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Spyridon Zafeiropoulos, Stefano Di Cairano

***Abstract*— A mechanically decoupled steering system enables autonomous or semi-autonomous vehicle steering by independently actuating the vehicle wheels and the steering wheel. In semi-autonomous operation the steering system should be controlled such that the vehicle wheels angle tracks a reference signal provided by the trajectory planner rapidly and safely, while guaranteeing that a certain alignment is maintained between the steering wheel and the vehicle wheels to avoid loss of "driver's panic". We develop a controller for a mechanically decoupled steering system that can achieve this by coordinating the steering column and the steering rack actuators, while enforcing constraints on the motion of the vehicle wheels, on the interaction between the steering wheel with the driver, and on the relative motion between steering wheel and vehicle wheels. Our design is based on a particular command governor, for which convergence is proven. The control strategy is simulated in closed loop with a detailed simulation model.**

I. Introduction

Advanced steering systems are fundamental components in future vehicles for enabling autonomous and semi-autonomous driving. A well studied technology for advanced steering system is Active Front Steering (AFS) [1]–[4]. Nonetheless, AFS cannot modify both the vehicle wheels *and* the steering wheel, since there is no actuator for the steering wheel. Thus, during semi-autonomous vehicle operation, when the vehicle is responding to the commands of a trajectory planning system with the driver still handling the steering wheel, there is no direct feedback to the driver on what the vehicle is currently doing. This may result in loss of drivability, i.e., loss of a predictable vehicle response to the driver commands, and significant misalignment between steering wheel and vehicle wheels. In torque-based steering assist systems, such as Electric Power Steering (EPS) [5], the actuator is connected to the steering wheel, which is mechanically coupled to the vehicle wheels. Thus, misalignment never occurs. However, the mechanical coupling limits the capabilities of improving vehicle cornering performance and lateral stabilization [6] due to the rigid mechanical interaction with the driver through the steering wheel.

In order to overcome the limitations of AFS and torque-based steering assist systems, mechanically decoupled steering systems, such as the steer-by-wire [7], [8], have been proposed. In these systems, the steering column (and hence the steering wheel) and the steering rack (and hence the vehicle wheels) are always mechanically decoupled. The steering wheel and the vehicle wheels are actuated by two motors, one at the steering column and one at the steering rack, respectively. The actuators are controlled by an electronic control unit (ECU) that coordinates them in order to achieve the desired vehicle and steering wheel motion. Due to the additional degrees of freedom, these steering systems have the potential of handling more driving situations and objectives, such as preservation and enhancement of the feel for the road. Furthermore, the separated actuation allows to operate the vehicle in semi-autonomous mode, when the vehicle wheels are controlled based on a trajectory planned by the autonomous system and the driver receives a non-damaging feedback on what the vehicle is doing. In addition, the misalignment between steering wheel and vehicle wheels can be limited, which improves drivability during semi-autonomous operation, as the driver is (approximately) informed on what the vehicle is doing and it feels it maintains partial control over it. Limiting, and eventually removing, the misalignment also simplifies the transition back to normal (driver-control) mode.

In this paper we propose a control system architecture and design for a mechanically decoupled steering system. The objective of the control system is to track a reference for the vehicle wheels angle supposedly provided by a trajectory planner for (semi)autonomous vehicle operation, while guaranteeing safe operation of the vehicle, of the steering wheel, and a limited misalignment between the steering wheel and the vehicle wheels. We design a governor strategy that commands the setpoints of the steering wheel and the vehicle wheels to achieve steady state tracking of the steering wheel angle reference signal and alignment between the steering wheel and the vehicle wheels, and to enforce all the aforementioned constraints including during the transients. The proposed design uses a slightly modified a cost function with respect to the standard ones, e.g., in [9], [10], and we prove that the main properties of the governor, i.e., constraints satisfaction and finite time convergence of the command to the actual reference, are maintained.

The paper is structured as follows. In Section II we model the mechanically decoupled steering system and we describe the control objectives. In Section III we describe the control architecture, and in Section IV we design the governor for coordinating the two steering subsystems and enforcing constraints, for which a proof of (finite time) convergence is provided. In Section V we present simulation results in closed-loop with a high-fidelity model developed in CarSim [11] for different maneuvers. Conclusions are

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summarized in Section VI.

