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We analyze demodulation methods for high-order QAM formats in the presence of quadrature angular skew caused by imperfect biasing of the transmitter. Proposed turbo demodulation improves skew tolerance of up to 33-degree angle for an SNR penalty of 1 dB for 1024QAM.

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# Turbo Demodulation for LDPC-Coded High-Order QAM in Presence of Transmitter Angular Skew

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**Abstract** We analyze demodulation methods for high-order QAM formats in the presence of quadrature angular skew caused by imperfect biasing of the transmitter. Proposed turbo demodulation improves skew tolerance of up to 33-degree angle for an SNR penalty of 1 dB for 1024QAM.

## Introduction

Thanks to the recent advancement of powerful forward-error correction (FEC) codes, such as low-density parity-check (LDPC) codes, the so-called turbo principle<sup>1-7</sup> has drawn much attention to cope with various impairments in optical communications. For example, Djordjevic *et al.* have investigated turbo equalization to mitigate linear and nonlinear distortions<sup>1,2</sup>. In an analogous context, the second-order statistics of nonlinear distortion has been considered for sliding-window turbo equalizers<sup>3,4</sup>. Wu *et al.* have studied turbo carrier recovery<sup>5</sup> with scattered pilots. Turbo differential decoding<sup>6</sup> has been used to mitigate error propagation in differential encoding. Cycle slip issues for blind carrier/phase estimators have been dealt with by turbo slip recovery<sup>7</sup> with hidden Markov model.

In this paper, we propose another turbo receiver, referred to as *turbo skew recovery*, to mitigate angular skew in high-speed optical modulators. Quadrature-amplitude modulation (QAM) formats are typically generated with a triple Mach-Zehnder structure. These modulators have in-phase (I) and quadrature (Q) arms, each of which is a Mach-Zehnder interferometer. The relative phase between the I and Q arms is set to 90° by biasing an electro-optic phase shifter, which may be controlled with external circuitry<sup>8</sup>. Its imperfect biasing is referred to as transmitter angular skew. This skew compromises the orthogonality of the I and Q components of the transmitted constellation. It should be noted that transmitter angular skew is considered as distinct from time-domain skew between the I and Q arms, which may be equalized by an appropriate filter<sup>9,10</sup>. Angular skew in the receiver (where the I and Q arms in the optical hybrid are not at 90°) has been studied in the literature<sup>11</sup>, with the use of Gram-Schmidt orthogonalization providing significant benefits. Mitigation of transmitter angular skew has also been considered<sup>12</sup> for high-order QAM. In this paper, we show a potential benefit of turbo demodulation, by comparing to those strategies.

## Quadrature angular skew problem

Let  $x$  be one of  $M$ -ary QAM constellations: e.g.,  $x \in \{\pm 1 \pm j\} / \sqrt{2}$  for 4QAM, where  $j$  is an imaginary unit. In presence of transmitter angular skew, the transmitting constellation becomes

$$x_\theta = \Re[x] + \sin(\theta)\Im[x] + j\cos(\theta)\Im[x], \quad (1)$$

where  $\Re[\cdot]$ ,  $\Im[\cdot]$ , and  $\theta$  are the real-part, the imaginary-part operators, and an I-Q skew angle, respectively. Fig. 1 depicts the 1024QAM constellation for  $\theta = 11.45^\circ$ . It is noted that the constellation points deviate from the ideal square-grid points according to the skew angle.

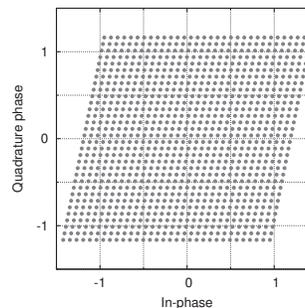


Fig. 1: 1024QAM constellation with skew angle of  $\theta = 11.45^\circ$ .

After several signal processing blocks such as dispersion compensation and carrier recovery, the signal before demodulation is expressed as

$$y = x_\theta + z, \quad (2)$$

where  $z$  is an additive noise (with variance  $\sigma^2$ ). Even without the noise source, a naïve demodulation strategy (ideal rectangular decision boundaries assuming no angular skew) suffers from a significant performance degradation in the presence of angular skew.

