE admittance model with enhancement. The model is transformed into a state space model after approximation. The final model is able to predict both temperature and humidity for general types of buildings with time steps of five minutes. To the best of our knowledge, it is first time that a high-accuracy grey-box building thermal model is reported with prediction time interval equal or less to 5 mins. The key advantages of using the the model proposed in this paper includes:

- Humidity prediction happens at a time step of 5 minutes, while existing humidity models are typically using one hour as time steps. This makes it possible for humidity control.
- The grey-box model has relatively simple mathematical format comparing with models such as neural network models. The mathematical format makes it easier for integration with more optimal control algorithms.
- Humidity prediction accuracy is improved dramatically with temperature prediction. The models fit better for control algorithm that provide control over temperature and humidity simultaneously, which are the 2 core factors for human thermal comfort.

In the remainder of this paper, section II describes the mathematical formulation of the building thermal model. The training procedure is described in section III. Results on laboratory tests and field tests are described and analyzed in section IV. A short conclusion and description on possible extension using this model is given in section V.

## II. PROBLEM DESCRIPTION AND MODELING

The target of our problem is an accurate model for building temperature and humidity prediction with short time steps. As shown in Figure 1, there are a few significant heat and moisture transmission processes affecting thermal states of a building envelope. These heat and moisture transmission process includes:

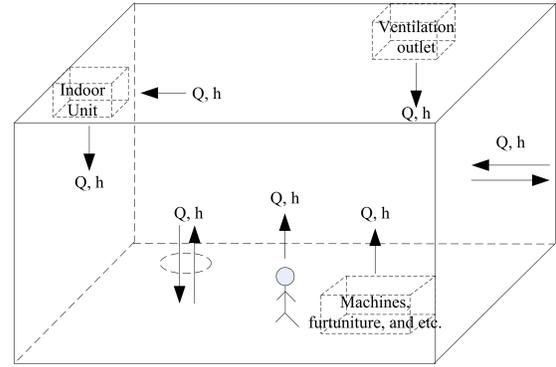


Fig. 1: Indoor Thermal Environment

- Heat and moisture transmission between zone air and outside air through building structure, including walls, windows, and etc.
- Heat and moisture transmission between zone air and inner fabrics including: inside wall surface, floor, ceiling, furniture and etc.
- Heat and moisture generation from human related activities including human moisture generation and machine utilization.
- Heat and moisture inputs from the ventilation system.
- Heat and moisture losses/gains from indoor units of the HVAC system.

### A. Notation

Necessary notation for the building thermal model is listed as following:

- $C_{Eo}$ , Thermal capacitance at outside wall surface (1)
- $C_{Ei}$ , Thermal capacitance at inside wall surface (1)
- $C_Z$ , Thermal capacitance for zone air (1)
- $E_{Outside}$ , Air enthalpy at outside environment ( $kJ \cdot kg^{-1}$ )
- $E_{vent}$ , Air enthalpy at outlets of ventilation system ( $kJ \cdot kg^{-1}$ )
- $E_Z$ , Zone air enthalpy ( $kJ \cdot kg^{-1}$ )
- $G_{Ne}$ , Coefficient for radiation heat factor's impact on outside wall surface temperature (1)
- $G_{Ni}$ , Coefficient for radiation heat factor's impact on inside wall surface temperature (1)
- $H_{r1}$ , Air specific humidity at inlet of evaporator ( $kg \cdot kg^{-1}$ )
- $H_{r2}$ , Air specific humidity at outlet of evaporator ( $kg \cdot kg^{-1}$ )
- $I_r$ , Coefficient for occupancy heat impact on zone air temperature and inside wall surface temperature (1)
- $I$ , Radiation heating factor ( $K \cdot s^{-1}$ )
- $O$ , Occupancy heating factor ( $K \cdot s^{-1}$ )
- $P_{sat}$ , Vapor saturation air pressure (Pa)
- $P_{atm}$ , Air pressure at atmosphere (Pa)
- $Q_L$ , Latent cooling rate ( $kg \cdot kg^{-1} \cdot s^{-1}$ )
- $Q_{source}$ , Moisture generation rate ( $kg \cdot s^{-1}$ )
- $Q_S$ , Sensible cooling rate ( $K \cdot s^{-1}$ )