Notation: We indicate the set of real, nonnegative real, and nonnegative integer numbers by $\mathbb{R}$, $\mathbb{R}_{0+}$, and $\mathbb{Z}_{0+}$, respectively. When a is a vector, $[a]_i$ is its i^{th} component, and $\|a\|_p$ is its p-norm, where if p is skipped, $p = 2$. $\mathcal{B}(\rho)$ is the 2-norm ball centered at the origin and with radius $\rho > 0$. Inequalities between vectors are intended componentwise. The notation $\mathrm{int}(\mathcal{X})$, where $\mathcal{X}$ is a set, indicates the interior of the set, and $\mathcal{X} \oplus \mathcal{Y}$ is the Minkowski sum of sets $\mathcal{X}$ and $\mathcal{Y}$. We use the shorthand notation $(x, y) = [x' \ y']'$.

II. Mechanically Decoupled Steering System

The mechanically decoupled steering system considered in this paper is composed of two subsystems, the steering rack subsystem and the steering column subsystem, that are mechanically disconnected and coordinated by a steering control unit, as shown in Figure 1.

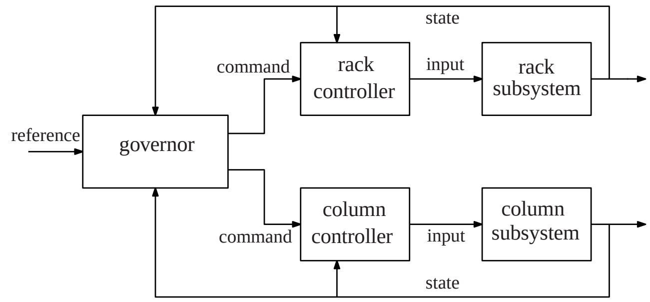


Fig. 1: Control architecture for the mechanically decoupled steering system.

The steering rack subsystem is composed of an electric motor (rack motor) that, through appropriate gearing, joints, and shafts, steers the vehicle wheels that are also affected by the road aligning moment. Here, we assume the rack angle to be equal to the vehicle wheels angle. The dynamics of the steering rack subsystem is described by

$$\dot{\delta}_r = \varphi_r, \tag{1a}$$

$$J_r \dot{\varphi}_r = -\beta_r \varphi_r + T_{\mathrm{mot},r} - T_{\mathrm{aln}}, \tag{1b}$$

where φ_r [rad/s] is the vehicle wheels (and steering rack) angular rate, δ_r [rad] is the vehicle wheels (and steering rack) angle, $T_{\mathrm{mot},r}$ [Nm] is the torque generated by the rack motor, J_r [kg m^2] is the moment of inertia of the steering rack, vehicle wheels, and connecting shafts, β_r [Nms/rad] is the lumped friction coefficient of the steering rack, and T_{aln} [Nm] is the aligning moment torque.

The steering column subsystem is composed of an electric motor (column motor) that, through appropriate gearing, joints, and shafts, can apply torque and possibly steer the steering wheel, where also the driver steering torque is applied. Here we assume the steering column angle to be equal to the steering wheel angle. The steering column dynamics are described by

$$\dot{\delta}_w = \varphi_w, \tag{2a}$$

$$J_w \dot{\varphi}_w = -\beta_w \varphi_w + T_{\mathrm{drv}} + T_{\mathrm{mot},w}, \tag{2b}$$

where φ_w [rad/s] is the steering wheel (and column) angular rate, δ_w [rad] is the steering wheel (and column) angle, $T_{\mathrm{mot},w}$ [Nm] is the torque generated by the column motor, J_w [kg m^2] is lumped moment of inertia of the steering wheel column, steering wheel, and connecting shafts, β_w [Nms/rad] is the friction coefficient of the steering wheel column, and T_{drv} [Nm] is the steering torque by the driver.

The steering system receives a reference angle r[rad] and controls the rack motor torque so that the vehicle wheels angle tracks such reference. During normal operation, the reference angle is provided by the position of the steering wheel, which is controlled by the driver. In semi-autonomous operation, the reference angle is provided by a trajectory planner, and the control unit must tracks such a reference with the vehicle wheels angle, while controlling the steering wheel to maintain some alignment between the steering wheel and the vehicle wheels. For the correct operation of the system a number of constraints needs to be enforced. For instance, the vehicle wheels angle and the steering wheel angle and their derivative need to remains in appropriate ranges to avoid the loss of stability of the vehicle and a negative interaction with the driver, respectively. Also, the misalignment angle, i.e., the difference between steering wheel angle and vehicle wheels angle, should be controlled to maintain drivability and to inform the driver of the current vehicle behavior. Summarizing, the control system should: (i) Rapidly track the reference angle with the vehicle wheels angle; (ii) Smoothly track the (steering-ratio scaled) vehicle wheels angle with the steering wheel angle; (iii) Enforce constraints on vehicle wheels angle dynamics, i.e., on the vehicle dynamics; (iv) Enforce constraints on the steering wheel angle dynamics, i.e., on the driver-vehicle interaction; (v) Enforce constraints on the misalignment angle.

