MITSUBISHI ELECTRIC RESEARCH LABORATORIES
http://www.merl.com

# Proportional–Integral Extremum Seeking for Vapor Compression Systems

Burns, D.J.; Laughman, C.R.; Guay, M.

TR2016-049 July 2016

**Abstract**

In this paper, we optimize vapor compression system power consumption through the application of a newlydeveloped proportional–integral extremum seeking controller (PI-ESC) that converges at the same timescale as the process. This method modifies the control law to include terms proportional to the estimated gradient, but this modification of the control law requires a more sophisticated gradient estimator in order to avoid bias. We develop a PI-ESC for which this bias is eliminated. PI-ESC is applied to the problem of compressor discharge temperature setpoint selection for a vapor compression system where setpoints are automatically determined so that power consumption is minimized. The vapor compression system operates with a regulating feedback controller configured to drive the compressor discharge temperature to setpoints selected by the PI-ESC, and we use a physics-based simulation model to demonstrate that power consumption is minimized dramatically faster than by traditional perturbation-based methods.

*2016 American Control Conference (ACC)*

# Proportional–Integral Extremum Seeking for Vapor Compression Systems

Daniel J. Burns†, Christopher R. Laughman, and Martin Guay

***Abstract*— In this paper, we optimize vapor compression system power consumption through the application of a newly-developed proportional–integral extremum seeking controller (PI-ESC) that converges *at the same timescale as the process.* This method modifies the control law to include terms proportional to the estimated gradient, but this modification of the control law requires a more sophisticated gradient estimator in order to avoid bias. We develop a PI-ESC for which this bias is eliminated. PI-ESC is applied to the problem of compressor discharge temperature setpoint selection for a vapor compression system where setpoints are automatically determined so that power consumption is minimized. The vapor compression system operates with a regulating feedback controller configured to drive the compressor discharge temperature to setpoints selected by the PI-ESC, and we use a physics-based simulation model to demonstrate that power consumption is minimized dramatically faster than by traditional perturbation-based methods.**

## I. Introduction

Vapor compression machines (Fig. 1A) move thermal energy from a low temperature zone to a high temperature zone, performing either cooling or heating depending on the configuration of the refrigerant piping. The relative simplicity of the machine and its effective and robust performance has enabled the vapor compression machine in various forms and packages to become widely deployed, and it is critical to modern comfort standards and the global food production and distribution industries.

In many control formulations for vapor compression machines the evaporator superheat temperature is selected as a regulated variable for cycle efficiency and equipment protection [1], [2], [3]. However, for the commercial system considered herein, a measurement of the evaporator superheat is not available. Instead, cycle efficiency is maintained through the regulation of the compressor discharge temperature to a setpoint that depends on disturbances such as the heat load and the outdoor air temperature. The discharge temperature is often measured for equipment protection making it a commonly available signal, and because the refrigerant state at this location in the cycle is always superheated, this signal is a one-to-one function of the disturbances over the full range of expected operating points [4]. Because discharge temperature changes with heat loads and outdoor air temperatures, its setpoint cannot be regulated to a constant, but instead must vary with external conditions. It is the aim of this paper to automate the generation of such setpoints in order to maximize energy efficiency.

More specifically, the optimization target for PI-ESC is the closed-loop system consisting of a vapor compression system and feedback controller (Fig. 1B). The feedback controller is configured to regulate the zone temperature to a setpoint determined by an occupant and the compressor discharge temperature to a setpoint determined by the extremum seeking controller. The feedback controller manipulates a variable speed compressor, the electronic expansion valve and both the evaporator and condenser fans. With this multivariable feedback controller, and because of the coupling between the system actuators and regulated variables, all actuators are changed as a result of changes to the discharge temperature setpoint, and therefore a energy consumption for the entire machine is minimized, given the set of outdoor air temperature and heat load disturbances.

However, determining these energy-optimal setpoints is not straightforward. Models of the vapor compression system that attempt to describe the influence of commanded inputs on thermodynamic behavior and power consumption are often low in fidelity, and while they may have useful predictive capabilities near the conditions at which they were calibrated, the environments into which these systems are deployed are so diverse as to render comprehensive calibration and model tuning intractable. Therefore, relying on model-based strategies for realtime optimization is tenuous.

Recently, model-free extremum seeking methods that operate in realtime and aim to optimize a cost have received increased attention and have demonstrated improvements in the optimization of vapor compression systems and other HVAC applications [5], [6], [7], [8]. To date, the dominant extremum seeking algorithm that appears in the HVAC research literature is the traditional perturbation-based algorithm first developed in the 1920s [9] and re-popularized in the late 1990s by an elegant proof of convergence for a general class of nonlinear systems [10].

