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EXTREMUM SEEKING-BASED INDIRECT ADAPTIVE CONTROL AND FEEDBACK GAINS AUTO-TUNING FOR NONLINEAR SYSTEMS

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Abstract

We present in this chapter some recent results on learning-based adaptive control for nonlinear systems. We first study the problem of adaptive trajectory tracking for nonlinear systems. We focus on the class of nonlinear systems with parametric uncertainties, which can be rendered integral Input-to-State Stable (iISS) w.r.t. the parameter estimation error. We argue, for this particular class of systems, that it is possible to merge together the integral Input-to-State stabilizing feedback controller and a model-free extremum seeking (ES) algorithm, to realize a learning-based adaptive controller. We investigate the performance of this approach in term of tracking error upper-bounds, for two different ES algorithms. Next, we consider the class of nonlinear systems affine in the control, and propose a learning-based approach to iteratively auto-tune the feedback gains for nonlinear stabilizing controllers.

Keywords: adaptive control, learning-based control, extremum-seeking, nonlinear systems, iterative feedback gains tuning

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1. Introduction

Extremum seeking (ES) is a well known approach which one can use to search for the extremum of a cost function, associated with a given process performance, without the need

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for a precise model of the process, e.g., [1–3]. Several ES algorithms have been proposed, e.g., [1–8], and many applications of ES algorithms have been reported, e.g., [9–13].

On the other hand, classical adaptive control deals with controlling partially unknown processes based on their uncertain models, i.e., controlling plants with parameter uncertainties. One can classify classical adaptive methods into two main approaches; ‘direct approaches’, where the controller is updated to adapt to the process, and ‘indirect approaches’, where the model is updated to better reflect the actual process. Many adaptive methods have been proposed over the years for linear and nonlinear systems. We could not possibly cite here all the design and analysis results that have been reported; instead, we refer the reader to e.g., [14, 15] and the references therein for more details. What we want to underline here, is that these results in ‘classical’ adaptive control are mainly based on the structure of the model of the system, e.g., linear vs. nonlinear model, linear uncertainties parametrization vs. nonlinear parameterizations, etc.

Another adaptive control paradigm is the one which uses ‘learning schemes’ to estimate the uncertain part of the process. Indeed, in this paradigm the learning-based controller, based either on machine learning theory, neural networks, fuzzy systems, etc., is trying either to estimate the parameters of an uncertain model, or the structure of a deterministic or a stochastic function representing part or all of the model. Several results have been proposed in this area as well; we refer the reader to e.g., [16] and the references therein for more details.

We want to concentrate in this chapter on the use of ES theory in the ‘learning-based’ adaptive control paradigm. Indeed, several results were recently developed in this direction, e.g., [9, 10, 12, 13, 17–20]. For instance, in [19, 20] an extremum seeking-based controller was proposed for nonlinear affine systems with linear parameters uncertainties. The controller drives the states of the system to unknown optimal states that optimize a desired objective function. The ES controller used in [19, 20] is not model-free, in the sense that it is based on the known part of the model, i.e., it is designed based on the objective function and the nonlinear model structure. A similar approach is used in [9, 10] when dealing with more specific examples. In [17, 18], the authors used a model-free ES, i.e., based on a desired cost function, to estimate parameters of a linear state feedback to compensate for unknown parameters for linear systems. In [12], the authors used, for the case of electromagnetic actuators, a model-free ES, i.e., only based on the cost function without the use of the system model. The model-free ES was used to learn the ‘best’ feedback gains of a passive robust state feedback. Similarly, in [13, 21] a backstepping controller was merged with a model-free ES to estimate the uncertain parameters of a nonlinear model for electromagnetic actuators.

In this context, we present here an ES-based indirect adaptive controller for a class of nonlinear systems. The results reported here are based on the work of the author introduced in [22, 23]. The idea is based on a modular design, where we first design a feedback controller which makes the closed-loop tracking error dynamic ISS (or iISS) w.r.t. the estimation errors. This ISS controller is then complemented with a mode-free ES algorithm that can minimize a desired cost function. The cost function is minimized by tuning, i.e., estimating, the unknown parameters of the model. This modular design simplifies the analysis of the total controller, i.e., ISS controller plus ES estimation algorithm. We propose this formulation in the general case of nonlinear systems.

We underline here that the main advantage w.r.t. classical ES results, which do not use any model to control a given system, is the fact that pure ES-based controllers are mainly meant for regulation control, not output trajectory tracking. Another point is that the pure ES controllers are slower to converge to the optimal control, compared to a modular approach which uses the known part of the model to design a model-based controller, and then complement it with a model-free ES algorithm to learn the unknown part of the model and improve the overall control performance. In other words, with the pure ES-based controller one assumes no knowledge at all of the controlled system, ignoring the physics of the system, and even if under some conditions, convergence of such algorithms has been proven, it is intuitive to expect the modular control design, which takes advantage of the physics of the system, to be able to converge to the optimal performance faster than the complete model-free ES control.

Another well known control problem, concerns iterative feedback gains tuning (IFT) for linear and nonlinear controllers. Indeed, the use of learning algorithm to tune feedback gains of nominal linear controllers to achieve some desired performances has been studied in several papers, e.g., [24–27]. We present here some results related to IFT for nonlinear systems. The results presented here were introduced by the author in [12, 28]. We consider a particular class of nonlinear systems, namely, nonlinear models affine in the control input, which are linearizable via static state feedback. We consider bounded additive model uncertainties with known upper bound function. We propose a simple modular iterative gains tuning controller, in the sense that we first design a passive robust controller, based on the classical Input-Output linearization method merged with a Lyapunov reconstruction-based control, e.g., [29, 30]. This passive robust controller ensures uniform boundedness of the tracking errors and their convergence to a given invariant set. Next, in a second phase we add a multi-variable extremum seeking algorithm to iteratively auto-tune the feedback gains of the passive robust controller to optimize a desired system performance, which is formulated in terms of a desired cost function minimization.