Next, we propose a control system design that addresses (i)–(v).

III. Control System Architecture

We consider a control architecture schematically depicted in Figure 1, where a governor receives the reference for the vehicle wheels and the current state of the entire steering system and sends commands of a target vehicle wheels angle and a target steering wheel angle to the steering subsystem rack and the steering column subsystem, respectively. The commands are received by two feedback controllers that actuate the rack motor and the column motor, based on the states of the respective subsystem, to track them.

In the considered steering system, the driver torque and the aligning torque are measured/estimated from appropriate sensors, and hence considered as measured disturbances. Thus, we define the net rack motor torque $T_r = T_{\mathrm{mot},r} - T_{\mathrm{aln}}$, and the net column motor torque $T_w = T_{\mathrm{mot},w} + T_{\mathrm{drv}}$. Then, we sample (2), (1) with period T_s and design the controllers

$$u_r = K_r x_r + H_r v_r, \tag{3a}$$

$$u_w = K_w x_w + H_w v_w, \tag{3b}$$

where $u_r = T_r \in \mathbb{R}$, $u_w = T_w \in \mathbb{R}$, and $v_r \in \mathbb{R}$, $v_w \in \mathbb{R}$

are the commands for δ_r and δ_w, respectively, resulting in the closed-loop systems

$$x_j(k+1) = A_j x_j(k) + B_j v_j(k), \tag{4a}$$

$$y_j(k) = C_j x_j(k), \qquad j \in \{r, v\} \tag{4b}$$

$j \in \{r, w\}$, where for the rack dynamics $x_r = [\delta_r\ \varphi_r]' \in \mathbb{R}^2$ is the state vector, and $y = \delta_r \in \mathbb{R}$ is the output, and $x_w = [\delta_w\ \varphi_w]' \in \mathbb{R}^2$ is the state vector, and $y_w = \delta_w \in \mathbb{R}$ is the output. Based on these models The feedback gains K_j, $j \in \{r, w\}$ are designed to stabilize the systems and achieve a desired performance, and the feedforward gains H_j, $j = \{r, w\}$ are designed to obtain unitary dc-gain for the closed-loop systems, that is, if $v_j(t) = v_j$ for all $t \geq \bar{t}$, $\lim_{t\to\infty} y_j(t) = v_j$.

Remark 1: We have assumed T_{drv} T_{aln}, to be measured, which can be achieved by a torque sensor on the steering column, and by measuring the rack motor voltage and current. If the sensor measurement noise and uncertainties are significantly large, the controllers (3) may be augmented with integral action.

The objectives of the steering controller in semi-autonomous operation (i.e., (i)–(v)) include enforcing constraints on vehicle, driver-vehicle interaction, and misalignment which are formulated next as functions of the steering system states and inputs. The constraints on the rack subsystem and column subsystem that we consider are

$$\delta_{j,\min} \leq \delta_j \leq \delta_{j,\max}, \tag{5a}$$

$$\varphi_{j,\min} \leq \varphi_j \leq \varphi_{j,\max}, \tag{5b}$$

$$T_{j,\min} \leq T_j \leq T_{j,\max}, \qquad j \in \{r, w\}. \tag{5c}$$

The constraints on the rack subsystem (i.e., $j = r$ in (5)) limit the vehicle wheels angle, the angular rate, and the net rack steering torque, which can also be seen as a proxy for the angular acceleration, due to the mechanical design of the steering system and to the impact of the steering motion on the vehicle dynamics. Similarly, the constraints on the column subsystem (i.e., $j = w$ in (5)) limit the steering wheel angle, angular rate, and the net column steering torque, due to the interaction of the steering system with the driver. In particular, the steering wheel motion is controlled to have limited range, velocity, and torque, to avoid excessively aggressive effects on the driver who is holding it.

