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GMI-Maximizing Constellation Design with Grassmann Projection for Parametric Shaping

Toshiaki Koike-Akino, David S. Millar, Kieran Parsons, Keisuke Kojima

Mitsubishi Electric Research Labs. (MERL), 201 Broadway, Cambridge, MA 02139, USA. *koike@merl.com*

Abstract: We introduce a new geometric shaping to optimize signal constellations maximizing generalized mutual information for both binary and nonbinary coding. With parametric projection of hyper cubes, we can perform seamless constellation shaping with near 1 dB gain.

OCIS codes: (060.4510) Optical communications, (060.1660) Coherent communications, (060.4080) Modulation.

1. Introduction

While constellation shaping has been widely investigated in the wireless communications for decades, it has only recently become of interest to the optical communications community [2–9]. By modifying the signal distribution to be Gaussian-like, the performance of the system is improved in an additive white Gaussian noise (AWGN) channel. There are two major approaches in the literature: probabilistic shaping [2–6] and geometric shaping [7–9].

The probabilistic shaping [2–6] adaptively modifies the probabilities of the constellation points to approximate a Gaussian distribution for a given constellation, which can be regular quadrature-amplitude modulation (QAM). Although conventional equalization and phase recovery algorithms can be used with minimal modification, an external entropy coding is required. In [3], a multi-level coded modulation scheme with trellis shaping is found to operate close to capacity. Shell mapping for a 4-dimensional (4D) QAM is presented in [4]. In [5], probabilistic shaping is implemented by a many-to-one mapping with iterative demapping. In [6], 15% capacity and 43% reach increase were experimentally verified by shaped 64QAM.

For the geometric shaping [7–9], the location of the constellation points is adaptively modified to approach Gaussian. While each constellation point is equally likely, the demodulation complexity can be increased as the constellation is irregular. In [7], a multi-ring construction is optimized by modifying the spacings of rings and the spacing of constellation points on the ring. To optimize an irregular constellation, an iterative Arimoto–Blahut algorithm is used in [8], showing more than 1 dB gain with iterative demodulation.

This paper proposes a novel geometric shaping based on Grassmann projection, allowing a parametric expression of modulation points to resolve one of the drawbacks of irregular constellations. We optimize the shaping not to approach Gaussian distribution (which is only optimal with nonbinary coding in AWGN channels) but to maximize the generalized mutual information (GMI) [10, 11] for bit-interleaved coded modulation (BICM) systems.

2. Grassmann Projection for Parametric Shaping

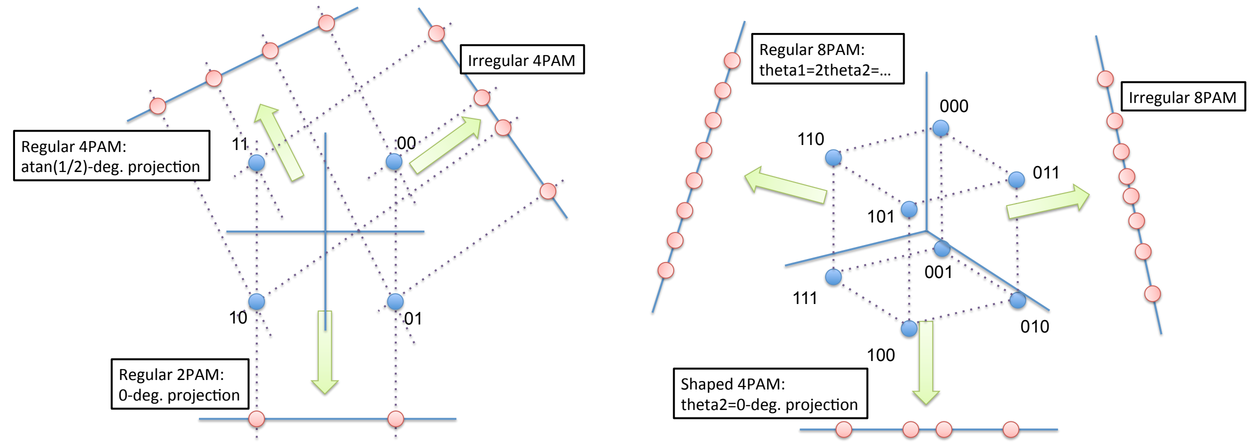
Regular QAMs can be generated by superposing multiple 2-ary pulse-amplitude modulations (PAM). For example, 16QAM uses independent 4PAMs for I and Q components, where 4PAM is a superposition of two 2PAMs as $2(-1)^{b_1} + (-1)^{b_1+b_2}$ (power normalization is omitted), where b_i is the i-th bit. The constellation points of 2^N-ary PAMs can be modified by changing the superposition scales as follows: $\sum_i^N P_i(-1)^{b_i}$, where P_i is the superposition coefficient (some bits can be Gray-coded, e.g., by substituting as $b_i \leftarrow \sum_{j\leq i} b_j$). This generalization can be thought of as a 1D projection of N-D hyper cubes formed by 2PAMs. This can be extended to any number of dimensional projections for high-dimensional modulation design by a linear projection as follows:

$$\boldsymbol{x} = \boldsymbol{P}\boldsymbol{s}, \quad \boldsymbol{s} = \left[(-1)^{b_1}, (-1)^{b_2}, \dots, (-1)^{b_N}\right]^{\mathrm{T}}, \tag{1}$$

where $\boldsymbol{x} \in \mathbb{R}^{M\times 1}$ is the M-dimensional constellation points, $\boldsymbol{P} \in \mathbb{R}^{M\times N}$ is a projection matrix, and $\boldsymbol{s} \in \mathbb{R}^{N\times 1}$ is the N-dimensional 2PAM hyper cube for N-bit modulation. We propose a shaping scheme based on a Grassmann projection: Any M linear subspaces of N-dimensional vector space (a.k.a. Grassmannian manifold) can be represented as follows:

$$\boldsymbol{P} = \begin{bmatrix}\boldsymbol{I}_M & \boldsymbol{0}_{N-M}\end{bmatrix} \exp(\boldsymbol{\Phi}),\ \boldsymbol{\Phi} = \begin{bmatrix}\boldsymbol{0}_M & \boldsymbol{\Theta} \\ -\boldsymbol{\Theta} & \boldsymbol{0}_{N-M}\end{bmatrix},\ \boldsymbol{\Theta} = \begin{bmatrix}\theta_1 & \theta_2 & \cdots & \theta_{N-M} \\ \vdots & & & \vdots \\ \theta_{(M-1)(N-M)+1} & \theta_{(M-1)(N-M)+2} & \cdots & \theta_{M(N-M)}\end{bmatrix}, \tag{2}$$

where $\boldsymbol{I}_M$ is the identity matrix of size M. There exist $M(N-M)$ degrees of freedom (i.e., parameters of θ_k) to determine any arbitrary subspaces. Fig. 1(a) shows an example of shaped 4PAM by the Grassmann projection: $\boldsymbol{P} = [\cos(\theta_1), \sin(\theta_1)]$ for $N = 2$ and $M = 1$. When $\theta_1 = 0$, the shaped 4PAM becomes regular 2PAM, while it becomes regular 4PAM when $\theta_1 = \mathrm{atan}(1/2)$. Fig. 1(b) depicts 1D subspace projection from 3D space for shaped 8PAM, where there are two parameters of θ_1 and θ_2. When $\theta_1 = 2\theta_2 = \sqrt{4/5}\ \mathrm{atan}(\sqrt{5/16})$, the projected constellation becomes the regular 8PAM. For $\theta_2 = 0$, the shaped 8PAM is identical to the shaped 4PAM as illustrated in Fig. 1(a). In consequence, the proposed Grassmann shaping allows seamless adaptation of modulation formats including regular QAMs.



(a) Shaped 4PAM from two 2PAMs

(b) Shaped 8PAM from three 2PAMs

Fig. 1: Grassmann projection shaping of hyper cubes: N-bit space onto M subspaces.

3. Parameter Optimization for Grassmann Shaping

We optimize θ_k to maximize the GMI after log-likelihood ratio (LLR) calculation at the demodulator. The normalized GMI is given by $1 - \mathbb{E}[\log_Q(\sum_i \exp(-L_i))]$, where $\mathbb{E}[\cdot]$ denotes the expectation and L_i is the i-th LLR value for Q-ary Galois field. The GMI of shaped 4PAM as a function of θ_1 is plotted in Fig. 2, where both binary and nonbinary systems are considered for different signal-to-noise ratio (SNR) of $-5, -4, \ldots, 10$ dB. It can be seen that regular 2PAM ($\theta_1 = 0$) is optimal for low SNR regimes, and regular Gray-coded 4PAM ($\theta_1 = \pm\mathrm{atan}(1/2)$) is best for high SNR regimes. There is a transition around an SNR of $2 \sim 5$ dB. Note that the nonbinary GMI is less sensitive against θ_1 for low SNR regimes. In particular, the GMI does not go to zero at $\theta_1 = \pi/2$ if nonbinary coding is used. Another peak of the nonbinary GMI at $\theta_1 = \mathrm{atan}(2)$ is also a regular 4PAM with irregular (non-Gray) labeling (illustrating the tolerance of nonbinary systems to non-Gray bit-labeling). The extracted best θ_1 vs. SNR is plotted in Fig. 3. It was found that the optimized θ_k can be simple piece-wise linear in terms of an SNR in dB, to seamlessly adapt the constellation from regular 2PAM to regular 4PAM for $2 \sim 5$ dB. The best θ_1 and θ_2 for 8PAM shaping is also present in Fig. 3, where there is another smooth transition from regular 4PAM to regular 8PAM for $7 \sim 12$ dB.