- $R_{Eo}$ , Thermal resistance between outside air and outside wall surface (s)
- $R_{Em}$ , Thermal resistance between outside wall surface and inside wall surface (s)
- $R_{Ei}$ , Thermal resistance between inside wall surface and zone air (s)
- $R_{Win}$ , Thermal resistance between outside air and zone air through windows (s)
- $R_{Oz}$ , Thermal resistance between focal zone air and other zone air (s)
- $S_d$ , Fan speed factor at evaporator of the indoor unit ( $s^{-1}$ )
- $S_{vent}$ , Ventilation air flow speed ( $m^3 \cdot s^{-1}$ )
- $S_1$ , Heating effect on outside wall surface ( $K \cdot s^{-1}$ )
- $S_2$ , Heating effect on inside wall surface ( $K \cdot s^{-1}$ )
- $S_3$ , Heating effect on zone air ( $K \cdot s^{-1}$ )
- $T_{Outside}$ , Outside air temperature (K)
- $T_{Osurf}$ , Outside wall surface temperature (K)
- $T_{Isurf}$ , Inside wall surface temperature (K)
- $T_Z$ , Zone air temperature (K)
- $T_{Oz}$ , Other zone temperature (K)
- $T_{e1}$ , Air temperature at inlet of evaporator (K)
- $T_{e2}$ , Air temperature at outlet of evaporator (K)
- $T_{vent}$ , Air temperature at outlet of ventilation system (K)
- $V_a$ , Zone air volume ( $m^3$ )
- $a$ , Ventilation humidity coefficient (1)
- $b$ , Ventilation humidity coefficient (1)
- $c$ , Ventilation humidity coefficient ( $kg \cdot kg^{-1}$ )
- $h_z$ , Zone air specific humidity or moisture content ( $kg \cdot kg^{-1}$ )
- $h_{zr}$ , Zone air relative humidity (%)
- $h_{sat}$ , Saturation specific humidity ( $kg \cdot kg^{-1}$ )
- $h_{vent}$ , Specific humidity of air from ventilation system outlet ( $kg \cdot kg^{-1}$ )
- $h_v$ , Vapor thermal constant ( $kg^{-1}$ )
- $h_{sat}$ , Air saturation specific humidity ( $kg \cdot kg^{-1}$ )
- $h_{Isurf}$ , Air specific humidity at inside wall surface ( $kg \cdot kg^{-1}$ )
- $h_{vent}$ , Air specific humidity at outlets of ventilation system ( $kJ \cdot kg^{-1}$ )
- $h_{Outside}$ , Air specific humidity at outside environment ( $kg \cdot kg^{-1}$ )
- $k_1$ , Humidity driving force factor ( $kg \cdot s^{-1}$ )
- $k_2$ , Temperature driving force factor ( $kg \cdot K \cdot s^{-1}$ )
- $\dot{m}$ , Air flow speed factor at evaporator of indoor unit (1)
- $n$ , Air exchanging rate factor between inside and outside air ( $s^{-1}$ )
- $t$ , Time (s)
- $\alpha$ , Moisture admittance factor ( $s^{-1}$ )
- $\beta$ , Moisture admittance factor ( $s^{-1}$ ) ]
- $\beta'$ , Moisture admittance factor after linearization of  $h_{sat}$  expression ( $kg \cdot kg^{-1} \cdot s^{-1} \cdot K^{-1}$ )
- $\gamma$ , Constant ( $kg \cdot kg^{-1} \cdot s^{-1}$ )
- $\delta$ , Ventilation admittance factor ( $m^{-3}$ )
- $\epsilon$ , Temperature admittance factor ( $kg \cdot kg^{-1} \cdot K^{-1}$ )

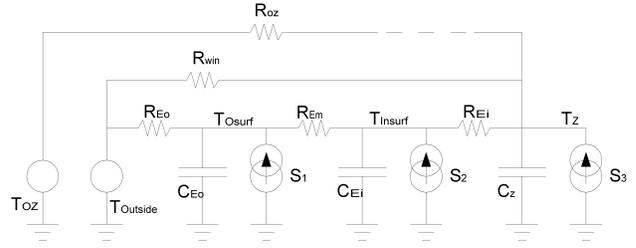


Fig. 2: Third Order Building Thermal Model

- $\epsilon$ , Temperature admittance factor ( $kg \cdot kg^{-1} \cdot K^{-1} \cdot s^{-1}$ )
- $\rho$ , Air density ( $kg \cdot m^{-3}$ )

### B. Temperature Model

We use a third order thermal circuit model to predict the zone air temperature changing under varying HVAC inputs and environment conditions. As shown in Figure 2, the temperature  $T_{Oz}$ ,  $T_{Outside}$ , and  $T_Z$  and their differences work as driving potential of heat flow. The capacitance units  $C_{Eo}$ ,  $C_{Ei}$ , and  $C_Z$  represent heat storage capacity at different components of the building. The resistance units  $R_{Oz}$ ,  $R_{Win}$ ,  $R_{Eo}$ ,  $R_{Em}$ , and  $R_{Ei}$  represent heat transmission channels. The heat generation by internal and external factors, such as radiation, human activity, and HVAC inputs, are represented as heat sources of  $S_1$ ,  $S_2$ , and  $S_3$ .