One demodulation strategy is the use of Gram-Schmidt orthogonalization process<sup>11</sup>, which makes an inverse skew for the received signal

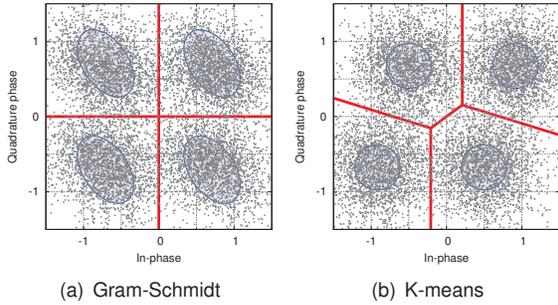


Fig. 2: Demodulation strategies (4QAM with  $\theta = 17.2^\circ$ ).

with the angle of  $-\theta$  as follows:

$$y_\theta = \Re[y] + \sin(-\theta)\Im[y] + j\cos(-\theta)\Im[y]. \quad (3)$$

Fig. 2(a) illustrates the anti-skewed received signal constellations for 4QAM at a noise variance of  $\sigma^2 = 0.25$ . Since the mean points are recovered to a regular 4QAM, it offers a better performance than the naïve method. However, as we can see, the noise becomes non-circularly symmetric, leading to a noise enhancement.

Another strategy is to use a K-means type method<sup>12</sup>, which determines the representative points for each cluster and data points are classified depending on which representative points are the closest. K-means method changes the decision boundary for demodulator as shown in Fig. 2(b). Because there is no noise enhancement, K-means type method offers better performance than the Gram-Schmidt method.

### Turbo demodulation for angle skew recovery

We propose the use of turbo demodulation to mitigate performance degradation due to transmitter angular skew. Fig. 3 shows a schematic of the proposed turbo skew recovery, where the soft-decision information is exchanged between the demodulator and LDPC decoder in a turbo loop.

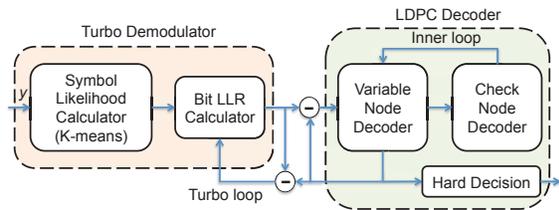


Fig. 3: Turbo skew recovery.

Provided that the additive noise follows the Gaussian distribution, the demodulator in Fig. 3 first calculates the symbol likelihood in the logarithmic domain as below (unnecessary constants discarded):

$$d(x_\theta) = \frac{-1}{\sigma^2} |y - x_\theta|^2. \quad (4)$$

This is based on the squared Euclidean distance between the received signal and the skewed QAM constellation.

The demodulator then calculates bit log-likelihood ratio (LLR) values from the distance metric and/or *a priori* information fed back from the LDPC decoder. The  $k$ -th bit LLR is calculated as

$$L_k = \ln \frac{\sum_{x_\theta: b_k=1} e^{d(x_\theta) + \frac{1}{2} \sum_i (-1)^{b_i} \lambda_i}}{\sum_{x_\theta: b_k=0} e^{d(x_\theta) + \frac{1}{2} \sum_i (-1)^{b_i} \lambda_i}}, \quad (5)$$

where  $\lambda_k$  is the soft-decision message from LDPC decoder. At the very first iteration, we have  $\lambda_k = 0$ . Here,  $b_k$  is the  $k$ -th bit. For numerical stability, we use the relation

$$\ln(e^a + e^b) = \max(a, b) + \ln(1 + e^{-|a-b|}). \quad (6)$$

Given the LLR messages, the LDPC decoder employs the belief propagation between variable-node decoders (VND) and check-node decoders (CND) in an inner loop. After several LDPC decoder iterations, the extrinsic information is fed back to the demodulator to improve the LLR calculations. After a given number of outer-loop iterations, a hard decision is performed to obtain data after LDPC decoding.

### Results

We used an irregular LDPC code [38400, 30832] (code rate: 0.803), whose degree distribution is optimized to achieve 12dB net coding gain by extrinsic information transfer (EXIT) chart. For all simulations, we used a total of 32 iterations for LDPC decoding. For turbo demodulation, we used 4 inner LDPC decoder iterations with 8 outer turbo iterations, resulting in a total of 32 LDPC decoder iterations.

Fig. 4 shows the post-LDPC bit-error rate (BER) performance as a function of SNR in the presence of skew angle  $\theta = 17.2^\circ$  for 4QAM. One can see that the naïve demodulation suffers from a penalty of 0.8dB at a BER of  $3 \times 10^{-3}$ . The Gram-Schmidt and K-means reduce the penalty to 0.4dB and 0.18dB, respectively. Turbo demodulation further reduces the penalty to 0.08dB.

It is noted that the performance degradation becomes larger for higher-order QAM, due to the reduction of phase margin. Fig. 5 shows the post-LDPC BER for 16QAM at a skew of  $\theta = 17.2^\circ$ . Here, the curve of the naïve demodulation is not present due to a large penalty of 4.5dB. The penalties of Gram-Schmidt, K-means, and turbo demodulation are 0.53, 0.26, and 0.17dB, respectively. Those are approximately 0.08dB worse than 4QAM case.

