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## Abstract

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## HEALTH-AWARE AND USER-INVOLVED BATTERY CHARGING MANAGEMENT FOR ELECTRIC VEHICLES USING LINEAR QUADRATIC CONTROL

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## ABSTRACT

This paper studies control-theory-enabled charging management for battery systems in electric vehicles (EVs). Charging is a crucial factor for the battery performance and life as well as EV users' anxiety. Existing methods run with two shortcomings: insufficiency of battery health awareness during charging, and failure to include the user into the charging loop. To address such issues, we propose to perform charging that deals with both health protection and user-specified charging needs or objectives. Capitalizing on the linear quadratic control theory, a set of charging strategies are developed. A simulation-based study demonstrates their effectiveness and potential. We expect that charging with health awareness and user involvement will improve not only the battery longevity but also user satisfaction.

## 1 Introduction

Holding the promise for reduced fossil fuel use and air pollutant emissions, electrified transportation has been experiencing a surge of interest in recent years. Battery systems are crucial for the performance of electric vehicles (EVs) and the consumer acceptance. To improve the operating performance, safety and longevity of batteries, a considerable amount of research and development effort has been made to advanced battery management, especially state-of-charge (SoC) estimation, state-ofhealth (SoH) estimation and thermal monitoring, see [1–7] and the references therein. Recently, there has been a growing attention to optimal charging strategies to reduce the charginginduced harm. Improper charging (e.g., charging with a high voltage or current density) can cause rapid buildup of internal stress and resistance, crystallization and other negative effects. The consequence is fast capacity fade and shortened life cycle, and finally, impaired consumer confidence [8–10].

Popular charging ways in industrial practice, especially for inexpensive lead-acid batteries used for cars and backup power systems, are to apply a constant voltage or force a constant current flow through the battery [2]. Such methods, though easy to implement, can lead to serious detrimental effects for the battery. One improvement is the constant-current/constant-voltage charging [1,2]. As illustrated in Figure 1, it applies a constant current, and when the voltage increases to a desired level, switches to the constant voltage mode with the current diminishing accordingly. A main issue with such methods is the lack of an effective feedback-based regulation mechanism. With an open-loop architecture, they simply take energy from power supply and put it into the battery. As a result, both the charging dynamics and the battery's status feedback information are not well exploited to control the charging process for improvements of efficiency and health protection. This calls for the deployment of closedloop model-based control. Constrained optimal control has thus been used in [8, 11, 12], in conjunction with electrochemical or equivalent circuit models, to address fast charging subject to input, state and temperature constraints for health. To mitigate the computational cost, a rule-based, easier-to-implement method is proposed in [13,14] to handle charging under constraints using an on/off strategy. An adaptive control scheme for energy-efficient fast charging is crafted in [15]. In [16–18], optimal EV charging pattern is designed with considerations of both the electricity cost and battery degradation. In spite of such works, the existing work still remains limited to date, leaving much capacity of feedback-controlled charging unexplored yet.

In this paper, we propose to perform control-based EV charging management in a *health-aware* and *user-involved* way. Since the battery system is the heart as well as the most expen-

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FIGURE 1: Constant-current/constant-voltage charging.

sive component of an EV, health protection during charging is of remarkable importance to prevent performance and longevity degradation. Incorporated in the form of various constraints, it has been a major design consideration in the controlled charging literature mentioned above. Furthermore, we put forward that user involvement will bring significant improvements to the charging strategies. The current practice excludes the EV user from the charging management indeed. However, it will create two-fold advantages if the user can give the controller commands or advisement about the charging objectives based on his/her immediate situation. First, the battery health will be protected better. The user's advisement can translate into information useful for the EV charger. For instance, given the user's prediction of parking time, a healthier way than simply fast charging can be adopted, as it may happen for charging while working or at home. In another scenario, the user can specify the charging objective as 50% full in 1 hour if he/she has a drive to the airport in 1 hour but only needs half capacity. The charger can make wiser, more health-oriented charging decisions while meeting the user specifications with such information. Second, a direct, positive impact on user satisfaction may result arguably, because offering a user options to meet his/her different charging needs is indicative of a better service quality and enhance his/her perception of level of involvement.