In addition, we consider the alignment constraint that bounds the misalignment angle and hence "virtually couples" the two subsystems,

$$M^{\delta}_{\min} \leq \delta_w - \varrho\delta_r \leq M^{\delta}_{\max}, \tag{6}$$

where ϱ is the steering (gear) ratio, i.e., the value such that in conventional steering systems $\varrho\delta_r = \delta_w$. Constraint (6) ensures that the vehicle lateral motion is not completely unexpected by the driver. A similar constraint could also be enforced on the commanded steering wheel angle and vehicle wheels angle, $M^{v}_{\min} \leq v_w - \varrho v_r \leq M^{v}_{\max}$. An additional alignment constraint can be formulated on the angular velocities, i.e,

$$M^{\varphi}_{\min} \leq \varphi_w - \varrho\varphi_r \leq M^{\varphi}_{\max}, \tag{7}$$

which bounds the maximum difference between the steering wheel angular rate and the vehicle wheels angular rate. The objective of (7) is to guarantee that the change in the vehicle lateral motion is not completely unexpected by the driver.

Indeed, if in (i)–(v) the constraint enforcement was not of concern, one could just set $v_r(t) = r(t)$, $v_w(t) = \varrho r(t)$ in (3) and if $r(t) = r$, for all $t \geq \bar{t}$, the angles would be asymptotically controlled to their desired values. Here, in order to enforce (5) and (6) (and, if desired, (7)), the feedforward commands in (4), v_r, v_w, are generated by a governor

$$v = g(x, r), \tag{8}$$

where $x = [x_r'\ x_w']'$ is the full system state, r is the reference for the vehicle wheels angle, and $v \in \mathbb{R}^2$, $[v]_1 = v_r$, $[v]_2 = v_w$ is the governor command. The purpose of the governor is to maintain the steady-state behavior, i.e., for the vehicle wheels to track the reference, and for the steering wheel to align, and in addition to enforce the constraints, including during the transients.

IV. Constraint Governor Design

Different governor exists for enforcing constraints, such as reference governor [10], [12], command governor [9], extended command governor [13], and virtual state governor [14]. In [15] reference and extended reference governors were applied to the AFS for enforcing constraints on lateral and roll vehicle dynamics, which dos not require the coordination of steering wheel and vehicle wheels. The coordination of the steering wheel and the vehicle wheels is a major difference requiring a (particular) multivariable governor.

In order to design a governor (8) that enforces the subsystem constraints (5), and the alignment constraints (6) and/or (7), we exploit the *maximum output admissible set* [12]. Given a system $x(k+1) = f(x(k))$, $x \in \mathbb{R}^n$, and a constrained output $z = h(x)$, $z \in \mathbb{R}^q$, such that $z \in \mathcal{Z} \subset \mathbb{R}^q$, an output admissible set $\mathcal{S}_\infty$ is a set such that

$$x(k) \in \mathcal{S}_\infty \Rightarrow h(x(t)) \in \mathcal{Z},\ \forall t \geq k \tag{9}$$

and the maximal output admissible set, $\mathcal{O}_\infty$, is the largest output admissible set, meaning that there exist no state value $x \in \mathbb{R}^n$ and output admissible set $\mathcal{S}_\infty$, such that $x \in \mathcal{S}_\infty$, and $x \notin \mathcal{O}_\infty$. Furthermore, $\mathcal{O}_\infty$ is an invariant set for $x(k+1) = f(x(k))$, that is, if $x(k) \in \mathcal{O}_\infty$, then $f(x(k)) \in \mathcal{O}_\infty$.

Result 1 ([12]): Consider the asymptotically stable linear system $x(k+1) = A_s x(k)$, $x \in \mathbb{R}^n$, with linear constrained outputs $z(k) = F_s x(k)$ and subject to the constraint $z \in \mathcal{Z}$, where $z \in \mathbb{R}^q$, (F_s, A_s) is observable and $\mathcal{Z}$ is a polytope, $\mathcal{Z} = \{z \in \mathbb{R}^q :\ H_s z \leq K_s\}$. The maximum output admissible set $\mathcal{O}_\infty$ is finitely determined as a polytope defined by a finite number of constraints

$$\mathcal{O}_\infty = \{x \in \mathbb{R}^n : H_\infty x \leq K_\infty\}. \tag{10}$$

$\square$

Consider the system $x(k+1) = f(x(k), v(k))$, $z(k) = h(x(k))$, $v(k) \in \mathbb{R}$, with performance output $y(k) = \kappa(x(k))$ which has dc-gain from v to y equal 1. At time $k \in \mathbb{Z}_{0+}$, given a desired reference value $r(k) \in \mathbb{R}$, a governor selects the "closest" actual command $v \in \mathbb{R}$ to $r(k)$ such that if $v(t) = v$ for all $t \geq k$, $z(t) \in \mathcal{Z}$, for all $t \geq k$.