One point worth mentioning at this stage, is that compared to model-free pure ES-based controllers, the ES-based IFT control has a different goal. Indeed, the available pure ES-based controllers are meant for output or state regulation, i.e., solving a static optimization problem. On the contrary, here we propose to use ES to complement a model-based nonlinear control to auto-tune its feedback gains, which means that the control goal, i.e., state or output trajectory tracking, is handled by the model-based controller. The ES algorithm are used to improve the tracking performance of the model-based controller, and once the ES algorithm has converged, one can carry on using the nonlinear model-based feedback controller alone, i.e., without the need of the ES algorithm. In other words, the ES algorithm is used here to replace the manual feedback gains tuning of the model-based controller, which is often done in real-life by some type of trial and error tests.

This chapter is organized as follows: In Section 2 we recall some notations and definitions that will be used in the sequel. In Section 3 we present the first indirect adaptive control approach, namely the ES-based learning adaptive controller for constant structured model uncertainties. In Section 4 we study the case of time-varying structured model uncertainties, using time-varying ES-based techniques. Section 5 is dedicated to the problem of passive robust nonlinear control with learning-based iterative feedback gains tuning. Finally, some summarizing remarks and open problems are given in Section 6.

2. Preliminaries

Throughout the chapter we will use $\|.\|$ to denote the Euclidean norm; i.e., for $x \in \mathbb{R}^n$ we have $\|x\| = \sqrt{x^T x}$. We will use $\dot{(.)}$ for the short notation of time derivative. We denote by C^k functions that are k times differentiable. A function is said analytic in a given set, if it admits a convergent Taylor series approximation in some neighborhood of every point of the set. A continuous function $\alpha : [0,a) \to [0,\infty)$ is said to belong to class $\mathcal{K}$ if it is strictly increasing and $\alpha(0) = 0$. A continuous function $\beta : [0,a) \times [0,\infty) \to [0,\infty)$ is said to belong to class $\mathcal{KL}$ if, for each fixed s, the mapping $\beta(r,s)$ belongs to class $\mathcal{K}$ with respect to r and, for each fixed r, the mapping $\beta(r,s)$ is decreasing with respect to s and $\beta(r,s) \to 0$ as $s \to \infty$. Let us now introduce few definitions that will be used in the remainder of this chapter.

Definition 2.1 (Local Integral Input-to-State Stability [31]). Consider the system

$$\dot{x} = f(t,x,u) \tag{1}$$

where $x \in \mathcal{D} \subseteq \mathbb{R}^n$ such that $0 \in \mathcal{D}$, and $f : [0,\infty) \times \mathcal{D} \times \mathcal{D}_u \to \mathbb{R}^n$ is piecewise continuous in t and locally Lipschitz in x and u, uniformly in t. The inputs are assumed to be measurable and locally bounded functions $u : \mathbb{R}_{\geq 0} \to \mathcal{D}_u \subseteq \mathbb{R}^m$. Given any control $u \in \mathcal{D}_u$ and any $\xi \in \mathcal{D}_0 \subseteq \mathcal{D}$, there is a unique maximal solution of the initial value problem $\dot{x} = f(t,x,u)$, $x(t_0) = \xi$. Without loss of generality, assume $t_0 = 0$. The unique solution is defined on some maximal open interval, and it is denoted by $x(\cdot,\xi,u)$. System (1) is locally integral input-to-state stable (LiISS) if there exist functions α, $\gamma \in \mathcal{K}$ and $\beta \in \mathcal{KL}$ such that, for all $\xi \in \mathcal{D}_0$ and all $u \in \mathcal{D}_u$, the solution $x(t,\xi,u)$ is defined for all $t \geq 0$ and

$$\alpha(\|x(t,\xi,u)\|) \leq \beta(\|\xi\|,t) + \int_0^t \gamma(\|u(s)\|)ds \tag{2}$$

for all $t \geq 0$. Equivalently, system (1) is LiISS if and only if there exist functions $\beta \in \mathcal{KL}$ and γ_1, $\gamma_2 \in \mathcal{K}$ such that

$$\|x(t,\xi,u)\| \leq \beta(\|\xi\|,t) + \gamma_1\left(\int_0^t \gamma_2(\|u(s)\|)ds\right) \tag{3}$$

for all $t \geq 0$, all $\xi \in \mathcal{D}_0$ and all $u \in \mathcal{D}_u$.

Remark 2.1. The use of the iISS definition is not a limitation of the ideas presented here. Indeed, we are presenting here a modular design, i.e., a model-based controller ensuring iISS stability and a model-free part to improve the performance of the model-based controller. However, instead of iISS one could easily use ISS definition or semi-global practical spISS etc. The main idea is to ensure some sort of safety (boundedness of the closed-loop signals) of the feedback system during the learning phase. The reason why we choose to use iISS here is that in real applications with complicated time-varying nonlinear models , e.g., [32], we found that proving iISS using dissipativity-based equivalence theorems, e.g., [33], is easier than proving ISS. The fact is iISS, ISS, or spISS will not change the general results presented here, it will solely change the details of the upper-bounds of the closed-loop signals.

Definition 2.2 (ε- Semi-global practical uniform ultimate boundedness with ultimate bound δ (($\varepsilon-\delta$)-SPUUB) [4]). Consider the system

$$\dot{x} = f(t,x) \tag{4}$$

with $\phi^{\varepsilon}(t,t_0,x_0)$ being the solution of (4) starting from the initial condition $x(t_0)=x_0$. Then, the origin of (4) is said to be (ε,δ)-SPUUB if it satisfies the following three conditions:
1-(ε,δ)-*Uniform Stability:* For every $c_2 \in]\delta,\infty[$, there exists $c_1 \in]0,\infty[$ and $\hat{\varepsilon} \in]0,\infty[$ such that for all $t_0 \in \mathbb{R}$ and for all $x_0 \in \mathbb{R}^n$ with $||x_0|| < c_1$ and for all $\varepsilon \in]0,\hat{\varepsilon}[$,