Most extremum seeking controllers can be viewed as a gradient descent optimization algorithm implemented as a feedback controller and therefore consists of two main functional components: (1) a estimation part that determines the local gradient of the performance metric with respect to the decision variables, and (2) a control law part that manipulates the decision variables to steer the system to the optimum of the performance metric. In the traditional perturbation-based method, a sinusoidal term is added to the input at a slower frequency than the natural plant dy-

D. J. Burns (burns@merl.com) and C. R. Laughman (laughman@merl.com) are with Mitsubishi Electric Research Laboratories, 201 Broadway, Cambridge, MA 02139.

M. Guay is with the Department of Chemical Engineering, Queen's University, Kingston, ON, Canada. email: martin.guay@chee.queensu.ca

† Corresponding author.

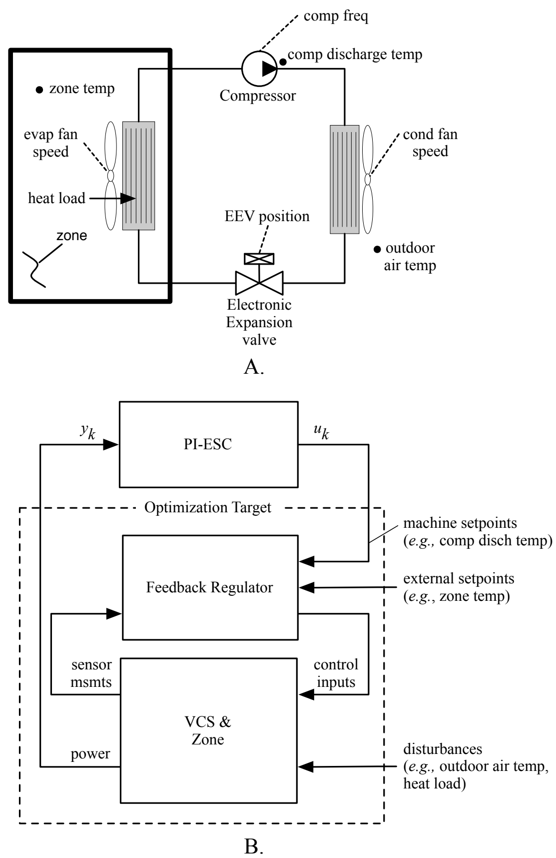



Fig. 1. A. The vapor compression system under study consists of a variable speed compressor, condensing heat exchanger, electronically controlled expansion valve, and evaporating heating exchanger. The inputs to the VCS that are manipulated by the control system include (i) the compressor frequency, (ii) the condenser fan speed, (iii) the electronic expansion valve position, and (iv) the evaporator fan speed. B. Actuator commands are computed by the feedback regulator, which drives the the vapor compression system toward setpoints that consist of external setpoints (*e.g.*, a desired zone temperature setpoint) and machine setpoints (*e.g.*, a compressor discharge temperature setpoint). The PI-ESC selects discharge temperature setpoints $u_k$ that minimize the system power consumption $y_k$ in the presence of disturbances such as changes in outdoor air temperature and heat load.

namics, inducing a sinusoidal response in the performance metric [11] and introducing a timescale slower than the process dynamics. The controller then filters and averages this signal to obtain an estimate of the gradient. Averaging the perturbation introduces yet another (and slower) time scale in the optimization process. Using a gradient estimate obtained in this way, the control law integrates the estimated gradient (with appropriate sign that depends on the optimization objective) to drive the gradient to zero.

As a result, the traditional perturbation-based extremum seeking converges to the neighborhood of the optimum at about two timescales slower than the plant dynamics due to inefficient estimation of the gradient, and slow (integral-action dominated) adaptation in the control law. For thermal systems such as vapor compression machines where the dynamics are already on the order of tens of minutes, the slow convergence properties of perturbation-based extremum seeking become impediments to wide-scale deployment.

However, convergence rates can be improved by addressing both components of the extremum seeking algorithm. A more efficient method for estimating gradients has been developed that treats the gradient as an unknown time-varying parameter to be identified. Time-varying extremum seeking (TV-ESC) uses adaptive filtering techniques to estimate the parameters of the gradient—eliminating the timescale associated with averaging perturbations [12]. Recently, the authors successfully applied this technique to vapor compression system optimization [4]. However, that method did not modify the control law, and while convergence was significantly improved compared to the perturbation-based method, the control law of TV-ESC is still integral-action dominated.