The governors in [9], [10] exploit the $\mathcal{O}_\infty$ set for the system dynamics augmented with the dynamics of a constant command, $v(k+1) = v(k)$, to select the control input. Specifically, for $x \in \mathbb{R}^n$ and $v, r \in \mathbb{R}$, the reference governor is defined as

$$g(x,r) = \quad \arg\min_{v} \quad \|r - v\|_2^2 \tag{11a}$$

$$s.t. \qquad (x, v) \in \mathcal{O}_\infty \tag{11b}$$

Indeed for the case where the $\mathcal{O}_\infty$-set is polyhedral, (11) is a quadratic program which finds the projection of r onto the section obtained for the current state x of $\mathcal{O}_\infty$. The simplest implementation of the command governor [9] has a definition similar to (11), whit the relaxed condition $v, r \in \mathbb{R}^m$, $m \in \mathbb{Z}_+$, and allowing a positive definite matrix weight in the cost function, yet still requiring r and v to have the same dimension. Additional details can be found in the tutorial [16]. A fundamental result on the reference governor is recalled next.

Result 2 ([10]): Consider the closed-loop system $x(k+1) = f(x(k), g(x(k), r(k)))$, $z(k) = h(x(k))$, $x \in \mathbb{R}^n$, $z \in \mathbb{R}^q$, and the constraint $z \in \mathcal{Z} \subset \mathbb{R}^q$. For some $\bar{k} \geq 0$, let $x(\bar{k})$ be such that a solution v for (11) exists. Then, for every $t \geq \bar{k}$, $z(t) \in \mathcal{Z}$, and if $r(t) = r$ for all $t \geq \bar{t} \geq \bar{k}$, there exists a finite time $t_1 \geq \bar{t}$ such that $v(t) = r$ for all $t \geq t_1$.

A. Governor for mechanically decoupled steering system

In order to design a governor that generates the command vector $v = [v_r \ v_w]'$ with the commands of both the vehicle wheels and the steering wheel, we consider the system

$$x(k+1) = Ax(k) + Bv(k) \tag{12a}$$

$$z(k) = Fx(k) \tag{12b}$$

$$z(k) \in \mathcal{Z}, \tag{12c}$$

where $x = [x_r' \ x_w']'$, (12a) is constructed from (4), (12b) and (12c) are constructed from (5), (6) and/or (7). Due to the linear nature of the constraints (5)–(7), the set $\mathcal{Z}$ is polyhedral, i.e., $\mathcal{Z} = \{z \in \mathbb{R}^q : H_z z \leq K_z\}$. According to Result 1, for (12) augmented with the constant command dynamics $v(k+1) = v(k)$, the maximum output admissible set is the polytope $\mathcal{O}_\infty = \{(x, v) : H_\infty^x x + H_\infty^v v \leq K_\infty\}$.

Thus, we define the governor for the mechanically decoupled steering system as

$$g_v(x,r) = \arg\min_{v \in \mathcal{V}} \ q\|r - [v]_1\|^2 + \|\varrho[v]_1 - [v]_2\|^2 \tag{13a}$$

$$\text{s.t. } (x, v) \in \mathcal{O}_\infty, \tag{13b}$$

where $\mathcal{O}_\infty$ is the maximum output admissible set computed from (12) and $v(k+1) = v(k)$, $q \geq 0$ is a cost function weight, and $\mathcal{V}$ is the (polytopic) set of allowed commands. The objective function (13a) aims at computing the vehicle wheel command that is closest to the reference, and the steering wheel command that is closest to the vehicle wheel command. In fact, the controller aims at rapidly reacting to the reference angle provided from a path planner, and at the same time aims at maintain alignment of the steering wheel with the vehicle wheels for drivability and for informing the driver of the current vehicle behavior. Often, it is not possible to achieve the optimum of both objectives, because the vehicle wheels can be moved faster than the steering wheel, due to the limitations imposed by the interaction with the driver encoded as constraints, and $q \geq 0$ trades off the two objectives.