We will build health-aware and user-involved charging strategies via investigating two problems. The first one is *charging with fixed terminal charging state*. In this case, the user will give target state-of-charge (SoC) and charging duration, which will be incorporated as terminal state constraint. The second problem is *tracking-based charging*, where the charging is implemented via tracking a charge trajectory. The trajectory is generated on the basis of user-specified objectives and battery conditions, developed on the basis of *linear quadratic optimal control*, will be presented as controlled charging laws expressed in explicit equations. The proposed methods differ from existing ones in [8, 11, 12] in two aspects. From the viewpoint of application, they keep into account both user specifications and battery health. Technically, they, though based on optimization of



FIGURE 2: The battery RC model.

quadratic cost functions, do not require real-time optimization and thus are computationally more attractive. We notice that the linear quadratic control is a fruitful area, with many established results and new progresses, e.g., [19–22], potentially deployable for the charging control problem.

The rest of the paper is organized as follows. Section 2 presents an RC model oriented towards charging dynamics. Section 3 offers the development of charging strategies, with Section 3.1 on charging with fixed terminal charging state decided by the user and Section 3.2 on tracking-based charging. Section 4 shows numerical results to illustrate the effectiveness of the design. Finally, concluding remarks are gathered in Section 5.

## 2 RC Model for Charging

While the energy storage within a battery results from complex electrochemical and physical processes, it has been useful to draw an analogy between the battery electrical properties and an equivalent circuit which consists of multiple linear passive elements such as resistors, capacitors, inductors and virtual voltage sources. Throughout the paper, we consider a second-order RC model shown in Figure 2.

Developed by Saft Batteries, Inc., this model was intended for the simulation of battery packs in hybrid EVs [23, 24]. It has been used in [25] to study battery model identification. The bulk capacitor  $C_b$  represents the battery's capability to store energy, and the capacitor  $C_s$  accounts for the surface effects, where  $C_b \gg C_s$ . The associated resistances are  $R_b$  and  $R_s$ , respectively, with  $R_b \gg R_s$ . Let  $Q_b$  and  $Q_s$  be the charge stored by  $C_b$  and  $C_s$ , respectively, and define them as the system states. The statespace representation of the model is given by (1). It can be verified that this system is controllable and observable, indicating the feasibility of controlled charging and status monitoring.

When a positive current is applied for charging, both  $Q_b$  and  $Q_s$  will grow. However, the voltage of  $C_s$ , denoted as  $V_s$ , increases at a rate much faster than the voltage of  $C_b$ , denoted as  $V_b$ . For a high current *I*, the terminal voltage *V*, which is largely dependent on the fast increasing  $V_s$ , will grow quickly. Then  $Q_s$  will reach the maximum in a short time and end the charging process, though  $Q_b$  still remains at a low level. This is in accordance

$$\begin{cases} \begin{bmatrix} \dot{Q}_b(t) \\ \dot{Q}_s(t) \end{bmatrix} = \begin{bmatrix} -\frac{1}{C_b(R_b + R_s)} & \frac{1}{C_s(R_b + R_s)} \\ \frac{1}{C_b(R_b + R_s)} & -\frac{1}{C_s(R_b + R_s)} \end{bmatrix} \begin{bmatrix} Q_b(t) \\ Q_s(t) \end{bmatrix} + \begin{bmatrix} \frac{R_s}{R_b} \\ \frac{R_b}{R_b + R_s} \end{bmatrix} I(t),$$

$$V(t) = \begin{bmatrix} \frac{R_s}{C_b(R_b + R_s)} & \frac{R_b}{C_s(R_b + R_s)} \end{bmatrix} \begin{bmatrix} Q_b(t) \\ Q_s(t) \end{bmatrix} + \left( R_o + \frac{R_b R_s}{R_b + R_s} \right) I(t).$$

$$(1)$$

of the "rate capacity effect", which means that the total charge absorbed by the battery goes down with the increase in charging current as stated by the Peukert's law. In addition, the RC model can also describe another concerning effect, the "recovery effect". That is, when the charging stops, the terminal voltage V will decrease by (1), due to the charge transfer from  $C_s$  to  $C_b$ .