In this paper, we apply a newly-developed PI-ESC to vapor compression systems (Fig. 1B) in which the algorithm estimates the gradient using the efficient time-varying approach, but also modifies the control law to include a term proportional to the value of the estimated gradient. This term drives the system toward the optimum operating point at the same timescale as the vapor compression system dynamics. However, naïve modification of the control law to include the proportional term will introduce bias in the estimated gradient, and therefore the combined estimator–control law structure is developed concurrently in order to eliminate bias.

The rest of the paper is as follows. In Section 2 we derive the new PI-ESC. In Section 3 the performance of PI-ESC is compared to both (i) perturbation-based and (ii) time-varying extremum seeking in a simple example. Section 4 describes the simulation model used for validation and present simulated results, and concluding remarks are offered in Section 5.

## II. Proportional–Integral Extremum Seeking Controller

This section outlines the development of an extremum seeking controller based on a time-varying estimate of the gradient of the cost and a PI control law to drive the system to its optimum operating point. See [13] for the full development and stability and convergence analysis in discrete time.

### A. PI-ESC Development

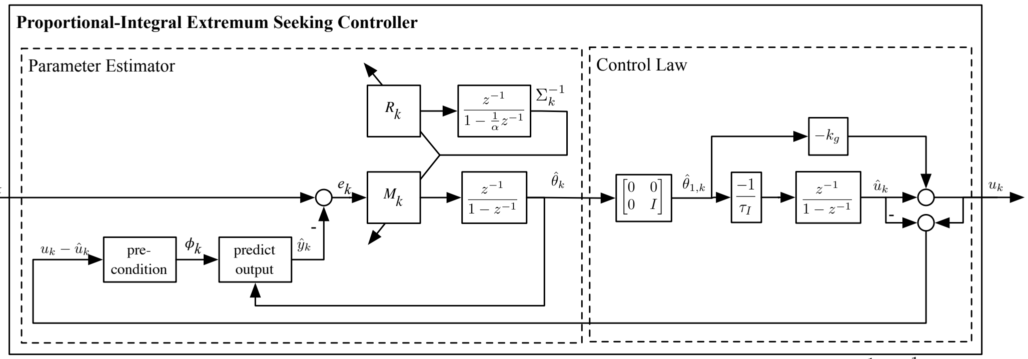
We consider a class of nonlinear systems of the form:

$$x_{k+1} = x_k + f(x_k) + g(x_k)u_k \tag{1}$$
$$y_k = h(x_k) \tag{2}$$

where $x_k \in \mathbb{R}^n$ is the vector of state variables at time $k$, $u_k$ is the input variable at time $k$ taking values in $\mathscr{U} \subset \mathbb{R}$ and $y_k \in \mathbb{R}$ is the objective function at step $k$, to be minimized. It is assumed that $f(x_k)$ and $g(x_k)$ are smooth vector valued functions and that $h(x_k)$ is a smooth function.

We assume that the cost $h(x)$ is relative order one and satisfies the optimality conditions:

1) $\frac{\partial h(x^*)}{\partial x} = 0$
2) $\frac{\partial^2 h(x)}{\partial x \partial x^T} > \beta I$, $\forall x \in \mathbb{R}^n$




pre-condition: $\phi_k = [1, (u_k - \hat u_k)^T]^T$

correct parameter estimate: $M_k = \dfrac{\frac{1}{\alpha}\Sigma_k^{-1}\omega_k}{1+\frac{1}{\alpha}\omega_k^T\Sigma_k^{-1}\omega_k}$

predict output: $\hat y_k = \hat y_{k-1} + Ke_{k-1} + \phi_{k-1}^T\hat\theta_{k-1} + \omega_{k-1}^T(\hat\theta_k - \hat\theta_{k-1})$

update covariance: $R_k = \dfrac{\frac{1}{\alpha^2}\Sigma_k^{-1}\omega_k^T\omega_k\Sigma_k^{-1}}{1+\frac{1}{\alpha}\omega_k^T\Sigma_k^{-1}\omega_k}$

Fig. 2. Overview of the PI-ESC algorithm.

where $\beta$ is a strictly positive constant.