$$||\phi^{\varepsilon}(t,t_0,x_0)|| < c_2,\ \forall t \in [t_0,\infty[$$

2-(ε,δ)-*Uniform ultimate boundedness:* For every $c_1 \in]0,\infty[$ there exists $c_2 \in]\delta,\infty[$ and $\hat{\varepsilon} \in]0,\infty[$ such that for all $t_0 \in \mathbb{R}$ and for all $x_0 \in \mathbb{R}^n$ with $||x_0|| < c_1$ and for all $\varepsilon \in]0,\hat{\varepsilon}[$,

$$||\phi^{\varepsilon}(t,t_0,x_0)|| < c_2,\ \forall t \in [t_0,\infty[$$

3-(ε,δ)-*Global uniform attractivity:* For all $c_1,\ c_2 \in (\delta,\infty)$ there exists $T \in]0,\infty[$ and $\hat{\varepsilon} \in]0,\infty[$ such that for all $t_0 \in \mathbb{R}$ and for all $x_0 \in \mathbb{R}^n$ with $||x_0|| < c_1$ and for all $\varepsilon \in]0,\hat{\varepsilon}[$,

$$||\phi^{\varepsilon}(t,t_0,x_0)|| < c_2,\ \forall t \in [t_0+T,\infty[$$

An impulsive dynamical system is said to be well-posed if it has well defined distinct resetting times, admits a unique solution over a finite forward time interval and does not exhibits any Zeno solutions, i.e., an infinitely many resetting of the system in finite time interval [34]. Finally, in the sequel when we talk about error trajectories boundedness, we mean uniform boundedness as defined in [29] (p.167, Definition 4.6) for nonlinear continuous systems, and in [34] (p. 67, Definition 2.12) for time-dependent impulsive dynamical systems.

In the next section, we first consider the case of nonlinear models with constant parametric uncertainties.

3. Extremum Seeking-based Indirect Adaptive Controller for the Case of Constant Model Uncertainties

Consider the system (1), with an additional argument $\Delta \in \mathbb{R}^p$, representing constant parametric uncertainties

$$\dot{x} = f(t, x, \Delta, u) \tag{5}$$

We associate with (5), the output vector

$$y = h(x) \tag{6}$$

where $h: \mathbb{R}^n \to \mathbb{R}^h$.
The control objective here is for y to asymptotically track a desired smooth vector time-dependent trajectory $y_{ref}: [0, \infty) \to \mathbb{R}^h$.
Let us now define the output tracking error vector as

$$e_y(t) = y(t) - y_{ref}(t) \tag{7}$$

We then assume the following

Assumption 3.1. There exists a robust control feedback $u_{iss}(t, x, \hat{\Delta}): \mathbb{R}^{\times}\mathbb{R}^n \times \mathbb{R}^p \to \mathbb{R}^m$, with $\hat{\Delta}$ being the dynamic estimate of the uncertain vector Δ, such that, the closed-loop error dynamics

$$\dot{e}_y = f_{e_y}(t, e_y, e_\Delta) \tag{8}$$

is iISS from the input vector $e_\Delta = \Delta - \hat{\Delta}$ to the state vector e_y.

Remark 3.1. Assumption 1 might seem too general, however, several control approaches can be used to design a controller u_{iss} rendering an uncertain system iISS (or ISS, spISS), for instance backstepping control approach has been shown to achieve such a property for parametric strict-feedback systems, e.g., [15]. We have also proposed in [35] a constructive control design which ensures ISS for the class of nonlinear systems affine in the control variable.

Let us define now the following cost function

$$Q(\hat{\Delta}) = F(e_y(\hat{\Delta})) \tag{9}$$

where $F: \mathbb{R}^h \to \mathbb{R},\ F(0) = 0,\ F(e_y) > 0$ for $e_y \neq 0$.
We need the following assumptions on Q.

Assumption 3.2. The cost function Q has a local minimum at $\hat{\Delta}^* = \Delta$.

Assumption 3.3. The initial error $e_\Delta(t_0)$ is sufficiently small, i.e., The original parameters' estimates vector $\hat{\Delta}$ is close enough to the actual parameters vector Δ.

Assumption 3.4. The cost function is analytic and its variation with respect to the uncertain variables is bounded in the neighborhood of Δ^*, i.e., $\|\frac{\partial Q}{\partial \Delta}(\tilde{\Delta})\| \leq \xi_2,\ \xi_2 > 0,\ \tilde{\Delta} \in \mathcal{V}(\Delta^*)$, where $\mathcal{V}(\Delta^*)$ denotes a compact neighborhood of Δ^*.

Remark 3.2. Assumption 3.2 simply means that we can consider that Q has at least a local minimum at the true values of the uncertain parameters.

Remark 3.3. Assumption 3.3 indicates that our result will be of local nature, meaning that our analysis holds in a small neighborhood of the actual values of the parameters.

We can now present the following Theorem.

Theorem 3.1. *Consider the system (5), (6), with the cost function (9), then under Assumptions 1 to 4, the controller u_{iss}, where $\hat{\Delta}$ is estimated with the multi-parameter extremum seeking algorithm*

$$\begin{aligned} \dot{x}_i &= a_i sin(\omega_i t + \tfrac{\pi}{2}) Q(\hat{\Delta}) \\ \hat{\Delta}_i &= x_i + a_i sin(\omega_i t - \tfrac{\pi}{2}),\ i \in \{1,...,p\} \end{aligned} \tag{10}$$

with $\omega_i \neq \omega_j$, $\omega_i + \omega_j \neq \omega_k$, $i,j,k \in \{1,...,p\}$, and $\omega_i > \omega^*$, $\forall i \in \{1,...,p\}$, with ω^* large enough, ensures that the norm of the error vector e_y admits the following bound

$$\|e_y(t)\| \leq \beta(\|e_y(0)\|,t) + \alpha(\int_0^t \gamma(\tilde{\beta}(\|e_\Delta(0)\|,t) + \|e_\Delta\|_{max}))ds$$

where $\|e_\Delta\|_{max} = \frac{\xi_1}{\omega_0} + \sqrt{\sum_{i=1}^{i=p} a_i^2}$, $\xi_1, \xi_2 > 0$, $e(0) \in \mathcal{D}_e$, $\omega_0 = \max_{i \in \{1,...,p\}} \omega_i$, $\alpha \in \mathcal{K}$, $\beta \in \mathcal{KL}, \tilde{\beta} \in \mathcal{KL}$ and $\gamma \in \mathcal{K}$.