The overall SoC is given by

$$SoC = \frac{Q_b - \underline{Q}_b + Q_s - \underline{Q}_s}{\overline{Q}_b - \underline{Q}_b + \overline{Q}_s - \underline{Q}_s},$$
(2)

where  $\underline{Q}_j$  and  $\overline{Q}_j$  with j = b, s denote the unusable and the maximum allowed charge.

To develop and apply digital control, the model in (1) can be discretized with a sampling period of  $T_s$ . The discrete-time model takes the following standard form:

$$\begin{cases} x_{k+1} = Ax_k + Bu_k, \\ y_k = Cx_k + Du_k. \end{cases}$$
(3)

where  $x = [Q_b Q_s]^{\top}$ , u = I, y = V, and *A*, *B*, *C* and *D* can be decided via applying a discretization method to (1).

For health consideration, we need to constrain the difference between  $V_b$  and  $V_s$  throughout the charging process. Here,  $\tilde{V} = V_b - V_s$  drives the migration of the charge from  $C_s$  to  $C_b$ . It, intuitively, delineates the gradient of the concentration of ions within the electrode. Created during charging, the concentration gradient induces the diffusion of ions. However, too large a gradient value will cause internal stress increase, heating, solidelectrolyte interphase (SEI) formation and other negative side effects [26–28]. Mechanical degradation of the electrode and capacity fade will consequently happen. Thus uniformity of the ion concentration should be pursued at the maximum possible level during charging. It is noteworthy that such a restriction should be implemented more strictly as the SoC increases, because the adverse effects of a large concentration difference would be stronger in this case.

Next, we will build the charging strategies on the basis of the RC model. The development will be laid out in the framework of linear quadratic control, taking into account both health awareness and user needs.

## 3 Health-Aware and User-Involved Charging Strategies

In this section, we develop charging strategies for two cases. The first one is concerned with the user defining the final charging state. It will be treated via linear quadratic control subject to fixed terminal state resulting from the user objective. In the second case, charging is managed via tracking a charging trajectory which is produced according to the user objective.

## 3.1 Charging with Fixed Terminal Charging State

A charging scenario that frequently arises is: according to the next drive need, a user will inform the charging management system of his/her objective in terms of target SoC and charging duration. This can occur for overnight parking at home, severalhour parking at the workplace, or when a drive to some place is needed in just half an hour. As discussed before, the objective offered by the user, if incorporated into the dynamic charging decision making process, would create support for health protection more effective than charging with maximum speed.

From the perspective of control design, the considered charging task can be formulated as an optimal control problem, which minimizes a cost function commensurate with the harm to health and subject to the user's goal. With the model in (3), the following linear quadratic control problem is of interest:

$$\min_{u_0,u_1,\cdots,u_{N-1}} \frac{1}{2} x_N^\top S_N x_N + \frac{1}{2} \sum_{k=0}^{N-1} \left( x_k^\top G^\top \mathcal{Q}_k G x_k + u_k^\top R u_k \right),$$
  
subject to  $x_{k+1} = A x_k + B u_k, x_0,$   
 $x_N = \bar{x}.$  (4)

where  $P \ge 0$ ,  $Q_k \ge 0$ , R > 0 and  $G = [1/C_b - 1/C_s]$ . In above,  $Gx_k$  is the voltage difference between  $C_b$  and  $C_s$  indeed. The quadratic cost function, defined over the user-specified time range  $[0, NT_s]$ , intends to constrain the voltage difference and magnitude of the charging current. The minimization is subject to the state equation and the fixed terminal state  $\bar{x}$  as a result of user's target SoC. In the final state, the battery should be at the equilibrium point with  $V_b = V_s$ . Together with (2),  $\bar{x}$ , can be determined from the specified SoC value. The weight coefficient  $Q_k$  should be chose in a way such that it increases over time, in order to reflect the truth that the stronger health protection is needed as the SoC builds up.