We let $\alpha(x_k,\hat u_k) = x_k + f(x_k) + g(x_k)\hat u_k$. The rate of change of the cost function $y_k = h(x_{k+1})$ is given by:

$$\begin{aligned}h(x_{k+1}) - h(x_k) = h(x_k + f(x_k) + g(x_k)u_k)\\ - h(\alpha(x_k)) + h(\alpha(x_k),\hat u_k) - h(x_k).\end{aligned}$$

Using a second order Taylor expansion on the first two terms we can rewrite the cost dynamics as:

$$y_{k+1} - y_k = \Psi_{0,k}(x_k,\hat u_k) + \Psi_{1,k}(x_k,u_k,\hat u_k)(u_k - \hat u_k).$$

where $\Psi_{0,k}(x_k,\hat u_k) = h(\alpha(x_k,\hat u_k)) - h(x_k)$, and $\Psi_{1,k}(x_k,u_k,\hat u_k) = (\nabla h(\alpha(x_k,\hat u_k))g(x_k) + \frac{1}{2}(u_k - \hat u_k)^\top g(x_k)^\top \nabla^2 h(\tilde y_k) g(x_k)$ where $\tilde y_k = \alpha(x_k,\hat u_k) + \theta g(x_k)(u_k - \hat u_k)$ for $\theta \in (0,1)$. By the relative order one assumption on $h(x)$, the system's dynamics can be decomposed and written as:

$$\xi_{k+1} = \xi_k + \psi(\xi_k, y_k) \tag{3}$$

$$y_{k+1} = y_k + \Psi_{0,k}(x_k,\hat u_k) + \Psi_{1,k}(x_k,u_k,\hat u_k)(u_k - \hat u_k) \tag{4}$$

where $\xi_k \in \mathbb{R}^{n-1}$ and $\psi(\xi_k, y_k)$ is a smooth vector valued function. By Equation (4), the cost function dynamics are parameterized as follows:

$$y_{k+1} = y_k + \theta_{0,k} + \theta_{1,k}(u_k - \hat u_k)$$

where the time-varying parameters $\theta_{0,k}$ and $\theta_{1,k}$ represent $\Psi_{0,k}$ and $\Psi_{1,k}$, and are to be identified. Importantly, in order to estimate the gradient $\theta_{1,k}$ without bias, $\theta_{0,k}$ must also be determined.

Let $\hat\theta_{0,k}$ and $\hat\theta_{1,k}$ denote the estimates of $\theta_{0,k}$ and $\theta_{1,k}$, respectively, and consider the following state predictor

$$\begin{aligned}\hat y_{k+1} &= \hat y_k + \hat\theta_{0,k} + \hat\theta_{1,k}(u_k - \hat u_k)\\ &+ K_k e_k - \omega_{k+1}(\hat\theta_k - \hat\theta_{k+1})\end{aligned} \tag{5}$$

where $\hat\theta_k = [\hat\theta_{0,k}, \hat\theta_{1,k}^T]^T$ is the vector of parameter estimates at time step $k$ given by any update law, $K_k$ is a correction factor at time step $k$, $e_k = y_k - \hat y_k$ is the state estimation error. We let $\phi_k = [1, (u_k - \hat u_k)^T]^T$. The variable $w_k$ is the following output filter at time step $k$

$$w_{k+1} = w_k + \phi_k - K_k w_k, \tag{6}$$

with $\omega_0 = 0$. Using the state predictor defined in (5) and the output filter defined in (6), the prediction error $e_k = y_k - \hat y_k$ is given by

$$\begin{aligned}e_{k+1} &= e_k + \phi_k\tilde\theta_{k+1} - K_k e_k\\ &+ \omega_{k+1}(\hat\theta_k - \hat\theta_{k+1}) + w_{k+1}(\theta_{k+1} - \theta_k)\\ e_0 &= y_0 - \hat y_0.\end{aligned} \tag{7}$$

An auxiliary variable $\eta_k$ is introduced which is defined as $\eta_k = e_k - w_k^T\tilde\theta_k$. Its dynamics are given by

$$\begin{aligned}\eta_{k+1} &= e_{k+1} - w_{k+1}\tilde\theta_{k+1}\\ \eta_0 &= e_0.\end{aligned} \tag{8}$$

Since $\eta_k$ is unknown, it is necessary to use an estimate, $\hat\eta$, which is generated by the recursion:

$$\hat\eta_{k+1} = \hat\eta_k - K_k\hat\eta_k \tag{9}$$

Let the identifier matrix $\Sigma_k$ be defined as

$$\Sigma_{k+1} = \alpha\Sigma_k + w_k^T w_k, \quad \Sigma_0 = \alpha I \succ 0 \tag{10}$$

with an inverse generated by the recursion

$$\begin{aligned}\Sigma_{k+1}^{-1} =& \Sigma_k^{-1} + \left(\frac{1}{\alpha} - 1\right)\Sigma_k^{-1}\\ &- \frac{1}{\alpha^2}\Sigma_k^{-1} w_k (1 + \frac{1}{\alpha} w_k^T \Sigma_k^{-1} w_k)^{-1} w_k^T \Sigma_k^{-1}\end{aligned} \tag{11}$$