The thermal dynamics can be described using an ordinary differential equation system with equations (1), (2), and (3). These equations are constructed using Kirchhoff's current law, Ohm's law, and electric characteristic of capacitors.  $S_1$ ,  $S_2$ , and  $S_3$  can be calculated using equations (4), (5), and (6), according to the thermal factors considered,

$$\frac{dT_{Osurf}}{dt} = \frac{T_{Outside} - T_{Osurf}}{R_{Eo}C_{Eo}} + \frac{T_{Isurf} - T_{Osurf}}{R_{Em}C_{Eo}} + \frac{S_1}{C_{Eo}} \quad (1)$$

$$\frac{dT_{Isurf}}{dt} = \frac{T_{Osurf} - T_{Isurf}}{R_{Em}C_{Ei}} + \frac{T_Z - T_{Isurf}}{R_{Ei}C_{Ei}} + \frac{S_2}{C_{Ei}} \quad (2)$$

$$\frac{dT_Z}{dt} = \frac{T_{Outside} - T_Z}{R_{Win}C_Z} + \frac{T_{Isurf} - T_Z}{R_{Ei}C_Z} + \frac{T_{Oz} - T_Z}{R_{Oz}C_Z} + \frac{S_3}{C_Z} \quad (3)$$

$$S_1 = G_{Ne}I \quad (4)$$

$$S_2 = G_{Ni}I + I_rO \quad (5)$$

$$S_3 = \dot{m}Q_S + (1 - I_r)O \quad (6)$$

### C. Humidity Model

The original BRE admittance model can be described in equation (7) according to [11]. In equation (7),  $-\alpha h_z + \beta h_{sat}$  represents the humidity balancing between zone air and interior fabrics in the moisture absorption/desorption process. The term  $n(h_z - h_{Outside})$  represents the impact on zone humidity from the air exchange between inside and outside air through building structures, where  $n$  is the factor of air exchanging factor. The term  $\frac{Q_{source}}{\rho V_a}$  represents the impact from indoor moisture sources.

$$\frac{dh_z}{dt} = -\alpha h_z + \beta h_{sat} - n(h_z - h_{Outside}) + \frac{Q_{source}}{\rho V_a} \quad (7)$$

BRE model has been widely used for general building humidity modeling, though problems are also reported [13]. This is because that BRE model was proposed under the assumption of near constant zone temperature and without HVAC systems.

The humidity balancing dynamics in the moisture absorption/desorption process, which is represented by  $-\alpha h_z + \beta h_{sat}$  in (7), is derived from a mass transfer equation (8). As it is also mentioned in the literature on coupled heat and moisture transportation within building components, temperature differences should also be considered as a driving potential in the mass transfer process. Temperature difference  $T_Z - T_{I_{surf}}$  will be significant in a HVAC actively controlled zone.

$$\frac{dm}{dt} = k_1(h_z - h_{I_{surf}}) \quad (8)$$

A new mass transfer equation (9) should be used in studying building humidity models when zone air temperature and inside wall surface temperature have significant differences. After similar derivation, the original BRE admittance model described in (7) can be updated to (10), considering fluctuating zone air temperature.

$$\frac{dm}{dt} = k_1(h_z - h_{I_{surf}}) + k_2(T_Z - T_{I_{surf}}) \quad (9)$$

$$\frac{dh_z}{dt} = -\alpha h_z + \beta h_{sat} - n(h_z - h_{Outside}) + \frac{Q_{source}}{\rho V_a} + \epsilon(T_Z - T_{I_{surf}}) \quad (10)$$

Meanwhile, modern buildings are usually equipped with HVAC systems that are actively ventilating and cooling/heating inside air using different controlling logic units. The ventilation system outlet humidity  $h_{vent}$  is typically affected by  $h_z$ ,  $h_{Outside}$ , and etc. With ventilation effect considered, the humidity model in (10) is updated to (11).

$$\frac{dh_z}{dt} = -\alpha h_z + \beta h_{sat} - n(h_z - h_{Outside}) + \frac{Q_{source}}{\rho V_a} + \epsilon(T_Z - T_{I_{surf}}) - \delta S_{vent}(h_z - h_{vent}) \quad (11)$$

During cooling season, condensation process would also affect the humidity for zone air. This is because the indoor unit of HVAC typically have lower temperature at evaporator than dew point of zone air during initial cooling process. The lower temperature would lead to a condensation process that would dehumidify the air that flows through the evaporator. Air humidity is reduced from condensation process at evaporators. The total amount of power consumed by HVAC on cooling is namely divided into two parts:  $Q_S$  and  $Q_L$ .  $Q_S$  is sensible cooling rate that causes air temperature drop.  $Q_L$  is latent cooling rate that causes condensation of moisture into water from the air. The humidity model can be further updated to (12), where term  $\frac{Q_L \dot{m}}{h_v \rho V_a}$  represents the humidity loss in the condensation process.

$$\frac{dh_z}{dt} = -\alpha h_z + \beta h_{sat} - n(h_z - h_{Outside}) + \frac{Q_{source}}{\rho V_a} + \epsilon(T_Z - T_{I_{surf}}) - \delta S_{vent}(h_z - h_{vent}) + \frac{Q_L \dot{m}}{h_v \rho V_a} \quad (12)$$

### III. MODEL TRAINING AND DATA PREPARATION

The humidity integrated thermal model described by equations (1), (2), (3), (4), (5), (6), and (12), is a nonlinear model. In the model, there are a few different types of variables: model parameters, measurable factors, predictable factors, state variables, and state based factors. State variables are temperature and humidity of the target building. Parameter variables are those that define building thermal characteristics such as: thermal resistance, thermal capacitance, coefficients for inputs and state variables. Others are input variables to be calculated, measured, or obtained from forecast.