A closed-form solution for (4) can be developed, which

will lead to the state-feedback-based charging strategy as follows [19]:

$$K_k = (B^{\top} S_N B + R)^{-1} B^{\top} S_{k+1} A,$$
(5)

$$S_k = A^{\top} S_{k+1} (A - BK_k) + Q_k,$$
 (6)

$$T_k = (A - BK_k)^{\top} T_{k+1}, \ T_N = I,$$
 (7)

$$P_{k} = P_{k+1} - T_{k+1}^{\top} B (B^{\top} S_{k+1} B + R)^{-1} B^{\top} T_{k+1}, P_{N} = 0, \quad (8)$$

$$K_k^u = \left(B^\top S_{k+1} B + R\right)^{-1} B^\top, \tag{9}$$

$$u_{k} = -\left(K_{k} - K_{k}^{u}T_{k+1}P_{k}^{-1}T_{k}^{\top}\right)x_{k} - K_{k}^{u}T_{k+1}P_{k}^{-1}\bar{x}.$$
 (10)

Since the state  $x_k$  is not measurable directly, it is necessary to convert the above strategy to be based on the output feedback. One straightforward avenue to achieve this would be to replace  $x_k$ by its prediction  $\hat{x}_k$  that minimizes another quadratic cost function. This is justifiable by the certainty equivalence principle, which allows the optimal output-feedback control design to be divided into the separate designs of an optimal state-feedback control an optimal estimator [29]. Here, we use the one-stepforward Kalman predictor given by

$$L_k = A \Sigma_k C^\top (C \Sigma_k C^\top + V)^{-1}, \tag{11}$$

$$\hat{x}_{k+1} = A\hat{x}_k + Bu_k + L_k(y_k - C\hat{x}_k - Du_k), \qquad (12)$$

$$\Sigma_{k+1} = A\Sigma_k A^\top + W - A\Sigma_k C^\top (C\Sigma_k C^\top + V)^{-1} C\Sigma_k A^\top, \quad (13)$$

where W and V symmetric positive definite matrices accounting for the covriances of the process and measurement noises. Note that the Kalman filter has been in wide use for battery SoC estimation, e.g., in our previous work [30–33]. Then the optimal control law in (10) changes to be:

$$u_{k} = -\left(K_{k} - K_{k}^{u}T_{k+1}P_{k}^{-1}T_{k}^{\top}\right)\hat{x}_{k} - K_{k}^{u}T_{k+1}P_{k}^{-1}\bar{x}.$$
 (14)

Putting together (5)-(9), (11)-(13) and (14), we achieve a complete description of the charging method via linear quadratic control with fixed terminal state, which is named LQCwFTS and illustrated in Table 1. The LQCwFTS method performs state prediction at each time instant, and then feeds the predicted value, which is a timely update about the battery's internal state, to generate the control input, i.e., the charging current to the battery. Much of the computation for LQCwFTS can be performed prior to the implementation of the control law. The sequences,  $K_k$ ,  $S_k$ ,  $T_k$ ,  $P_k$  and  $K_k^u$  can be computed offline, and then  $K_k$ ,  $K_k^u T_{k+1} P_k^{-1} T_k^{\top}$  and  $K_k^u T_{k+1} P_k^{-1}$  are stored for use when the control is applied. On the side of the Kalman prediction, offline computation and storage of  $L_k$  can be done. Then the only work