Using (5), (6), and (9), the parameter update law is

$$\hat\theta_{k+1} = \hat\theta_k + \frac{1}{\alpha}\Sigma_k^{-1}\omega_k^T\left(I + \frac{1}{\alpha}w_k\Sigma_k^{-1}w_k^T\right)^{-1}(e_k - \hat\eta_k) \tag{12}$$

And to ensure that the parameter estimates remain within the constraint set $\Theta_k$, we use a projection operator [12], [14]

$$\bar{\hat\theta}_{k+1} = \mathrm{Proj}\{\hat\theta_k + \Sigma_k^{-1}w_k^T\left(I + w_k\Sigma_k^{-1}w_k^T\right)^{-1}(e_k - \hat\eta_k), \Theta_k\}. \tag{13}$$

The algorithm for the projection operator must be designed to ensure that estimates are bounded within the constraint set and guarantee stability. The design of projection algorithms in discrete-time systems must be done with care—it cannot be designed as in the continuous-time case. A suitable discrete-time projection algorithm is presented in [13].

Finally, the proposed control law is given by:

$$u_k = -k_g \hat{\theta}_{1,k} + \hat{u}_k \tag{14}$$

$$\hat{u}_{k+1} = \hat{u}_k - \frac{1}{\tau_I}\hat{\theta}_{1,k}. \tag{15}$$

where $k_g$ and $\tau_I$ are positive constants to be assigned. One aspect of the proposed adaptive controller (14) is that the quantity $\hat{\theta}_{1,k}$ estimates the term $\theta_{1,k}$ which itself depends on the input. As a result, once estimation of $\theta_{1,k}$ is achieved, the resulting control action defines a recursive map that converges to the implicitly defined state-feedback controller $u_k = \alpha(x_k, \hat{u}_k)$. The proof of convergence provided in [13] demonstrates how the effect of the dependence of $\theta_{1,k}$ on $u_k$ can be handled in the design of the PI-ESC.

### B. PI-ESC Summary

The final PI-ESC algorithm consists of a time varying parameter estimation routine for determining $\hat{\theta}_{0,k}$ and $\hat{\theta}_{1,k}$ and consists of Equations (5), (6), (9), (11), and (13) with tuning parameters $K$ and $\alpha$. The control law is given by Equations (14) and (15) and contains terms proportional to the estimated gradient and with integral action necessary to identify optimal equilibrium conditions, and is tuned using the parameters $k_g$ and $\tau_I$. A block diagram of the PI-ESC algorithm summarizing signal flow is shown in Figure 2.

## III. Comparison of Extremum Seeking Methods

This section presents the convergence performance of three extremum seeking controllers applied to a previously published example problem [4]. Traditional perturbation-based extremum seeking control [15], time-varying extremum seeking control [12] and proportional–integral extremum seeking control [13] are each applied to the problem of finding input values to a simple Hammerstein system that minimize its output, where the controllers have no knowledge of the plant model (see Fig. 3A). The system equations are

$$\begin{aligned} x_{k+1} &= 0.8x_k + u_k \\ y_k &= (x_k - 3)^2 + 1, \end{aligned}$$

which has a single optimum point at $u^* = 0.6$, $y^* = 1$.

The pole near the unit circle represents the dominant process dynamics and establishes a fundamental limit on convergence rate. Reasonable effort is made to obtain parameters for all ESC methods that achieve the best possible convergence rates. The perturbation ESC parameters are

$$\begin{aligned} d_k &= 0.2\sin(0.1k) \\ \omega_{LP} &= 0.03 \\ K &= -0.005 \end{aligned}$$