Consider the closed loop-system (12), (13)

$$x(k+1) = Ax(k) + Bv(k) \tag{14a}$$

$$v(k) = g_v(x(k), r(k)) \tag{14b}$$

$$z(k) = Fx(k) \tag{14c}$$

$$z(k) \in \mathcal{Z}. \tag{14d}$$

The following corollary follows from Result 2.

Corollary 1: Consider the closed-loop system (14). For some $\bar{k} \geq 0$, let $x(\bar{k})$ be such that a solution v for $g_v(x(\bar{k}), r(\bar{k}))$ exists. Then, for every $t \geq \bar{k}$, $z(t) \in \mathcal{Z}$. $\square$

The proof is immediate due to the use of $\mathcal{O}_\infty$ and it follows the same steps as that in [10] for the (standard) governor (11), since both exploit $\mathcal{O}_\infty$ to guarantee (recursive) constraints satisfaction. While indeed (13) uses the maximum output admissible invariant set and results in a quadratic program, it is different from the classical governors [9], [10] because of the cost function (13a). In particular, while Result 2 for (11) is obtained by the properties of projection (i.e., v is the projection of r onto the section of $\mathcal{O}_\infty$ obtained for the current state), see, e.g. [10], this is not the case for (13) because (13a) does not model a (standard) projection. Hence, next we prove that the second part of Result 2, that is, finite time convergence of the command, holds also for (13).

Let $J_r(x)$ be the value function of (13), i.e., the optimum of (13) for given x and r. For the simplicity of notation we define $x^+ = Ax + Bv$, $v^+ = g(x^+, r)$, $\Delta v_i = [v^+]_i - [v]_i$, $i = \{1, 2\}$, and along the trajectories of the system we use the shorthand notation $J_r(k) = J_r(x(k))$.

Lemma 1: Let $v = g_v(x, r)$, and $v^+ = g_v(x^+, r)$, where $x^+ = Ax + Bw$. Then, $J_r(x) - J_r(x^+) \geq q([v^+]_1 - [v]_1)^2 + (([v^+]_1 - [v]_1) - \varrho([v^+]_2 - [v]_2))^2$.

The proof of Lemma 1 is omitted due to space limitations, and it is based on proving that $J_r(x) - J_r(x^+) \geq q([v^+]_1 - [v]_1)^2 + (([v^+]_1 - [v]_1) - \varrho([v^+]_2 - [v]_2))^2$ by showing that $q(r - [v]_1)^2 + (\varrho[v]_1 - [v]_2)^2 \geq q(r - [v^+]_1)^2 + (\varrho[v^+]_1 - [v^+]_2)^2 + \varrho(\Delta v_1)^2 + (\varrho\Delta v_1 - \Delta v_2)^2$ by exploiting feasibility of $v^+ = v$ and optimality of the actual v^+.

Theorem 1: Let $\mathcal{V}$ contain only commands that are strictly steady state admissible, i.e., for all $v \in \mathcal{V}$, $(x_e(v), v) \in \text{int}(\mathcal{O}_\infty)$, where $x_e(v)$ is the equilibrium of (12) for $v(k) = v$. Let $r(k) = r$ for all $k \geq 0$, and $[r \ gr]' \in \mathcal{V}$. For the governor based on (13), let $(x(0), v) \in \mathcal{O}_\infty$, then there exists a finite index $\bar{k} \in \mathbb{Z}_{0+}$ such that $v_1(k) = r$ and

$v_2(k) = \varrho v_1(k)$ for all $k \geq \bar{k}$.

The proof of Theorem 1 is only sketched, due to space limitations.

Due to the properties of $\mathcal{O}_\infty$, the closed-loop system is recursively feasible, and hence $J_r(k+1) \leq J_r(k)$ and $\lim_{k\to\infty} J_r(k) = J_r^\infty$. Using Lemma 1 we can prove that $\lim_{k\to\infty} \Delta v_i(k) = 0$, $i = \{1,2\}$, and hence, $\lim_{k\to\infty} v(k) = v_\infty$. Thus, by the asymptotic stability of (12), $\lim_{k\to\infty} x(k) = x_e \in \mathrm{int}(\mathcal{O}_\infty)$.