We use the optimized θ_k of 8PAM Grassmann shaping for 64QAM. Fig. 4(a) shows the optimized shaping constellation of 64QAM for different SNRs. It is observed that the constellation changes across the SNR with soft transition between regular QAMs. Note that the shaped constellation is Gaussian-like (denser points near origin) only for nonbinary coding case. Fig. 4(b) shows the required SNR loss from Shannon limit to achieve the target GMI. The regular 64QAM has more than 1 dB loss, while the Grassmann-shaped 64QAM offers up to 0.5 dB improvement in low-GMI regimes. Although we can optimize random constellation without Grassmann projection to improve performance, the modulator and demodulator become complicated for non-structured shaping. The joint use of nonbinary coding and Grassmann shaping also provides additional gain by up to 0.5 dB, approaching to near the Shannon limit.

4. Conclusions

We propose a new geometric shaping which uses a Grassmann projection of hyper-cube 2PAMs, achieving 1 dB gain at maximum. The advantages of the Grassmann shaping include that the irregular constellations can be well parameterized by a limited number of factors θ_k, optimal parameters can be approximated by a well-known function such as

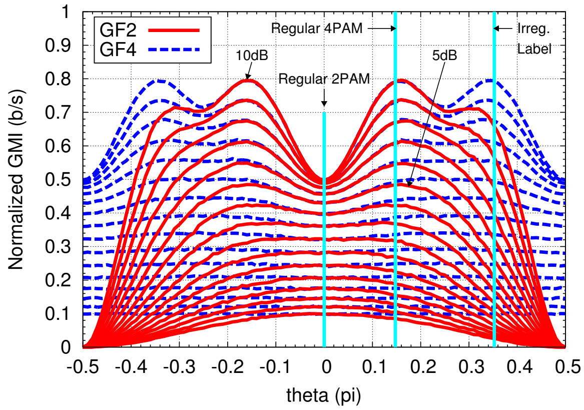


Fig. 2: Norm. GMI vs. θ_1 for Grassmann-shaped 4PAM.

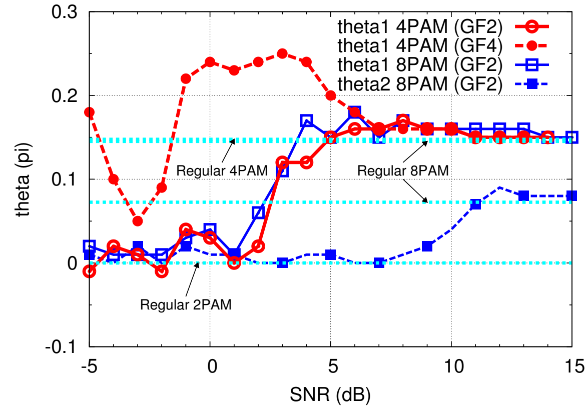
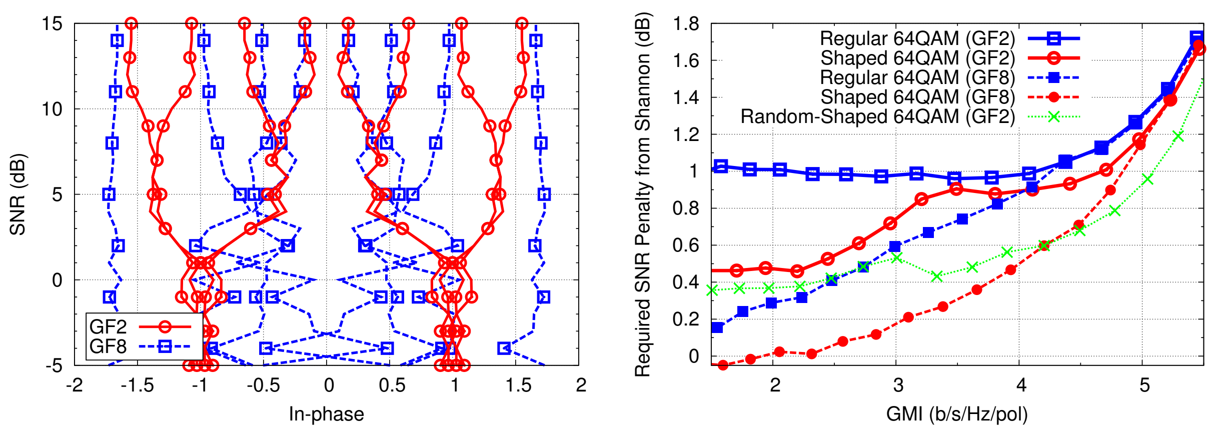


Fig. 3: Optimal factor for Grassmann-shaped 4/8PAM.



(a) Shaped Constellation

(b) Required SNR penalty from Shannon

Fig. 4: Optimized constellation and GMI performance of Grassmann-shaped 64QAM.

piece-wise linear, the shaped constellation can be generated by superposing 2PAMs, and the seamless adaptation of modulation schemes across regular QAMs is possible.

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