#### A. Measurable and Predictable Factors

The measurable and predictable factors in the model are those that can be measured by specific sensors or can be calculated from data measured by sensors. The directly measurable and predictable factors include outside air temperature  $T_{Outside}$ , radiation level  $I$ , indoor occupancy level  $O$ , indoor air volume  $V_a$ , air specific humidity level at ventilation outlet  $h_{vent}$ , air temperature at ventilation outlet  $T_{vent}$ , fan speed at evaporator  $S_d$ , fan speed at ventilation system  $S_{vent}$ , air temperature at inlet and outlet of evaporator  $T_{e1}$  and  $T_{e2}$ , and air specific humidity at inlet and outlet of evaporator  $H_{r1}$  and  $H_{r2}$ .

At the same time, there are a few factors need to be estimated from measured data. The moisture generation rate from human activities can be estimated from occupancy level using equation (13).

$$Q_{source} = \epsilon O \quad (13)$$

Sensible heating/cooling can be estimated from the heating/cooling effect on zone air using equation (14).

$$Q_S = S_d(T_{e2} - T_{e1}) \quad (14)$$

Latent cooling has nonzero value only during cooling season, and can be estimated using equation (15). The estimation is based on the condensation effect on specific humidity change of air flow passing through a evaporator.

$$Q_L = \min(0, S_d(H_{r2} - H_{r1})) \quad (15)$$

#### B. Linear Approximation

Saturation specific humidity  $h_{sat}$  in equation (12) has a nonlinear relation with  $T_Z$  and can be described using equation (16) and (17). This nonlinear relation leads to higher requirement on computational effort when fitting parameters with historical data. By using  $P_{atm} = 101325 Pa$ , the nonlinear relation between  $T_Z$  and  $h_{sat}$  show good linearity within range of normal zone air temperature ( $20^\circ C \sim 30^\circ C$ ). With this property available, we can approximate the relation between  $h_{sat}$  and  $T_Z$  using a linear function (18). By using this linear approximation, the humidity model is updated to equation (19).

$$P_{sat} = e^{\frac{77.345 + 0.0057T_Z - 7235/T_Z}{T_Z^2}} \quad (16)$$

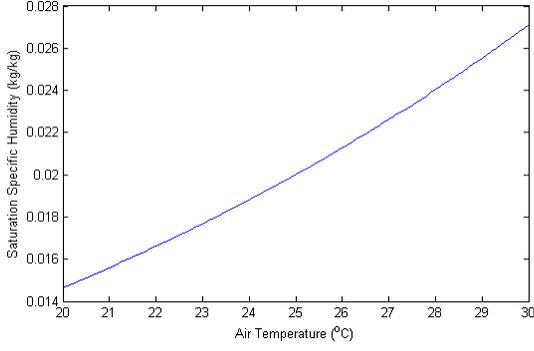


Fig. 3:  $h_{sat} - T$  Chart

$$h_{sat} = \frac{0.62198P_{sat}}{P_{atm} - P_{sat}} \quad (17)$$

$$h_{sat} = \frac{\beta'}{\beta}T_Z + \frac{\gamma}{\beta} \quad (18)$$

$$\begin{aligned} \frac{dh_z}{dt} = & -\alpha h_z + \beta' T_Z + \gamma - n(h_z - h_{Outside}) \\ & + \frac{Q_{source}}{\rho V_a} + \epsilon(T_Z - T_{Insurf}) \\ & - \delta S_{vent}(h_z - h_{vent}) + \frac{Q_L \dot{m}}{h_v \rho V_a} \end{aligned} \quad (19)$$

### C. Ventilation

For natural ventilation system,  $h_{vent} = h_{Outside}$  in (19). However, modern ventilation systems usually have enthalpy recovery design. For example, Mitsubishi Lossnay ventilation system has such design and is part of the ventilation system used in the field test.  $h_{vent}$  for a ventilation system with enthalpy recovery design would be a function depends on both zone air thermal characteristics and outside air thermal characteristics. In an example of such ventilation system, the temperature and enthalpy can be approximately calculated using linear equations. For example, (20) and (21) are the equations for calculation ventilation outlet air temperature and enthalpy for Lossnay system.

$$T_{vent} = T_{Outside} - 0.75(T_{Outside} - T_Z) \quad (20)$$

$$E_{vent} = E_{Outside} - 0.68(E_{Outside} - E_Z) \quad (21)$$

*Proposition 1:* It can be derived that  $h_{vent}$ ,  $h_{Outside}$ , and  $h_z$  also have a linear approximation as shown in equation (23), where coefficient  $a$ ,  $b$ , and  $c$  are constant based on outside climate conditions with fluctuating  $T_{Outside}$  and  $T_Z$ . When  $T_{Outside}$  and  $T_Z$  are constant  $15^\circ C$ ,  $h_{vent}$  can be described in equation (22).

$$h_{vent} = 0.32h_{Outside} + 0.68h_z \quad (22)$$

$$h_{vent} = ah_{Outside} + bh_z + c \quad (23)$$

*Proof:* Using enthalpy calculation formula that can be stated as (24), we can have a function of  $h_{vent}$  using  $E_{vent}$  and  $T_{vent}$  as (25).