Offline backward computation (from time N to 0)  

$$K_{k} = (B^{\top}S_{N}B + R)^{-1}B^{\top}S_{k+1}A$$

$$S_{k} = A^{\top}S_{k+1}(A - BK_{k}) + Q_{k}$$

$$T_{k} = (A - BK_{k})^{\top}T_{k+1}, T_{N} = I$$

$$P_{k} = P_{k+1} - T_{k+1}^{\top}B(B^{\top}S_{k+1}B + R)^{-1}B^{\top}T_{k+1}, P_{N} = 0$$

$$K_{k}^{u} = (B^{\top}S_{k+1}B + R)^{-1}B^{\top}$$
Online forward computation (from time 0 to N)  
Battery state prediction  

$$L_{k} = A\Sigma_{k}C^{\top}(C\Sigma_{k}C^{\top} + V)^{-1}$$

$$\hat{x}_{k+1} = A\hat{x}_{k} + Bu_{k} + L_{k}(y_{k} - C\hat{x}_{k} - Du_{k})$$

$$\Sigma_{k+1} = A\Sigma_{k}A^{\top} + W - A\Sigma_{k}C^{\top}(C\Sigma_{k}C^{\top} + V)^{-1}C\Sigma_{k}A^{\top}$$
Charging decision  

$$u_{k} = -\left(K_{k} - K_{k}^{u}T_{k+1}P_{k}^{-1}T_{k}^{\top}\right)\hat{x}_{k} - K_{k}^{u}T_{k+1}P_{k}^{-1}\bar{x}$$

**TABLE 1**: The LQCWFTS charging strategy (Linear Quadratic Control with Fixed Terminal State).

to do during charging is to compute the optimal state prediction and control by (12) and (14), thus reducing the computational burden.

#### 3.2 Charging Based on Tracking

For user-involved charging, it will be beneficial if a desired path is generated in advance on the basis of user-specified objectives for the charging process to follow. In this case, the path can serve as the references for the controller to track. The path generation can be conducted with prior experience or knowledge of the battery electrochemistries and present conditions, which, in turn, will enhance the health awareness through charging. An EV manufacturer can design path generation algorithms and embed them into BMSs, from which the user can select the one that best fits the needs when he/she intends to charge the EV. While how to compute an optimal charging path will make a topic of future research, we focus on developing the charging method to track the path here.

Suppose that the user describes the target SoC and duration for charging, which is translated into the final state  $\bar{x}$ . Then a reference trajectory  $r_k$  for  $k = 0, 1, \dots, N$  is calculated with  $r_N = \bar{x}$ . Note that the trajectory constrains the difference between  $V_b$ and  $V_s$  to guarantee health. The linear quadratic state-feedback Offline backward computation (from time *N* to 0)  $K_{k} = (B^{\top}S_{k+1}B + R)^{-1}B^{\top}S_{k+1}A$   $K_{k}^{s} = (B^{\top}S_{k+1}B + R)^{-1}B^{\top}$   $S_{k} = A^{\top}S_{k+1}(A - BK_{k}) + Q$   $s_{k} = (A - BK_{k})^{\top}s_{k+1} + Qr_{k}, s_{N} = S_{N}r_{N}$ Online forward computation (from time 0 to *N*) *Battery state prediction*   $L_{k} = A\Sigma_{k}C^{\top}(C\Sigma_{k}C^{\top} + V)^{-1}$   $\hat{x}_{k+1} = A\hat{x}_{k} + Bu_{k} + L_{k}(y_{k} - C\hat{x}_{k} - Du_{k})$   $\Sigma_{k+1} = A\Sigma_{k}A^{\top} + W - A\Sigma_{k}C^{\top}(C\Sigma_{k}C^{\top} + V)^{-1}C\Sigma_{k}A^{\top}$  *Charging decision*  $u_{k} = -K_{k}\hat{x}_{k} + K_{k}^{s}s_{k+1}$ 