Proof. Consider the system (5), (6), then under Assumption 1, the controller u_{iss} ensures that the tracking error dynamic (8) is iISS between the input e_Δ and the state vector e_y, which by Definition 1, implies that there exist functions $\alpha \in \mathcal{K}$, $\beta \in \mathcal{KL}$ and $\gamma \in \mathcal{K}$, such that, for all $e(0) \in \mathcal{D}_e$ and $e_\Delta \in \mathcal{D}_{e_\Delta}$, the norm of the error vector e_Δ admits the following bound

$$\|e_y(t)\| \leq \beta(\|e_y(0)\|,t) + \alpha(\int_0^t \gamma(\|e_\Delta\|))ds \tag{11}$$

for all $t \geq 0$.
Now, we need to evaluate the bound on the estimation vector $\tilde{\Delta}$, to do so we use the results presented in [7]. First, based on Assumption 4.1, the cost function is locally Lipschitz, i.e., $\exists \eta_1 > 0,\ s.t.\ |Q(\Delta_1) - Q(\Delta_2)| \leq \eta_1 \|\Delta_1 - \Delta_2\|$, $\forall \Delta_1, \Delta_2 \in \mathcal{V}(\Delta^*)$. Furthermore, since Q is analytic it can be approximated locally in $\mathcal{V}(\Delta^*)$ with a quadratic function, e.g., Taylor series up to second order. Based on this and on Assumptions 3.2 and 3.3, we can write the following bound ([7], pages 436-437):

$$\begin{aligned} \|e_\Delta(t)\| - \|d(t)\| \leq \|e_\Delta(t) - d(t)\| &\leq \tilde{\beta}(\|e_\Delta(0)\|,t) + \tfrac{\xi_1}{\omega_0} \\ \Rightarrow \|e_\Delta(t)\| &\leq \tilde{\beta}(\|e_\Delta(0)\|,t) + \tfrac{\xi_1}{\omega_0} + \|d(t)\| \\ \Rightarrow \|e_\Delta(t)\| &\leq \tilde{\beta}(\|e_\Delta(0)\|,t) + \tfrac{\xi_1}{\omega_0} + \sqrt{\textstyle\sum_{i=1}^{i=p} a_i^2} \end{aligned}$$

with $\tilde{\beta} \in \mathcal{KL}$, $\xi_1 > 0$, $t \geq 0$, $\omega_0 = \max_{i \in \{1,...p\}} \omega_i$, $d(t) = [a_1 sin(\omega_1 t + \frac{\pi}{2}), ..., a_p sin(\omega_p t + \frac{\pi}{2})]^T$, which together with the bound (11) completes the proof. □

So far we have dealt with the case of nonlinear models with constant parametric uncertainties. However, in real application it is often the case that the change in the parameters' values happens slowly overtime, for instance due to aging of the system. To deal with this scenario, we consider in the next section the case of nonlinear models with time-varying parametric uncertainties.

4. Extremum Seeking-based Indirect Adaptive Controller for the Case of Time-varying Model Uncertainties

Consider the system (5), with the time-varying parametric uncertainties $\Delta(t):\ \mathbb{R} \to \mathbb{R}^p$, and the output vector (6). We consider the same control objective here, which is for y to asymptotically track a desired smooth vector time-dependent trajectory $y_{ref}:\ [0,\infty) \to \mathbb{R}^h$. Let us define now the following cost function

$$Q(\hat{\Delta},t) = F(e_y(\hat{\Delta}),t) \tag{12}$$

where $F:\ \mathbb{R}^h \times \mathbb{R}^+ \to \mathbb{R}^+$, $F(0,t) = 0$, $F(e_y,t) > 0$ for $e_y \neq 0$.
In this case, we introduce the following additional assumptions on Q.

Assumption 4.1. $|\frac{\partial Q(\hat{\Delta},t)}{\partial t}| < \rho_Q$, $\forall t \in \mathbb{R}^+$, $\forall \hat{\Delta} \in \mathbb{R}^p$.

We can now state the following result.

Theorem 4.1. *Consider the system (5), (6), with the cost function (12), then under Assumptions 3.1, 3.2 and 4.1, the controller u_{iss}, where $\hat{\Delta}$ is estimated with the multi-parameter extremum seeking algorithm*

$$\dot{\hat{\Delta}}_i = a\sqrt{(}\omega_i)cos(\omega_i t) - k\sqrt{\omega_i}sin(\omega_i t)Q(\hat{\Delta}),\ i \in \{1,...,p\} \tag{13}$$

with $a > 0$, $k > 0$, $\omega_i \neq \omega_j$, $i,j,k \in \{1,...,p\}$, and $\omega_i > \omega^*$, $\forall i \in \{1,...,p\}$, with ω^* large enough, ensures that the norm of the error vector e_y admits the following bound

$$\|e_y(t)\| \leq \beta(\|e_y(0)\|,t) + \alpha(\int_0^t \gamma(\|e_\Delta(s)\|)ds$$

where $\alpha \in \mathcal{K}$, $\beta \in \mathcal{KL}$, $\gamma \in \mathcal{K}$, and $\|e_\Delta\|$ satisfies:

1-$(\frac{1}{\omega},d)$-Uniform Stability: For every $c_2 \in]d,\infty[$, there exists $c_1 \in]0,\infty[$ and $\hat{\omega} > 0$ such that for all $t_0 \in \mathbb{R}$ and for all $x_0 \in \mathbb{R}^n$ with $||e_\Delta(0)|| < c_1$ and for all $\omega > \hat{\omega}$,

$$||e_\Delta(t,e_\Delta(0))|| < c_2,\ \forall t \in [t_0,\infty[$$

2-$(\frac{1}{\omega},d)$-Uniform ultimate boundedness: For every $c_1 \in]0,\infty[$ there exists $c_2 \in]d,\infty[$ and $\hat{\omega} > 0$ such that for all $t_0 \in \mathbb{R}$ and for all $x_0 \in \mathbb{R}^n$ with $||e_\Delta(0)|| < c_1$ and for all $\omega > \hat{\omega}$,

$$||e_\Delta(t,e_\Delta(0))|| < c_2,\ \forall t \in [t_0,\infty[$$

3-$(\frac{1}{\omega},d)$-Global uniform attractively: For all c_1, $c_2 \in (d,\infty)$ there exists $T \in]0,\infty[$ and $\hat{\omega} > 0$ such that for all $t_0 \in \mathbb{R}$ and for all $x_0 \in \mathbb{R}^n$ with $||e_\Delta(0)|| < c_1$ and for all $\omega > \hat{\omega}$,

$$||e_\Delta(t,e_\Delta(0))|| < c_2,\ \forall t \in [t_0+T,\infty[$$

where d is given by: $d = min\{r \in]0,\infty[:\ \Gamma_H \subset B(\Delta,r)\}$, with $\Gamma_H = \{\hat{\Delta} \in \mathbb{R}^n:\ ||\frac{\partial Q(\hat{\Delta},t)}{\partial \hat{\Delta}}|| < \sqrt{\frac{2\rho_Q}{ka\beta_0}}\}$, $0 < \beta_0 \leq 1$, and $B(\Delta,r) = \{\hat{\Delta} \in \mathbb{R}^n:\ ||\hat{\Delta} - \Delta|| < r\}$.

Remark 4.1. Theorem 4.1 shows that the estimation error is bounded by a constant c_2 which can be tightened by making the constant d small. The d constant can be tuned by tuning the cardinal of the set Γ_H, which in turns can be made small by choosing large values for the coefficients a and k of the ES algorithm (13).

Proof. Consider the system (5), (6), then under Assumption 1, the controller u_{iss} ensures that the tracking error dynamic (8) is iISS between the input e_Δ and the state vector e_y, which by Definition 1, implies that there exist functions $\alpha \in \mathcal{K}$, $\beta \in \mathcal{KL}$ and $\gamma \in \mathcal{K}$, such that, for all $e(0) \in \mathcal{D}_e$ and $e_\Delta \in \mathcal{D}_{e_\Delta}$, the norm of the error vector e_Δ admits the following bound

$$\|e_y(t)\| \leq \beta(\|e_y(0)\|, t) + \alpha(\int_0^t \gamma(\|e_\Delta\|))ds \tag{14}$$

for all $t \geq 0$.
Now, we need to evaluate the bound on the estimation vector $\tilde{\Delta}$, to do so we use the results presented in [4]. Indeed, based on Theorem 3 of [4], we can conclude under Assumption 4.1, that the estimator (13), makes the local optimum of Q; $\Delta^* = \Delta$ (see Assumption 3.2), $(\frac{1}{\omega}, d)$-SPUUB, where $d = min\{r \in]0, \infty[:\ \Gamma_H \subset B(\Delta, r)\}$, with $\Gamma_H = \{\hat{\Delta} \in \mathbb{R}^n :\ |\frac{\partial Q(\hat{\Delta}, t)}{\partial \hat{\Delta}}| < \sqrt{\frac{2\rho_Q}{ka\beta_0}}\}$, $0 < \beta_0 \leq 1$, and $B(\Delta, r) = \{\hat{\Delta} \in \mathbb{R}^n :\ ||\hat{\Delta} - \Delta|| < r\}$, which by Definition 2 implies that $\|e_\Delta\|$ satisfies the three conditions: $(\frac{1}{\omega}, d)$-Uniform Stability, $(\frac{1}{\omega}, d)$-Uniform ultimate boundedness, and $(\frac{1}{\omega}, d)$-Global uniform attractively. □

Remark 4.2. The upper-bounds of the estimated parameters used in Theorem 3.1, and Theorem 4.1 are correlated to the choice of the extremum seeking algorithm (10) and (13). However, these bounds can be easily changed by using other ES algorithms, e.g., [40], which is due to the modular design of the controller, that uses the iISS robust part to ensure boundedness of the error dynamics and the learning part to improve the tracking performance.

Remark 4.3. We want to underline here that one of the main advantages of using a mode-free algorithm to estimate the uncertain parameters of the model w.r.t. using classical model-based adaptive control, is that classical adaptive control relies on the structure of the model and the the structure of the model uncertainties, i.e., linear vs. nonlinear uncertainties and linear vs. nonlinear model dynamics. For example it is shown in [36] that using gradient descent-based filters to estimate the unknown parameters of electromagnetic actuators is efficient but because the filters are based on the structure of the model dynamics, it is not possible to use them to estimate multiple uncertainties at the same time. On the other hand, we show in [32] that by using the approach presented here, it is possible to estimate multiple uncertainties at the same time, and even estimate nonlinear parametric uncertainties as shown in [37].

Let us move now to the second problem studied in this chapter, namely, the problem of auto-tuning of feedback gains for nonlinear systems, also referred to in the control community as iterative feedback tuning problem.

5. Learning-based Feedback Gains Auto-tuning for Nonlinear Robust Control

5.1. Class of Systems Under Study

We consider here affine uncertain nonlinear systems of the form

$$\begin{array}{c} \dot{x} = f(x) + \Delta f(x) + g(x)u,\ x(0) = x_0 \\ y = h(x) \end{array} \tag{15}$$

where $x \in \mathbb{R}^n, u \in \mathbb{R}^{n_a}, y \in \mathbb{R}^m$ $(n_a \geq m)$, represent respectively the state, the input and the controlled output vectors, x_0 is a known initial condition, $\Delta f(x)$ is a vector field representing additive model uncertainties. The vector fields f, Δf, columns of g and function h satisfy the following assumptions.

Assumption 5.1. $f : \mathbb{R}^n \to \mathbb{R}^n$ and the columns of $g : \mathbb{R}^n \to \mathbb{R}^{n \times n_a}$ are $\mathbb{C}^\infty$ vector fields on a bounded set X of $\mathbb{R}^n$ and $h(x)$ is a $\mathbb{C}^\infty$ function on X. The vector field $\Delta f(x)$ is $\mathbb{C}^1$ on X.