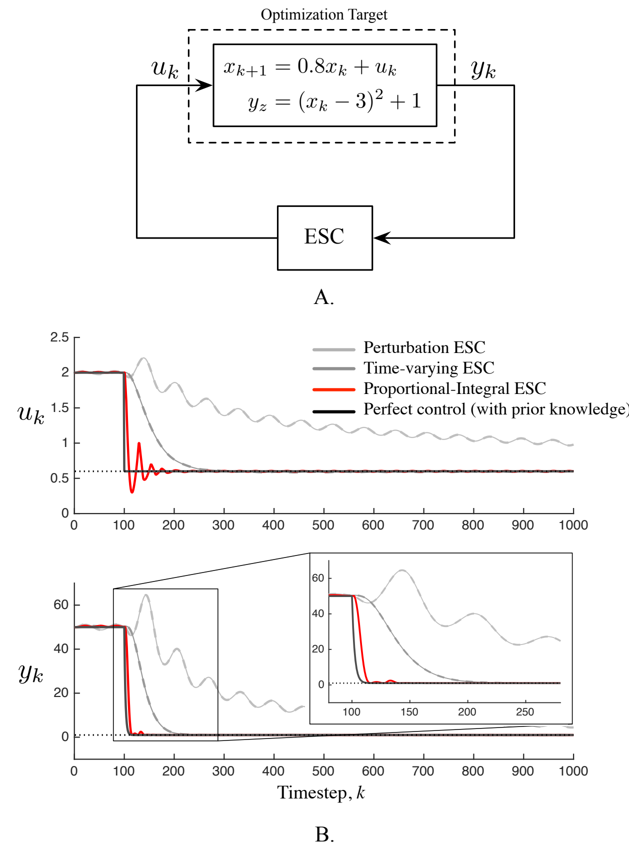



Fig. 3. A. A simple Hammerstein system under extremum seeking control. B. A comparison of the convergence performance of three extremum seeking algorithms. The top plot shows the control variable $u_k$ and the bottom plot shows the performance metric $y_k$. The perturbation-based ESC (light gray) converges to the optimum after about 4000 steps (not shown), Time-varying ESC (gray) converges in about 100 steps, while the PI-ESC (red) converges in about 15 steps—roughly 10 steps longer than the bang-bang method (black) with prior knowledge of the optimizer. The inset figure in the bottom plot shows a detailed view of the convergence.

Where $d_k$ is the sinusoidal perturbation, $\omega_{LP}$ is the cutoff frequency for a first-order low-pass averaging filter, and $K$ is the (integral-action) adaptation gain. A high-pass washout filter is not used as convergence rate is improved without it.

The parameters used for the TV-ESC are

$$\begin{aligned} d_k &= 0.001\sin(0.1k) & k_i &= 0.001 \\ \alpha &= 0.1 & \varepsilon &= 0.4, \end{aligned}$$

where $k_i$ is the (integral-action) adaptation gain, $\alpha$ is the estimator forgetting factor, and $\varepsilon$ is the estimator timescale separation parameter.

The parameters used for the PI-ESC are

$$\begin{aligned} d_k &= 0.001\sin(0.2k), & \tau_I &= 60, \\ \alpha &= 0.5, & k_g &= 0.0003, \quad K = 0.1, \end{aligned}$$

where $\tau_I$ is the integral time constant, $k_g$ is the proportional gain and is computed from the relationship $k_g = 1/(\tau_I^2)$, $\alpha$ is the estimator forgetting factor, and $K$ is the estimation gain.

Simulations are performed starting from an initial input value of $u_0 = 2$ and the ESC methods are turned on after

100 steps. The resulting simulations are shown in Fig. 3B. The perturbation ESC method converges to a neighborhood around the optimum in about 4000 steps (not shown in the figure), the TV-ESC method converges in about 100 steps, while the PI-ESC method converges in about 15 steps. The resulting controller performance is compared to the response obtained from a controller that has *a priori* knowledge of the system optimizer and applied directly in one time step, for which the output settles in about 10 steps.

The fast convergence characteristic of PI-ESC is well suited to the optimization of thermal systems with their associated long time constants. In the next section, we apply the PI-ESC algorithm to the problem of selecting setpoints for the discharge temperature of a vapor compression machine and present simulation results.

## IV. PI-ESC for Vapor Compression Systems

While the proposed algorithm is general and applicable to a wide range of problems, the fast convergence property is particularly appropriate for thermal systems and their characteristically slow dynamics. To demonstrate, we apply PI-ESC to the problem of determining a setpoint for discharge temperature such that power consumption is minimized, while other feedback loops maintain the setpoints of regulated variables in the presence of disturbances such as changes in outdoor air temperature and heat load (see Fig. 1B). In this section, we briefly describe a realistic nonlinear physics-based model of the vapor compression cycle that models the thermofluid dynamics, and then present the results of applying this PI-ESC method to determine the optimal compressor discharge setpoint for this vapor compression cycle model, and compare these results to the performance of a traditional perturbation-based extremum seeking method.

### A. Model Description

A detailed model describing the nonlinear dynamics of the vapor compression cycle is developed using the equation-oriented modeling language Modelica [16]. Physics-based models are constructed for the four principal components: the evaporating and condensing heat exchangers, the compressor, and the electronic expansion valve. Algebraic models are used for the compressor and the expansion valve because the dynamics of these components are much faster than that of the heat exchangers. The partial differential equations representing the mass, momentum, and energy balances for the refrigerant in the heat exchangers are discretized into 48 volumes along the direction of flow using the 1-D finite volume method. A real gas model of the refrigerant R410a was used for the primary working fluid, and a moist air model was used to describe the changes in the temperature and relative humidity of the secondary working fluid due to heat transfer between the refrigerant and the air through the discretized tube wall. Additional details for this model are found in [17].