$$E = 1.01(T - 273.15) + h[2501 + 1.85(T - 273.15)] \quad (24)$$

$$h_{vent} = \frac{E_{vent} - 1.01(T_{vent} - 273.15)}{2501 + 1.85(T_{vent} - 273.15)} \quad (25)$$

Using (20) and (21) for  $E_{vent}$  and  $T_{vent}$ , we can get:

$$h_{vent} = \frac{0.32E_{Outside} + 0.68E_Z - 0.2525T_{Outside}}{1995.6725 + 0.4625T_{Outside} + 1.3875T_Z} - \frac{0.7575T_Z + 1.01 \times 273.15}{1995.6725 + 0.4625T_{Outside} + 1.3875T_Z} \quad (26)$$

Using (24) for  $E_{Outside}$  and  $E_Z$ , we can get:

$$h_{vent} = \frac{0.0707T_{Outside} + h_{Outside}(638.6152 + 0.592T_{Outside})}{1995.6725 + 0.4625T_{Outside} + 1.3875T_Z} - \frac{0.0707T_Z + h_z(1357.0573 + 1.258T_Z)}{1995.6725 + 0.4625T_{Outside} + 1.3875T_Z} \quad (27)$$

Let  $T_{Outside} = 288.15 \pm \Delta_{Outside}$ ,  $T_Z = 288.15 \pm \Delta_Z$ , we can get:

$$\begin{aligned} h_{vent} = & \frac{809.2h_{Outside} + 1719.5499h_z}{2528.75 \pm 0.4625\Delta_{Outside} \pm 1.3875\Delta_Z} \\ & + \frac{0.0707\Delta_{Outside} \mp 0.0707\Delta_Z}{2528.75 \pm 0.4625\Delta_{Outside} \pm 1.3875\Delta_Z} \\ & + \frac{\pm 0.592h_{Outside}\Delta_{Outside} \pm 1.258h_z\Delta_Z}{2528.75 \pm 0.4625\Delta_{Outside} \pm 1.3875\Delta_Z} \end{aligned} \quad (28)$$

When  $\Delta_{Outside} = 0$ ,  $\Delta_Z = 0$ , we can get:

$$\begin{aligned} h_{vent} & = \frac{809.2h_{Outside} + 1719.5499h_z}{2528.75} \\ & = 0.32h_{Outside} + 0.68h_z \end{aligned} \quad (29)$$

Assuming outside and inside temperature fluctuation less than  $20^\circ C$ , we have  $\Delta_{Outside} \leq 10$  and  $\Delta_Z \leq 10$ , then we have:

$$\begin{aligned} 0.3174h_{Outside} + 0.6745h_z - 0.000563 & \leq h_{vent} \\ & \leq 0.3247h_{Outside} + 0.6899h_z + 0.000563 \end{aligned} \quad (30)$$

Since Lossnay's enthalpy recovery equation (21) is also an approximation, we can use  $h_{vent} = 0.32h_{Outside} + 0.68h_z$  as an approximation for  $h_{vent}$  when outside and indoor air temperature average are at  $15^\circ C$ . When there is temperature fluctuations for  $T_{Outside}$  and  $T_Z$  within normal range, a general function (23) can be used, where the coefficients' values can be updated according the predicted temperature fluctuation information. ■

### D. Model Training Procedure

Since the temperature model and the humidity model are coupled via the parameter  $\dot{m}$  and state variables  $T_Z$  and  $T_{Insurf}$ , the training process can be accelerated in a two step procedure by decoupling the temperature model and the humidity model:

- 1) Train the parameter in temperature model and obtain parameter values and temperature state values.
- 2) Train the parameter in humidity model by using the parameter value and temperature state values as known information.

According to the data collection frequency and easy computation, we discretize the humidity integrated thermal model using zero order approximation with time step of 5 minutes. After discretization, the humidity integrated thermal model can be written as (31). Where,

- $x_t$  is a vector for thermal state including:  $T_{Osurf}$ ,  $T_{Insurf}$ ,  $T_Z$ , and  $h_z$  at time  $t$ ,
- $u_t$  is the inputs from environment and HVAC systems including:  $I$ ,  $O$ ,  $Q_S$ ,  $Q_L$ ,  $h_{Outside}$ ,  $S_{vent}$ , and  $h_{vent}$ .

- $A$  and  $B$  are coefficient matrices based on model structure.

$$x_{t+1} = Ax_t + Bu_t \quad (31)$$

When enthalpy recovery ventilation system is considered, the humidity integrated thermal model can not be written in a form shown in (31), due to the term  $\delta S_{vent}(h_z - h_{vent})$ . In the second step of the training procedure, an iterative procedure is carried out to overcome this by iteratively updating  $(h_z - h_{vent})$ .

- 1) Assume  $S_{vent}$  always at maximum speed, and the model can be written into the form shown in (31),
- 2) Train the humidity model and obtain an output of  $h_z$ ,
- 3) Use  $h_z - h_{vent}$  as known inputs information to the model,
- 4) Train the humidity model and obtain an output specific humidity value  $h_{z,new}$ ,
- 5) If  $\frac{|h_z - h_{z,new}|}{h_{z,new}} \leq 0.001$ , stop, otherwise let  $h_z = h_{z,new}$  and go to step 3.