In Fig. 6, we plot the required SNR penalty as a function of skew angle  $\theta$  at a post-LDPC BER of  $3 \times 10^{-3}$  for 1024QAM. The naïve demodulation shows a poor tolerance to skew, while Gram-Schmidt method provides some gain, but degrades rapidly beyond  $5^\circ$ . Turbo demodulation offers the highest tolerance against the angular

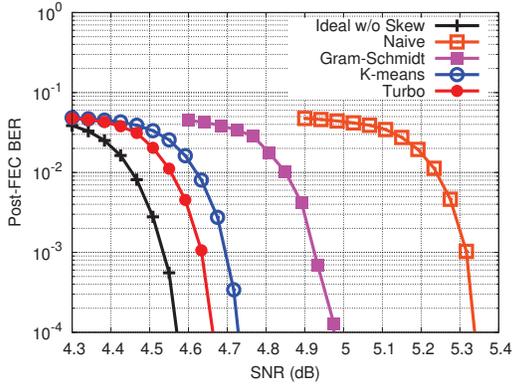


Fig. 4: Post-LDPC BER vs. SNR for 4QAM (skew  $\theta = 17.2^\circ$ ).

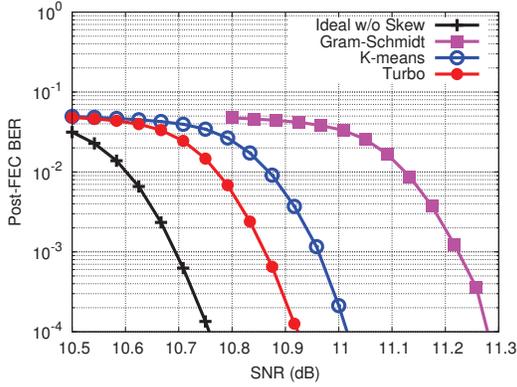


Fig. 5: Post-LDPC BER vs. SNR for 16QAM (skew  $\theta = 17.2^\circ$ ).

skew. For a skew angle of  $\theta = 10^\circ$ , turbo demodulation performs better than k-means by 0.1 dB, whereas the gain is increased to 0.3 dB at  $\theta = 25^\circ$ . Fig. 7 shows the skew margin to achieve below 0.5 dB or 1.0 dB loss for required SNR as a function of modulation size from 4QAM to 1024QAM. Naïve demodulation and Gram-Schmidt orthogonalization are both limited strategies for lower density modulation. Turbo demodulation outperforms k-means under all cases considered here, and appears to perform better for higher-order modulation formats.

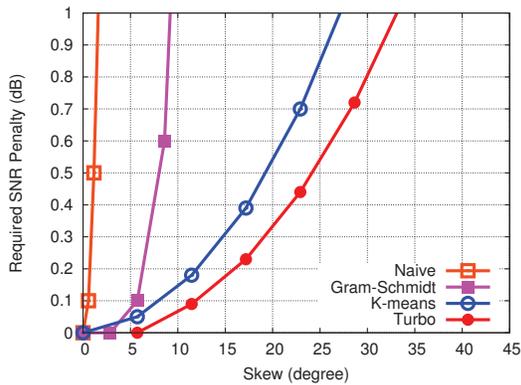


Fig. 6: Required SNR penalty vs. skew for 1024QAM at a BER of  $3 \times 10^{-3}$ .

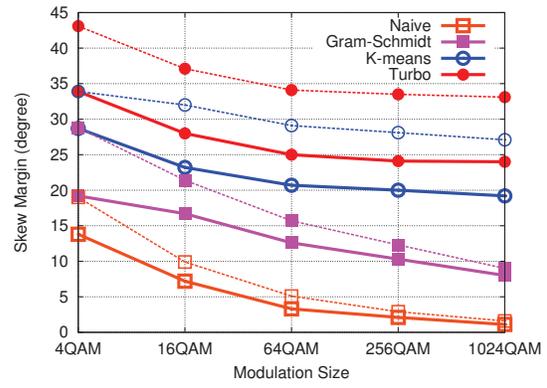


Fig. 7: Skew tolerance vs. modulation size for a BER of  $3 \times 10^{-3}$  (Solid lines are results for an SNR margin of 0.5 dB, and dashed lines are for 1 dB).

## Conclusions

We have investigated demodulation strategies for LDPC-coded QAM signals in the presence of transmitter angular skew. We have shown that naïve and Gram-Schmidt strategies perform poorly in particular for larger skew and higher-order modulation. K-means demodulation was found to provide a significant gain for skew beyond  $10^\circ$ . Turbo demodulation showed the best performance for all cases, with a considerable advantage over k-means demodulation.

## Acknowledgments

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