The PI-ESC algorithm is implemented in the Modelica language for the purposes of testing the PI-ESC control method on the vapor compression cycle. Because this algorithm is cast in discrete-time while the model of the vapor compression cycle is in continuous-time, the `Modelica.Synchronous` library is used to interface between these two paradigms by using a clocked approach. The resulting cycle and control system comprise a set of 8,663 differential algebraic equations with 124 state variables, and is compiled and simulated using the Dymola 2016 compiler [18] running on an i7 desktop machine with 8GB of memory.

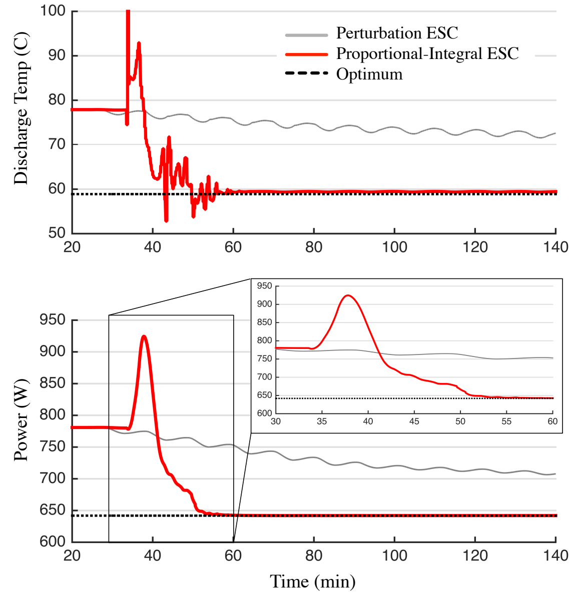



Fig. 4. A simulation of discharge temperature setpoint optimization using PI-ESC and perturbation ESC. Starting with a compressor temperature hotter than optimal for the particular set of boundary conditions, PI-ESC discovers values that minimize the power consumption. PI-ESC drives the system to its optimum operating point about 20 minutes after the algorithm is switched on, whereas perturbation-based ESC settles in about 900 min (not shown).

### B. Simulation results

This section describes simulation results wherein PI-ESC is compared to perturbation-based ESC for the problem of determining compressor discharge temperature setpoints that minimize power consumption. The simulation model described in the previous section is used for evaluation.

Initially, the discharge temperature setpoint is slowly ramped from 82°$C$ to about 42°$C$ in order to obtain the steady state mapping of power vs. Td setpoint and demonstrate convexity (black line of Fig. 5). The room temperature tracking error and discharge temperature tracking errors are monitored during the ramp to ensure no dynamics are excited. Note that the steady state map has a unique global minimum, several local minima, regions of very small gradients, and sharp changes in power (at least one of which corresponding to a loss of superheating in the evaporator).

Simulations with the two extremum seeking algorithms are initialized with the discharge temperature setpoint at 78 °$C$,

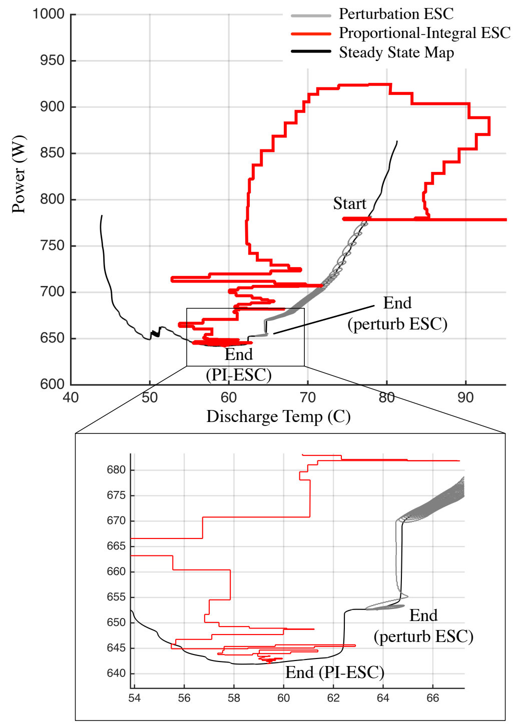



Fig. 5. Using the transient data shown in Fig. 4, the power is plotted against the decision variable. Trajectories for both PI-ESC and perturbation ESC are shown with the steady state map (black). After a transient, PI-ESC quickly converges to the true minimum power, whereas perturbation ESC converges to a local minimum for these simulation conditions.

and the system was run to steady state. At $t = 35$ min, both extremum seeking algorithms are bumplessly switched on. The transient responses are shown in Fig. 4, and the evolution of optimization is shown with steady state map in Fig. 5.