In terms of time range, data are collected for 7 consecutive days. Data from the first 6 days are used for model training while the last day is used for model prediction test.

#### IV. EXPERIMENT AND RESULTS ANALYSIS

The computational platform is a PC with 3.4GHz Intel Pentium 4 CPU with 2 GB RAM. The training and prediction errors are evaluated using rooted mean square error (RMSE), normalized rooted mean square error (NRMSE), and mean absolute percentage error (MAPE).

The laboratory test is performed in a test chamber with 30  $m^3$  volume and equipped with inner decoration to simulate furniture and inner fabrics of a normal business type building. In the laboratory test, ventilation systems are turned off. There are two data sets used in model validation: a cooling season data set and a heating season data set.

Training data are collected and used to fit model parameters as shown in section III-D. After training, all parameter values are fixed. Prediction procedure is carried out using the fitted model, which predicts temperature and humidity as a multi-step-ahead predictor. The predicted temperature and humidity and corresponding measured values in the next 24 hours are compared to obtain the accuracy of the model prediction error. In Figure

##### A. Laboratory Test

In the first step of training procedure, temperature values are obtained. As shown in the first row of Table I, the estimation error is small in the first training step when comparing the temperature outputs from thermal model and the measure zone air temperature. In the second step of the training, we compare two different versions of BRE admittance models, where original BRE model is named for equation (7) as in [11], enhanced BRE model is named for equation (19).

In Table I,  $h_z$ (Original BRE model) and  $h_{zr}$ (Original BRE model) represents the MAPE on humidity prediction on specific humidity and relative

TABLE I: Experiment Results during Cooling Season

	RMSE	NRMSE	MAPE(%)
$T_Z$	4.64	0.49	1.41
$h_z$ (Original BRE model)	0.0011	0.0785	6.80
$h_z$ (Enhanced BRE model)	0.0003	0.0212	1.56
$h_{zr}$ (Original BRE model)	4.989	0.00007	6.65
$h_{zr}$ (Enhanced BRE model)	1.238	0.00001	1.39

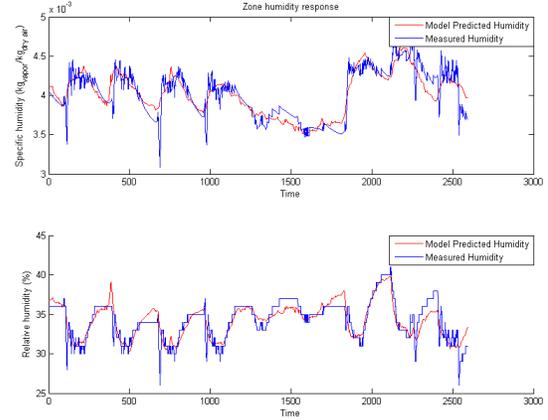


Fig. 4: Zone Humidity Training in Field Test

humidity when using original BRE model, where the prediction MAPE is around 7%.  $h_z$ (Enhanced BRE model) and  $h_{zr}$ (Enhanced BRE model) represent the MAPE on MAPE on humidity prediction on specific humidity and relative humidity when using humidity integrated thermal model proposed in this paper, where the prediction MAPE is reduced to less than 2%.

##### B. Field Test

The field test is performed in a physical business building that is in use. The building has 10 office zones and 4 separate ventilation systems which are working coordinately with indoor units. One testing data set from Jan 31 to Feb 8 2012 is implemented. The estimation results verify that our proposed model is able to capture the dynamics well. The test results are also shown in Table II.

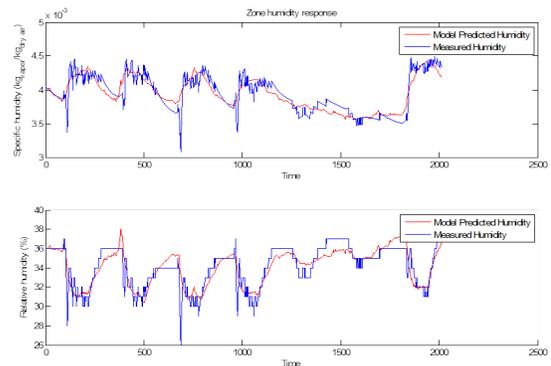


Fig. 5: Training Results for 2012/01/31~2012/02/06

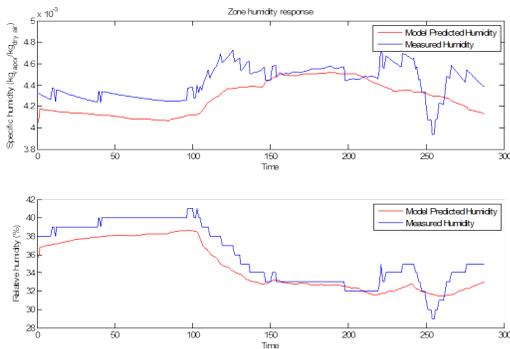


Fig. 6: Prediction Results for 2012/02/07

TABLE II: Field Test Results on Estimation Humidity During Winter

	RMSE	NRMSE	MAPE(%)
$h_z$ (Enhanced BRE model)	1.439	0.081	2.7
$h_{zr}$ (Enhanced BRE model)	1.199	0.00001	2.6