Assumption 5.2. System (15) has a well-defined (vector) relative degree $\{r1, \ldots, rm\}$ at each point $x^0 \in X$, and the system is linearizable, i.e., $\sum_{i=1}^{i=m} ri = n$ (see e.g., [38]).

Assumption 5.3. The uncertainty vector Δf is s.t., $|\Delta f(x)| \leq d(x)\ \forall x \in X$, where $d : X \to \mathbb{R}$ is a smooth nonnegative function.

Assumption 5.4. The desired output trajectories y_{id} are smooth functions of time, relating desired initial points y_{i0} at $t = 0$ to desired final points y_{if} at $t = t_f$, and s.t., $y_{id}(t) = y_{if}$, $\forall t \geq t_f$, $t_f > 0$, $i \in \{1, ..., m\}$.

5.2. Control Objectives

Our objective is to design a feedback controller $u(x, K)$, which ensures for the uncertain model (15) uniform boundedness of a tracking error, and for which the stabilizing feedback gains vector K is iteratively auto-tuned, to optimize a desired performance cost function.
We stress here that the goal of the gain auto-tuning is not stabilization but rather performance optimization. To achieve this control objective, we proceed as follows: We design a 'passive' robust controller which ensures boundedness of the tracking error dynamics, and we combine it with a model-free learning algorithm to iteratively (resting from the same initial condition at each iteration) auto-tune the feedback gains of the controller, and optimize online a desired performance cost function.

5.3. Controller Design

5.3.1. Step One: Passive Robust Control Design

Under Assumption 5.2 and nominal conditions, i.e., $\Delta f = 0$, system (15) can be written as [38]

$$y^{(r)}(t) = b(\xi(t)) + A(\xi(t))u(t) \tag{16}$$

where

$$
\begin{aligned}
& y^{(r)}(t) \triangleq (y_1^{(r_1)}(t), \ldots, y_m^{(r_m)}(t))^T \\
& \xi(t) = (\xi^1(t), \ldots, \xi^m(t))^T \\
& \xi^i(t) = (y_i(t), \ldots, y_i^{(ri-1)}(t)), \ 1 \leq i \leq m
\end{aligned}
\tag{17}
$$

b, A write as functions of f, g, h, and A is non-singular in X ([38], pp. 234-288).
At this point we introduce one more assumption on the system.

Assumption 5.5. We assume that the additive uncertainties Δf in (15) appear as additive uncertainties in the linearized model (16), (17), as follows

$$
y^{(r)} = b(\xi) + \Delta b(\xi) + A(\xi)u \tag{18}
$$

where Δb is $\mathbb{C}^1$ on $\tilde{X}$, and s.t., $|\Delta b(\xi)| \leq d_2(\xi)\ \forall \xi \in \tilde{X}$, where $d_2 : \tilde{X} \to \mathbb{R}$ is a smooth nonnegative function, and $\tilde{X}$ is the image of the set X by the diffeomorphism $x \to \xi$ between the states of (15) and (16).

Remark 5.1. Assumption 5.5, can be ensured under the so-called 'matching conditions' ([39], p. 146).

If we consider the nominal model (16) first, we can define a virtual input vector v as

$$
b(\xi(t)) + A(\xi(t))u(t) = v(t) \tag{19}
$$

Combining (16) and (19), we obtain the linear (virtual) Input-Output mapping

$$
y^{(r)}(t) = v(t) \tag{20}
$$

Based on the linear system (20), we propose the stabilizing output feedback for the nominal system (18) with $\Delta b(\xi) = 0$, as

$$
\begin{aligned}
& u_{nom} = A^{-1}(\xi)(v_s(t,\xi) - b(\xi)),\ v_s = (v_{s1}, \ldots, v_{sm})^T \\
& v_{si} = y_{i_d}^{(ri)} - K_{ri}^i (y_i^{(ri-1)} - y_{i_d}^{(ri-1)}) - \ldots - K_1^i (y_i - y_{id}) \\
& i \in \{1, \ldots, m\}
\end{aligned}
\tag{21}
$$

Denoting the tracking error vector as $e_i(t) = y_i(t) - y_{i_d}(t)$, we obtain the tracking error dynamics

$$
e_i^{(r_i)}(t) + K_{r_i}^i e_i^{(r_i-1)}(t) + \ldots + K_1^i e_i(t) = 0,\ i = 1, \ldots, m \tag{22}
$$

and by tuning the gains $K_j^i,\ i = 1, \ldots, m,\ j = 1, \ldots, r_i$ such that all the polynomials in (22) are Hurwitz, we obtain global asymptotic stability of the tracking errors $e_i(t),\ i = 1, \ldots m$, to zero. To formalize this condition let us state the following assumption.

Assumption 5.6. We assume that there exist a nonempty set $\mathcal{K}$ of gains $K_j^i,\ i = 1, \ldots, m,\ j = 1, \ldots, r_i$, such that the polynomials (22) are Hurwitz.

Remark 5.2. Assumption 5.6 is well know in the Input-Output linearization control literature. It simply states that we can find gains that stabilize the polynomials (22), which can be done for example by pole placements.

Next, if we consider that $\Delta b(\xi) \neq 0$ in (18), the global asymptotic stability of the error dynamics will not be guarantied anymore due to the additive error vector $\Delta b(\xi)$, we then choose to use Lyapunov reconstruction technique (e.g., [30]) to obtain a controller ensuring practical stability of the tracking error. This controller is presented in the following Theorem.