The rest of the proof follows the standard approach of, e.g., [9], [17], by showing that given any $v \in \mathcal{V}$, the set $x_e(v) \oplus \mathcal{B}(\sigma)$, where $\sigma > 0$ is small yet finite and $(x_e(v) \oplus \mathcal{B}(\sigma), v) \in \mathrm{int}(\mathcal{O}_\infty)$, is reached in finite time, and that for all $x \in x_e(v) \oplus \mathcal{B}(\sigma)$ there exists $\gamma_1^\Delta \geq \epsilon_v > 0$, for arbitrarily small yet finite ϵ_v, such that $v + v^\Delta$ with $\|v^\Delta\| \leq \gamma_1^\Delta$ is feasible for x. Since $J(x) \geq 0$ for all x, and $J(0)$ is finite, in finite time $\bar{k}$ it has to occur that $\|v(\bar{k}) - [r\ gr]'\| \leq \gamma_1^\Delta$, and hence, $v(\bar{k}+1) = [r\ gr]'$, $J(\bar{k}+1) = 0$.

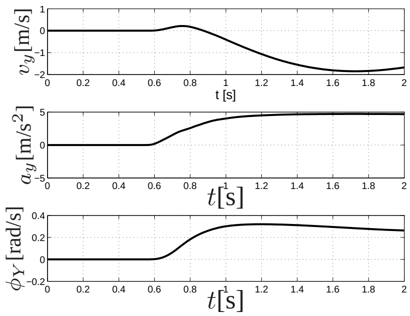
Remark 2: In order to improve robustness with respect to abrupt driver actions or disturbances, (13b) can be substituted by $(Ax + Bv, v) \in \mathcal{O}_\infty$ thus allowing a one step to recover feasibility after a disturbance.

V. Simulation Results

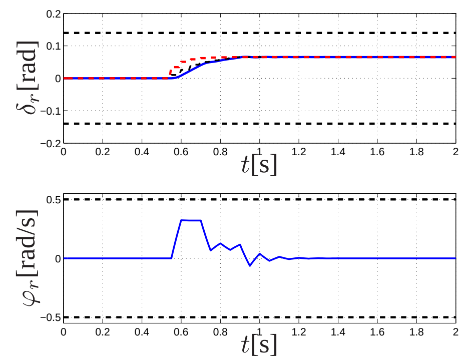
In this section we show the behavior of the control system, and in particular of the governor, for the mechanically decoupled steering system. We discuss the behavior in different maneuvers, namely a step-steer, a slalom, and a double lane change. The control architecture described in Section III is implemented with $T_s = 50$ms, and enforcing (5) and the coordination constraint (6) in the governor. The governor cost function in (13a) is designed to favor a fast response of the steering rack angle. For the simulations we use a proprietary model of the steering system connected with the model of a compact car implemented in CarSim, which provides a high fidelity and reliable simulation platform. Our control architecture is implemented in Simulink. The tests are executed for a constant longitudinal velocity of 60km/h on normal road (friction coefficient $\mu = 0.8$). For every test we show also the time history of the lateral acceleration a_y[m/s^2], the lateral velocity v_y[m/s], and the yaw rate ϕ_Y[rad/s]. We denote by $\Delta\delta_{rw}$[deg] the difference between the steering wheel angle and the (scaled) vehicle wheels angle ($\Delta\delta_{rw} = \delta_w - \varrho\delta_r$).

A step-steering of 60 degrees at the steering wheel is shown in Figure 2. In this maneuver the vehicle wheels are actuated to rapidly track the reference, while the steering wheel is actuated more slowly, so that its motion is acceptable for the driver. In the last part of the maneuver, the difference between the vehicle wheels angle and the steering wheel angle reaches the constraint, and hence the angular velocity of the rack decreases, since it cannot anymore rotate faster than the steering wheel.

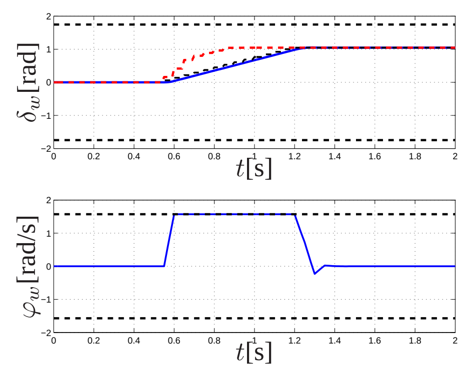
In Figure 3 we show a double lane change maneuver with 4.5m lateral amplitude. Also in this case the constraint (6) enforces a certain alignment to be maintained between steering wheel and vehicle wheels. In this case, the vehicle wheels move rapidly to track the reference signal, thus providing a fast response and good performance in executing the double



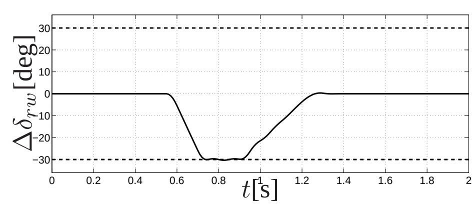
(a) Vehicle dynamics variables during the simulation.