Another set of data are also collected during spring and summer in 2012. As HVAC system gets into cooling mode, vapor condensation and latent cooling become significant. At the same time, due the lack of humidity and temperature sensors at evaporators and ventilation systems in the field test environment, a few variables need to be estimated such as:  $T_{e1}$ ,  $T_{e2}$ ,  $H_{r1}$ ,  $H_{r2}$ ,  $S_d$ . Thus, the sensible cooling and latent cooling loads can not be accurately estimated from temperature drop, humidity drop, and fan speed at evaporators. By assume the condensation rate and fan speed to be constant, these variables that are supposed to be measured by sensors are obtained during training process. By using a couple of assumptions to roughly estimate the sensible cooling and latent cooling loads, we derived a couple of models:

- BRE\_CC: A model version derived based on assumption that fan speed is constant and condensation rate is constant.
- BRE\_CC\_VFS: A model version derived based on assumption that fan speed is variant and condensation rate is constant, where fan speed is proportional to total power consumption rate of HVAC system.
- BRE\_VC\_VFS: A model version derived based on assumption that fan speed is variant and condensation rate is also variant, where both fan speed and condensation rate are proportional to total power consumption rate of HVAC system.

The field test on three summer weeks are executed using the three models, which are BRE\_CC, BRE\_CC\_VFS, BRE\_VC\_VFS. The three data sets contain one week data in each of them, where the first six days' data in one data set is used as training data and the seventh day's data is used for prediction experiments. As shown in Table III, the MAPE for the three models are all within 9%. To illustrate, Figure 7 shows the training results and corresponding based on data range from 2012/07/04 to 2012/07/09. The blue line

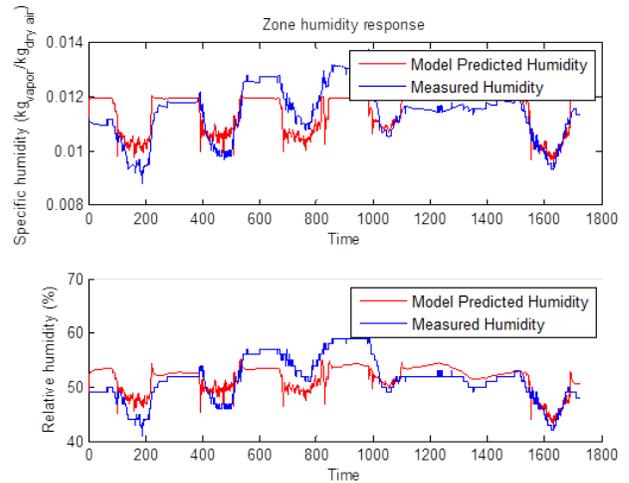


Fig. 7: Training Results for 2012/07/04~2012/07/09

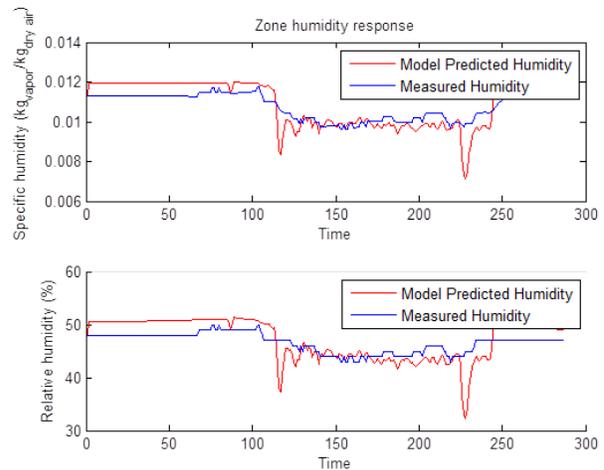


Fig. 8: Prediction Results for 2012/07/10

in the figure is the measured humidity, while the read line in the figure is model output for training stage. After models are fitted based on data from 2012/07/04 to 2012/07/09, the model is used to predict humidity on date 2012/07/10. Figure 8 shows the prediction result. The blue line shows the actual measured humidity on 2012/07/10, while the red line is model predicted humidity. In both figures, the time step is 5 minutes.

## V. CONCLUSION

This paper proposes a humidity integrated building thermal model that is able to recapture the temperature and humidity dynamic within short time range. The model is based on modifying and coupling an existing thermal circuit model and a BRE admittance model. With enhancements and approximations, the model we propose could obtain very accurate estimation and prediction with small computational requirements. The results show that this model can provide prediction MAPE less than 10% in field tests.