Theorem 5.1. *Consider the system (15) for any $x_0 \in \mathbb{R}^n$, under Assumptions 5.1 to 5.6, with the feedback controller*

$$\begin{array}{l} u = A^{-1}(\xi)(v_s(t,\xi) - b(\xi)) - A^{-1}(\xi)(\frac{\partial V}{\partial z}_{ind})' k\, d_2(e) \\ k > 0,\ \ v_s = (v_{s1},...,v_{sm})^T \\ v_{si} = y_{i_d}^{(ri)} - K^i_{ri}(y_i^{(ri-1)} - y_{i_d}^{(ri-1)}) - ... - K^i_1(y_i - y_{id}) \end{array} \tag{23}$$

Where, $K^i_j \in \mathcal{K}$, $j = 1,...,ri$, $i = 1,...,m$, and $\frac{\partial V}{\partial z}_{ind} = (\frac{\partial V}{\partial z(r1)},...,\frac{\partial V}{\partial z(rm)})$, $V = z^T P z$, $P > 0$ such that $P\tilde{A} + \tilde{A}^T P = -I$, with $\tilde{A}$ being an $n \times n$ matrix defined as

$$\tilde{A} = \begin{pmatrix} 0,1,0,...............................,0 \\ 0,0,1,0,.............................,0 \\ \ddots \\ -K^1_1,...,-K^1_{r1},0,.................,0 \\ \ddots \\ 0,..................,0,1,0,..........,0 \\ 0,..................,0,0,1,..........,0 \\ \ddots \\ 0,..............,0,-K^m_1,.....,-K^m_{rm} \end{pmatrix} \tag{24}$$

and $z = (z^1,...,z^m)^T$, $z^i = (e_i,...,e_i^{r_i-1})$, $i = 1,...,m$. Then, the vector z is uniformly bounded and reached the positive invariant set $S = \{z \in \mathbb{R}^n |\ 1 - k\,|\frac{\partial V}{\partial z}_{ind}| \geq 0\}$.

Proof. [28]. □

5.3.2. Iterative Tuning of the Feedback Gains

In Theorem 5.1, we showed that the passive robust controller (23) leads to bounded tracking errors attracted to the invariant set S for a given choice of the feedback gains K^i_j, $j = 1,...,ri$, $i = 1,...,m$. Next, to iteratively tune the feedback gains of (23), we define a desired cost function, and use a multi-variable extremum seeking to iteratively auto-tune the gains and minimize the defined cost function. We first denote the cost function to be minimized as $Q(z(\beta))$ where β represents the optimization variables vector, defined as

$$\beta = [\delta K^1_1,...,\delta K^1_{r1},...,\delta K^m_1,...,\delta K^m_{rm},\delta k]^T \tag{25}$$

such that the updated feedback gains write as

$$\begin{array}{c} K^i_j = K^i_{j-nominal} + \delta K^i_j,\ j = 1,...ri,\ i = 1,...,m \\ k = k_{nominal} + \delta k,\ \ k_{nominal} > 0 \end{array} \tag{26}$$

where $K^i_{j-nominal}$, $j = 1,...ri$, $i = 1,...,m$ are the nominal initial values of the feedback gains chosen such that Assumption 5.6 is satisfied.

Remark 5.3. The choice of the cost function Q is not unique. For instance, if the controller tracking performance at the time specific instants It_f, $I = 1,2,3...$ is important for the targeted application, one can choose Q as

$$Q(z(\beta)) = z^T(It_f)C_1 z(It_f),\ C_1 > 0 \tag{27}$$

If other performance needs to be optimized over a finite time interval, for instance a combination of a tracking performance and a control power performance, then one can choose for example the cost function

$$\begin{array}{l} Q(z(\beta)) = \int_{(I-1)t_f}^{It_f} z^T(t)C_1 z(t)dt + \int_{(I-1)t_f}^{It_f} u^T(t)C_2 u(t)dt \\ I = 1,2,3...,\ C_1,\ C_2 > 0 \end{array} \tag{28}$$

The gains variation vector β is then used to minimize the cost function Q over the iterations $I \in \{1,2,3,...\}$.

Following multi-parametric extremum seeking theory [2], the variations of the gains are defined as

$$\begin{array}{l} \dot{x}_{K^i_j} = a_{K^i_j} sin(\omega_{K^i_j} t - \frac{\pi}{2})Q(z(\beta)) \\ \delta\hat{K}^i_j(t) = x_{K^i_j}(t) + a_{K^i_j} sin(\omega_{K^i_j} t + \frac{\pi}{2}),\ j = 1,...ri,\ i = 1,...,m \\ \dot{x}_k = a_k sin(\omega_k t - \frac{\pi}{2})Q(z(\beta)) \\ \delta\hat{k}(t) = x_k(t) + a_k sin(\omega_k t + \frac{\pi}{2}) \end{array} \tag{29}$$

where $a_{K^i_j},\ j = 1,...ri,\ i = 1,...,m,\ a_k$ are positive tuning parameters, and

$$\begin{array}{l} \omega_1 + \omega_2 \neq \omega_3, \text{ for } \omega_1 \neq \omega_2 \neq \omega_3, \\ \forall \omega_1, \omega_2, \omega_3 \in \{\omega_{K^i_j}, \omega_k,\ j = 1,...ri,\ i = 1,...,m\} \end{array} \tag{30}$$

with $\omega_i > \omega^*$, $\forall \omega_i \in \{\omega_{K^i_j}, \omega_k,\ j = 1,...ri,\ i = 1,...,m\}$, ω^* large enough.
To study the stability of the learning-based controller, i.e., controller (23), with the varying gains (26) and (29), we first need to introduce some additional Assumptions.

Assumption 5.7. We assume that the cost function Q has a local minimum at β^*.

Assumption 5.8. We consider that the initial gain vector β is sufficiently close to the optimal gain vector β^*.

Assumption 5.9. The cost function is analytic and its variation with respect to the gains is bounded in the neighborhood of β^*, i.e., $|\frac{\partial Q}{\partial \beta}(\tilde{\beta})| \leq \Theta_2$, $\Theta_2 > 0$, $\tilde{\beta} \in \mathcal{V}(\beta^*)$, where $\mathcal{V}(\beta^*)$ denotes a compact neighborhood of β^*.

We can now state the following result.