(b) Steering rack. Upper plot: Reference angle (red), vehicle wheels angle command (black), vehicle wheels angle (blue), and constraints. Lower plot: Vehicle wheels angular rate, and constraints.

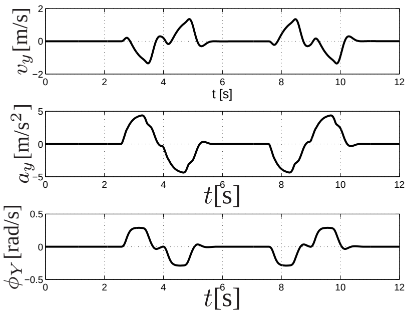


(c) Steering column. Upper plot: Scaled vehicle wheels angle command (red), steering wheel angle command (black), steering wheel angle (blue). Lower plot: steering wheel angular rate.

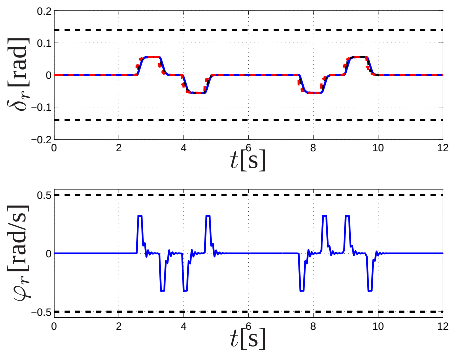


(d) Misalignment between vehicle wheels and steering wheel, and constraints.

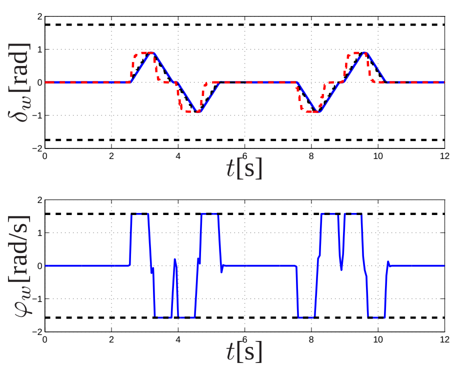
Fig. 2: Simulation of a 60 degrees step-steer maneuver.



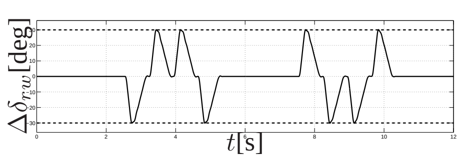
(a) Vehicle dynamics



(b) Steering rack. Upper plot: Reference angle (red), vehicle wheels angle command (black), vehicle wheels angle (blue), and constraints. Lower plot: Vehicle wheels angular rate, and constraints.



(c) Steering column. Upper plot: Scaled vehicle wheels angle command (red), steering wheel angle command (black), steering wheel angle (blue). Lower plot: steering wheel angular rate.



(d) Misalignment between vehicle wheels and steering wheel, and constraints.

Fig. 3: Simulation of a double lane change with 4.5m lateral amplitude.

lane change. The steering moves more slowly thus informing the driver of the current behavior of the vehicle without having an aggressive effect on the driver himself.

VI. Conclusions

We have proposed a design for coordinating vehicle wheels and steering wheel in mechanically decoupled steering systems in semi-autonomous vehicle operations, and in particular when a reference trajectory for the vehicle wheels angle is given. Our design is based on a governor that has the objectives of tracking the reference for the vehicle wheels angle and of aligning the steering wheel and the vehicle wheels, while enforcing constraints on rack subsystem and on steering wheel subsystem, and on the misalignment between vehicle wheels and steering wheel. The controller has been simulated in closed-loop with a CarSim vehicle model showing that the desired behavior is achieved. The properties of the governor have been proved, by appropriate modifications of the proofs of existing governors.

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