**TABLE 2**: The LQT charging strategy (Linear Quadratic Track-ing).

tracking for charging can be considered as:

$$\min_{u_{0},u_{1},\cdots,u_{N-1}} \frac{1}{2} (x_{N} - r_{N})^{\top} S_{N} (x_{N} - r_{N}) 
+ \frac{1}{2} \sum_{k=0}^{N-1} \left[ (x_{k} - r_{k})^{\top} Q (x_{k} - r_{k}) + u_{k}^{\top} R u_{k} \right], \quad (15)$$

subject to  $x_{k+1} = Ax_k + Bu_k, x_0,$ 

where  $S_N \ge 0$ ,  $Q \ge 0$  and R > 0. The optimal solution to the above problem is expressed as follows [19]:

$$K_k = (B^{\top} S_{k+1} B + R)^{-1} B^{\top} S_{k+1} A, \qquad (16)$$

$$K_k^s = (B^{\top} S_{k+1} B + R)^{-1} B^{\top}, \qquad (17)$$

$$S_k = A^{\top} S_{k+1} (A - BK_k) + Q,$$
 (18)

$$s_k = (A - BK_k)^{\top} s_{k+1} + Qr_k, \ s_N = S_N r_N,$$
 (19)

$$u_k = -K_k x_k + K_k^s s_{k+1}.$$
 (20)

Following lines analogous to the development of LQCwFTS, the output-feedback tracker for charging can be created based on (16)-(20) through the employment of the Kalman predictor in (11)-(13). Specifically, (20) will use  $\hat{x}_k$  rather than  $x_k$ , i.e.,

$$u_k = -K_k \hat{x}_k + K_k^s s_{k+1}.$$
 (21)

Summarizing (16)-(19), (11)-(13) and (21) will yield the linear quadratic tracking strategy, or LQT, for charging, see Table 2.

Similar to the aforeproposed LQCwFTS, the LQT can have much computation completed offline. Then only the Kalman state prediction and optimal tracking control (21) need to be computed during the actual control run.

It is noted that the control run of the LQT strategy will lead to a steady state where the gains  $K_k$  and  $K_k^s$  will be fixed. The steady state can be computed prior with knowledge of the discrete algebraic Riccati equation (DARE). In this case, the steady-state LQT strategy is named SS-LQT. The SS-LQT will enjoy further simplicity and computational efficiency in terms of its time and space complexities, thus more desirble for practical use.

## 4 Numerical Illustration

In this section, we present simulation examples to evaluate the performance of the proposed charging strategies. Let us consider a lithium-ion battery with known RC model parameters. Assume  $C_b = 82$ KF,  $R_b = 1.1$ m $\Omega$ ,  $C_s = 4.074$ KF,  $R_s = 0.4$ m $\Omega$ , and  $R_o = 1.2$ m $\Omega$  [23]. It has a nominal capacity of 7Ah. The initial SoC is 30%. The user specifies that certain SoC must be achieved within certain duration.

*Example 1 - Application of* LQCwFTS: Suppose that charging should be completed in 2 hours. A series of target SoC values, 55%, 65%, 75%, 85% and 95%, are set for the simulation purpose. The sampling period  $t_s = 1$ s, so the number of data points is N = 7200. We apply the LQCwFTS method to carry out the charging tasks. For the control run,  $Q_k = 0.1 \cdot (5 \times 10^7)^{k/N}$  and R = 0.1. The exponential increase of  $Q_k$  illustrates increasing emphasis on health as the charging goes on.

The computational results are illustrated in Figure 3. It is observed from Figure 3a that the different target SoCs are satisfied when the charging ends. The SoC increases approximately proportionally with time for the first 1.25 hours. Then the rate slows down gradually to zero as the charging objective is being approached. This is because of the large weight  $Q_k$  in the later stage for health protection. The charging current is kept at almost a constant level initially during each charging implementation, as illustrated in Figure 3b. For a higher target SoC, the magnitude is larger accordingly. However, the current drops quickly as the SoC grows further. The concerned health indicator, voltage different between  $C_s$  and  $C_b$  is characterized in Figure 3c. For each case,  $V_s - V_b$  remains around a constant value in the first hour, despite high-frequency fluctuations due to noise. However, it decreases drastically as more charge is pumped into the battery, maximizing the health of the battery's internal structure. As a comparison, we force a constant current of 2.275A through the battery for 2 hours to reach 95% of the capacity. The voltage difference, as shown in Figure 3d, will be kept at a fixed level unsurprisingly. This, however, will cause more serious detrimental effects, because the battery's tolerance to the voltage difference will decrease rapidly with the SoC growth. Thus with this comparison, we argue that the LQCwFTS can offer a stronger health



