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Understanding Contact Bounce

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Abstract

Understanding contact bounce requires consideration not only the energy dissipation at the contact interface, but the distribution of energy (both strain and kinetic energy) within the contact structure. We study the relationship between various system parameters for bounce suppression.

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Understanding Contact Bounce

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To understand the physics of contact bounce, it is instructive to consider the simplified model shown in Figure 1. This is a piecewise linear system which is similar to the much studied repeated impact of a ball with a sinusoidally vibrating table [3, 4, 7, 5, 1, 6].

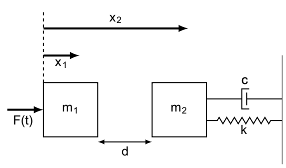

Figure 1. 2-DOF Model

The dynamics of this two-body model can be described by three main stages:

(I) before impact, $t < t_{impact}$, m_2 is stationary and m_1 is accelerated by $F(t)$,

(II) at impact, $t = t_{impact}$, dissipation occurs at the contact surface, the velocities of the two masses become equal,

(III) after impact, $t_{impact} < t \leq t_{separation}$, the masses stay in contact until the instant of first separation $t_{separation}$.

An illustration of this sequence is shown in Figure 2.

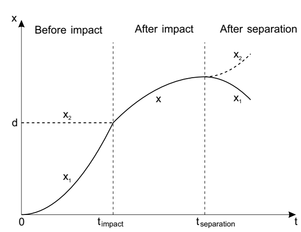

Figure 2. Behavior of two-body bounce model

Impact occurs as m_1 with velocity v_0 strikes the initially motionless m_2 (see Figure 3).

Whether the bodies start moving together after impact depends on the masses, materials, velocities, and the external force. We will assume that all strain energy generated by the impact is dissipated and the two masses will start to move together. First we establish scaling relations between system parameters when there is no damping, i.e. $c = 0$. By using dimensional analysis (Buckingham's Π-theorem, see [2]) we find the following

$$F = \sqrt{km}v_0\Phi\left(\frac{m_1}{m_1+m_2}\right) = \sqrt{km}v_0\Phi(\lambda). \tag{1}$$

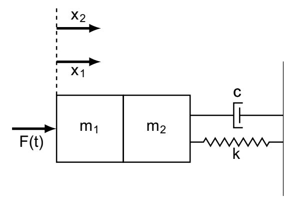

Figure 3. 2-DOF Model after impact, before separation

where $\lambda = \frac{m_1}{m_1+m_2}$. To determine the unknown function $\Phi(\lambda)$ we solve the equations of motions. This yields $\Phi(\lambda) = \frac{\lambda}{\sqrt{1-\lambda^2}}$. Introducing the mass ratio $\varepsilon = \frac{m_2}{m_1} = \frac{1}{\lambda} - 1$ we have

$$\underbrace{\frac{F}{\sqrt{km}v_0}}_{Bo} = \frac{1}{\sqrt{\varepsilon(2+\varepsilon)}}. \tag{2}$$

The equations of motion were solved numerically for (units are not displayed) $v_0 = 0.2$, $m_1 = 1, m_2 = 0.2, k = 1$. The total mass is $m = 1.2$, the mass ratio $\varepsilon = 0.2$, the critical Bouncy number is $Bo_{critical} = 1.51$, and the critical force is $F_{critical} = 0.33$. Simulation results performed with $F = 0.1, 0.33, 0.6$ are shown in Figure 4. The curve for the critical Bouncy number (equation (2)) is also shown.

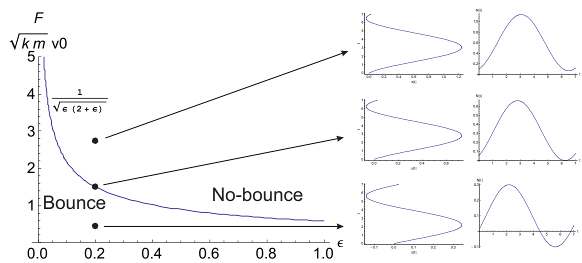

Figure 4. Bounce chart: a, $F = 0.1$ b, $F = 0.33$ c, $F = 0.6$

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