TABLE III: Field Test Results on Humidity Prediction in Spring and Summer

2012/06/06-2012/06/12				
Data Range	Specific Humidity		Relative Humidity	
Factor	RMSE( $10^{-4}$ )	MAPE	RMSE( $10^{-4}$ )	MAPE
Error Type				
BRE_CC	6.438	5.94%	3.0447	5.87%
BRE_CC.VFS	10.1358	7.42%	4.8038	7.36%
BRE_VC.VFS	10.626	8.51%	5.0389	8.43%
2012/06/10-2012/06/16				
Data Range	Specific Humidity		Relative Humidity	
Factor	RMSE( $10^{-4}$ )	MAPE	RMSE( $10^{-4}$ )	MAPE
Error Type				
BRE_CC	8.8760	7.53%	4.2309	7.42%
BRE_CC.VFS	6.9586	5.95%	3.3146	5.86%
BRE_VC.VFS	28.919	2.33%	1.3775	2.29%
2012/07/04-2012/07/10				
Data Range	Specific Humidity		Relative Humidity	
Factor	RMSE( $10^{-4}$ )	MAPE	RMSE( $10^{-4}$ )	MAPE
Error Type				
BRE_CC	5.7810	4.78%	2.4439	4.68%
BRE_CC.VFS	5.1178	4.29%	2.1527	4.19%
BRE_VC.VFS	6.477	4.97%	2.780	4.89%

## REFERENCES

[1] Y. Pan, M. Zuo, and G. Wu, "Whole building energy simulation and energy saving potential analysis of a large public building," *Journal of Building Performance Simulation*, vol. 4, no. 1, pp. 37–47, 2011.

[2] M. J. Jimenez and H. Madsen, "Models for describing the thermal characteristics of building components," *Building and Environment*, vol. 43, no. 2, pp. 152–162, 2008.

[3] M. J. Jimenez, H. Madsen, and K. K. Andersen, "Identification of the main thermal characteristics of building components using matlab," *Building and Environment*, vol. 43, no. 2, pp. 170–180, 2008.

[4] R. H. Dodier and G. P. Henze, "Statistical analysis of neural networks as applied to building energy prediction," *Journal of Solar Energy Engineering*, no. 1, 2004.

[5] N. Mendes, G. H. C. Oliveria, and H. X. D. Araujo, "Building thermal performance analysis by using matlab/simulink," in *7th International IBPSA Conference*, 2001.

[6] D. Nikovski, J. Xu, and M. Nonaka, "A method for computing optimal set-point schedules for hvac systems," in *Proceedings of the 11th REHVA World Congress CLIMA*, 2013.

[7] M. Wetter, "Modelica-based modelling and simulation to support research and development in building energy and control systems," *Journal of Building Performance Simulation*, vol. 2, no. 2, pp. 143–161, 2009.

[8] M. Steeman, K. Goethals, J. Laverge, A. Janssens, and M. D. Paepe, "On modelling moisture buffering when evaluating humidity controlled hvac systems," in *Eleventh International IBPSA Conference*, 2009.

[9] J. Lstiburek, "Moisture control for buildings," *ASHRAE Journal*, vol. 42, no. 2, pp. 36–41, 2002.

[10] R. Jones, "Indoor humidity calculation procedures," *Building Services Engineering Research and Technology*, vol. 16, no. 3, pp. 119–126, 1995.

[11] T. Oreszczyn and S. E. C. Pretlove, "Condensation targeter ii: Modelling surface relative humidity to predict mould growth in dwellings," *Building Services Engineering Research and Technology*, vol. 20, no. 3, pp. 143–153, 1999.

[12] S. M. Cornick and M. K. Kumaran, "A comparison of empirical indoor relative humidity models with measured data," *Journal of Building Physics*, vol. 31, no. 3, pp. 243–268, 2008.

[13] P. Roppel, W. C. Brown, and M. Lawton, "Modeling of uncontrolled indoor humidity for ham simulations of residential buildings," in *Proceedings, Thermal Performance of the Exterior Envelopes of Whole Buildings X*, 2007.

[14] F. Tariku, K. Kumaran, and P. Fazio, "Transient model for coupled heat, air and moisture transfer through multilayered porous media," *International Journal of Heat and Mass Transfer*, vol. 53, no. 15-16, pp. 3035–3044, 2010.

[15] M. Qin, R. Belarbi, A. Ait-Mokhtar, and L. Nilsson, "Coupled heat and moisture transfer in multi-layer building materials," *Construction and Building Materials*, vol. 23, no. 2, pp. 967–975, 2009.

[16] H. Janssen, B. Blocken, and J. Carmeliet, "Moisture control for buildings," *Conservative modelling of the moisture and heat transfer in building components under atmospheric excitation*, vol. 50, no. 5-6, pp. 1128–1140, 2007.

[17] M. Qin, R. Belarbi, A. Ait-Mokhtar, and A. Seigneurin, "An analytical method to calculate the coupled heat and moisture transfer in building materials," *International Communications in Heat and Mass Transfer*, vol. 33, no. 1, pp. 39–48, 2006.

[18] M. P. Deru and A. T. Kirkpatrick, "Ground-coupled heat and moisture transfer from buildings part1 - analysis and modeling," *Journal of Solar Energy Engineering*, vol. 124, no. 1, pp. 10–16, 2002.

[19] P. I. Daskalov, "Prediction of temperature and humidity in a naturally ventilated pig building," *Journal of Agricultural Engineering Research*, vol. 68, no. 4, pp. 329–339, 1997.