The perturbation based extremum seeking control system settles at about $t = 900$ min (Fig. 4) and ultimately becomes trapped in a local minima (Fig. 5). In contrast, the PI-ESC converges to the correct minimum in about only 20 minutes (shown in red in Figs. 4 and 5). This period is on the same order as the dominant plant time constant, representing convergence rates that enable realtime application of extremum seeking for thermodynamic applications where previous algorithms acted too slowly.

## V. Conclusion

We have developed a new extremum seeking algorithm that converges at the same timescale as the dominant plant dynamics, which requires an estimation routine designed in conjunction with the proportional-integral control law. With the gradient appropriately estimated, convergence can proceed much faster than alternative approaches. The PI-ESC algorithm is applied to the problem of compressor temperature setpoint selection such that the power consumption is minimized at timescales that can ultimately enable deployment beyond controlled laboratory conditions.

## References


[1] R. J. Otten, "Superheat control for air conditioning and refrigeration systems: Simulation and experiments," Master's thesis, University of Illinois at Urbana-Champaign, 2010.

[2] M. S. Elliott and B. P. Rasmussen, "On reducing evaporator superheat nonlinearity with control architecture," *International Journal of Refrigeration*, vol. 33, no. 3, pp. 607–614, 2010.

[3] K. Vinther, H. Rasmussen, R. Izadi-Zamanabadi, and J. Stoustrup, "Single temperature sensor superheat control using a novel maximum slope-seeking method," *International Journal of Refrigeration*, vol. 36, no. 3, pp. 1118–1129, 2013.

[4] D. Burns, W. Weiss, and M. Guay, "Realtime setpoint optimization with time-varying extremum seeking for vapor compression systems," in *American Control Conference*, 2015.

[5] D. Burns and C. Laughman, "Extremum seeking control for energy optimization of vapor compression systems," in *International Refrigeration and Air Conditioning Conference*, 2012.

[6] M. Guay and D. Burns, "A comparison of extremum seeking algorithms applied to vapor compression system optimization," in *American Control Conference*, 2014.

[7] P. Li, Y. Li, and J. E. Seem, "Efficient Operation of Air-Side Economizer Using Extremum Seeking Control," *Journal of Dynamic Systems, Measurement, and Control*, vol. 132, no. 3, May 2010.

[8] V. Tyagi, H. Sane, and S. Darbha, "An extremum seeking algorithm for determining the set point temperature for condensed water in a cooling tower," in *American Control Conference*, 2006.

[9] M. Leblanc, "Sur l'*é*lectrification des chemins de fer au moyen de courants alternatifs de fr*é*quence *é*lev*é*e," Revue G*é*n*é*rale de l'Electricit*é*, 1922.

[10] M. Krstic, "Performance Improvement and Limitations in Extremum Seeking Control," *Systems & Control Letters*, vol. 39, no. 5, pp. 313–326, April 2000.

[11] Y. Tan, W. Moase, C. Manzie, D. Nesic and, and I. Mareels, "Extremum seeking from 1922 to 2010," in *29th Chinese Control Conference (CCC)*, 2010.

[12] M. Guay, "A time-varying extremum-seeking control approach for discrete-time systems," *Journal of Process Control*, vol. 24, no. 3, pp. 98 – 112, 2014.

[13] M. Guay and D. Burns, "A proportional integral extremum-seeking control approach for discrete-time nonlinear systems," in *IEEE Conference on Decision and Control (CDC)*, 2015.

[14] G. Goodwin and K. Sin, *Adaptive Filtering Prediction and Control*. Dover Publications, Incorporated, 2013.

[15] N. J. Killingsworth and M. Krstic, "PID tuning using extremum seeking: online, model-free performance optimization," *IEEE Control Systems Magazine*, vol. 26, no. 1, pp. 70–79, Feb. 2006.

[16] Modelica Association. (2015) Modelica specification, version 3.3r1. [Online]. Available: www.modelica.org

[17] C. Laughman, H. Qiao, V. Aute, and R. Radermacher, "A comparison of transient heat pump cycle models using alternative flow descriptions," *Science and Tech. for the Built Env.*, vol. 21, no. 5, 2015.

[18] Dassault Systemes, AB. (2015) Dymola.