**FIGURE 3**: Example 1 - Application of LQCwFTS to charge the battery from 30% to 55%, 65%, 75%, 85% and 95%: (a) the SoC trajectories; (b) the charging current profiles; (c) the voltage differences as health indicator; (d) voltage difference due to constant current charging to 95% SoC.

protection during charging.

*Example 2 - Application of* SS-LQT: We consider the use of SS-LQT for charging in this example. The problem setting is the same as in Example 1. The charging trajectories are generated prior based on the objectives. For simplicity and convenience, we assume that the desired trajectory for  $x_1$  and  $x_2$ , denoted as  $r_b$  and  $r_s$ , is

$$r_{j,k} = \frac{1 - e^{-kt_s/\tau_j}}{1 - e^{-Nt_s/\tau_j}} (r_{j,N} - r_{j,0}) + r_{j,0},$$

for j = b or  $s, k = 1, 2, \dots, N-1$ , where  $r_{j,0}$  is the initial charge,  $r_{j,N}$  the target charge, and  $\tau_j$  the time coefficient for j = b or s. Note that  $r_{j,0}$  and that  $r_{j,N}$  can be calculated from the initial SoC and user-specified target SoC. The resultant trajectories have a steep increase followed by a gentle slope, which are reasonable in view of health protection. Letting  $\tau_b = \tau_s = Nt_s/4$ ,  $V_s$  and  $V_b$ are enforced to be equal. Thus at the trajectory design stage, we put the minimization of the detrimental effects well into consideration.

With the reference trajectories available, the SS-LQT strategy is applied to charging. The increase of the actual SoC over time is demonstrated in Figure 4a. All the targets are met. In each case, the SoC grows at a fast rate when the SoC is at a low level but at a slower rate when the SoC becomes higher. Figure 4b shows the current produced by SS-LQT. The current usually begins with a large magnitude but decreases quickly. The voltage difference, given in Figure 4c, has the similar trend. It is relatively high when the charging starts, but reduces fast. The state tracking for the task of 95% SoC is shown in Figures 4d and 4e. It is observed that tracking of  $r_b$  by  $x_1$  exhibits high accuracy. Tracking of  $r_s$  by  $x_2$  becomes accurate increasingly, despite deviation in the first hour. Meanwhile, the further the target SoC is approached, the smaller the tracking error becomes.

In Examples 1 and 2, different charging current profiles are noticed for the same charging task. This is caused by the charging trajectories adopted for the SS-LQT and the selection of the weight matrices Q and R. Such a difference does not compromise



**FIGURE 4**: Example 2 - Application of LQT to charge the battery from 30% to 55%, 65%, 75%, 85% and 95%: (a) the SoC trajectories; (b) the charging current profiles; (c) the voltage differences; (d) tracking of  $x_1$  (i.e.,  $Q_b$ ) for 95% target SoC; (e) tracking of  $x_2$  (i.e.,  $Q_s$ ) for 95% target SoC.

the value of the proposed charging strategies. Further experimental evaluation and validation of the strategies will be pursued in our future work.

## 5 Conclusions

Effective battery charging management is of vital importance for the development of EVs, though it has not received attention deserved. In recent years, fast charging control has gained some interest. However, the problem of health-aware and user-involved charging has not been explored in the literature. In this paper, we propose a set of novel charging strategies, which aim to accomplish user-defined charging objectives with awareness of the harms to health. They are developed in the framework of linear quadratic control. Compared with most existing fast charging techniques, they do not require the time-consuming real-time optimization. The usefulness of the proposed strategies is evaluated via a simulation study. This work can also find uses in consumer electronics and other applications and will provide further incentives for the study of intelligent charging management.

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