Theorem 5.2. *Consider the system (15) for any $x_0 \in \mathbb{R}^n$, under Assumptions 5.1 to 5.9 with the feedback controller*

$$\begin{array}{l} u = A^{-1}(\xi)(v_s(t,\xi) - b(\xi)) - A^{-1}(\xi)(\frac{\partial V}{\partial z}_{ind})' k(t)\ d_2(e) \\ k > 0,\ \ v_s = (v_{s1},...,v_{sm})^T \\ v_{si}(t,\xi) = \hat{y}_{id}^{(ri)} - K^i_{ri}(t)(y_i^{(ri-1)} - \hat{y}_{id}^{(ri-1)}) - ... \\ -K^i_1(t)(y_i - \hat{y}_{id}),\ i = 1,...,m \end{array} \tag{31}$$

Where, the state vector is reset following the resetting law $x(It_f) = x_0$, $I \in \{1,2,...\}$, the desired trajectory vector is rest following $\hat{y}_{id}(t) = y_{id}(t-(I-1)t_f)$, $(I-1)t_f \leq t < It_f$, $I \in \{1,2,...\}$, and $K^i_j(t) \in \mathcal{K}$, $j = 1,...,ri$, $i = 1,...,m$ are piecewise continues gains switched at each iteration I, $I \in \{1,2,...\}$, following the update law

$$\begin{array}{l} K^i_j(t) = K^i_{j-nominal} + \delta K^i_j(t) \\ \delta K^i_j(t) = \delta \hat{K}^i_j((I-1)t_f),\ (I-1)t_f \leq t < It_f \\ k(t) = k_{nominal} + \delta k(t),\ \ k_{nominal} > 0 \\ \delta k(t) = \delta \hat{k}((I-1)t_f),\ (I-1)t_f \leq t < It_f,\ I = 1,2,3... \end{array} \tag{32}$$

where $\delta\hat{K}^i_j, \delta\hat{k}$ are given by (29), (30) and whereas the rest of the coefficients are defined similarly to Theorem 5.1. Then, the obtained closed-loop impulsive time-dependent dynamic system (15), (29), (30), (31) and (32), is well posed, the tracking error z is uniformly bounded, and is steered at each iteration I towards the positive invariant set $S_I = \{z \in \mathbb{R}^n |\ 1 - k_I\ |\frac{\partial V}{\partial z}_{ind}| \geq 0\}$, $k_I = \beta_I(n+1)$, where β_I is the value of β at the Ith iteration. Furthermore, $|Q(\beta(It_f)) - Q(\beta^*)| \leq \Theta_2\big(\frac{\Theta_1}{\omega_0} + \sqrt{\sum_{i=1,...,m\ j=1,...,ri} a_{K^i_j}{}^2 + a_k{}^2}\big)$, Θ_1, $\Theta_2 > 0$, for $I \to \infty$, where $\omega_0 = Max(\omega_{K^1_1},...,\omega_{K^m_{rm}},\omega_k)$, and Q satisfies Assumptions 5.7, 5.8 and 5.9. Wherein, the vector β remains bounded over the iterations s.t., $|\beta((I+1)t_f) - \beta(It_f)| \leq 0.5t_f Max(a_{K^1_1}{}^2,...,a_{K^m_{rm}}{}^2,a_k^2)\Theta_2 + t_f\omega_0\sqrt{\sum_{i=1,...,m\ j=1,...,ri} a_{K^i_j}{}^2 + a_k{}^2}$, $I \in \{1,2,...\}$, and satisfies asymptotically the bound $|\beta(It_f) - \beta^*| \leq \frac{\Theta_1}{\omega_0} + \sqrt{\sum_{i=1,...,m\ j=1,...,ri} a_{K^i_j}{}^2 + a_k{}^2}$, $\Theta_1 > 0$, for $I \to \infty$.

Proof. [28]. □

Remark 5.4. It is worth mentioning here that the proposed ES-based nonlinear IFT method differs from the existing model-free iterative learning control (ILC) algorithm by two main points: First, the proposed method aims at auto-tuning a given vector of feedback gains associated with a nonlinear model-based robust controller. Thus, once the gains are tuned, the optimal gains obtained by the ES tuning algorithm can be used in the sequel without the need of the ES algorithm. Second, the available model-free ILC algorithms does not require any knowledge about the controlled system. In other words, ILC in essence is a model-free control which does not need any knowledge of the system's physics. This can be appealing when the model of the system is hard to obtain, however, it comes at the expense of a large number of iterations, needed to learn all the dynamics of the system (although indirectly, via the learning of a given optimal feedforward control signal). In this conditions, we believe that our approach is faster in terms of the number of iterations needed to improve the overall performance of the closed-loop system. Indeed, our approach is based on the idea of using all the available information about the system's model to design, in a first phase, a model-based controller, and then in a second phase improve the performance of this controller by tuning its gains to compensate for the unknown or the uncertain part of the model. Since, unlike the complete model-free ILC, we do not start from scratch, and do use some knowledge about the system's model, we expect to converge to an optimal performance faster than the model-free ILC algorithms.

6. Conclusion

Adaptive control and iterative feedback gains tuning, are well know challenging control problems. Indeed, much work has been dedicated to these problems during the past decade. However, many real challenges still remain unsolved. We summarized in this Chapter some of the new results dealing with adaptive control and feedback gains auto-tuning for nonlinear systems. The proposed method for indirect adaptive nonlinear control, is based on a modular approach. First, a model-based nonlinear robust controller is designed to ensure (integral) input-to-state stability between a defined tracking error output and a defined uncertain parameter estimation error. Next, the nonlinear robust controller is complemented with a model-free extremum seeking algorithm to estimate the uncertainties of the system. The combination leads to an indirect adaptive nonlinear controller. Similarly, for the feedback gains auto-tuning problem, we reported a new method based on the combination of a robust nonlinear controller with an extremum seeking model-free optimization algorithm, to auto-tune online the feedback gains of the nonlinear robust controller. Due to their modular design, and due to the fact that the adaptive part of both approaches is based on model-free algorithms, the proposed approaches seem to be well suited to handle a larger class of nonlinear systems compared to the available model-based adaptive controllers. To do so, the results reported here could be further improved. For instance, the model-free model estimation part can be improved, by using extremum seekers algorithms that have semi-global or global convergence results, e.g., [40, 41]. Another model-free learning approach which could be investigated in this context is the reinforcement learning method, e.g., [42].

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