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Nonlinear Adaptive Control for Electromagnetic Actuators

Mouhacine Benosman* Gökhan M. Atınç†

Abstract

We study here the problem of robust 'soft-landing' control for electromagnetic actuators. The soft landing requires accurate control of the actuators moving element between two desired positions. We present here two nonlinear adaptive controllers to solve the problem of robust trajectory tracking for the moving element. The first controller is based on classical nonlinear adaptive technique. We show that this controller ensures bounded tracking errors of the reference trajectories and bounded estimation error of the uncertain parameters. Second, we present a controller based on the so-called Input-to-State Stability (ISS), merged with gradient descent estimation filters to estimate the uncertain parameters. We show that it ensures bounded tracking errors for bounded estimation errors, furthermore, due to the ISS results we conclude that the tracking errors bounds decrease as function of the estimation errors. We demonstrate the effectiveness of these controllers on a simulation example.

1 Introduction

In many practical applications such us valves of combustion engines or artificial hearts, electromagnetic actuators are preferred to other type of actuators. In this work we concentrate on a particular control problem of nonlinear electromagnetic actuator called 'soft landing' problem. The soft landing requires accurate control of the moving element of the actuator between two desired positions. This 'soft-landing' performance has to be guaranteed over long period of time during which the actuator components may age. The main objective is to attain small contact velocity, which in turn ensures low component-wear operation of the actuator. Due to these practical constraints we have developed a robust control algorithm that aims for a zero impact velocity, and adapts to the actuator aging parts. We present here the results of this study.

Many papers have been dedicated to the soft-landing problem for electromagnetic actuators, e.g. [1, 2, 3, 4, 5, 6, 7, 8, 9]. Several controllers have been developed in [1, 4, 5, 9] based on linear models of the system. Linear models allow a relatively easy design of the control but due to their linearity, are not valid for a full operation range of the actuator. To control the system over a larger operating state space, the controller has to be based on more complex nonlinear models of the actuators. Different nonlinear controllers have been used in [2, 3, 6, 8, 10, 11]. For example in [6], the authors proposed a nonlinear controller to solve the problem of armature stabilization for an electromechanical valve actuator. The authors proved a global asymptotic stability result using Sontag's nonlinear controller. However, this approach did not solve the problem of armature trajectory tracking and did not consider robustness of the controller with respect to system's uncertainties and changes in parameters over time. In [2], the authors studied the problem of electromagnetic valve actuator control in an internal combustion engine. The solution proposed by the author is based on iteratively solving a constrained nonlinear optimal problem using Nelder-Mead algorithm. The robustness of this feedforward-based approach has neither been proven nor tested. In [11], the authors designed a backstepping based controller for electromagnetic actuators position regulation. However, robustness w.r.t. uncertainties in parameters of the system are not considered in this paper. In [8], a nonlinear sliding mode approach was used to solve the problem of trajectory tracking for an electromagnetic valve actuator. The authors used a nonlinear model to design the sliding mode control. The reported results showed good tracking performances, however, this sliding mode controller does not ensure robustness with respect to model uncertainties. In [3], the authors used a single parameter extremum seeking learning method to solve the problem of soft landing for an electromechanical valve actuator. In [12] a

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multiparameter extremum seeking-based control was presented. The authors first designed a nonlinear controller based on Lyapunov redesign technique and then added a multiparameter extremum seeking algorithm to tune the feedback gains for the controller. Although the learning algorithms in [3, 12] were not directly tailored to ensure robustness of the controller to model uncertainties or parameters drift over time, one could argue that this robustness is intrinsic due the iterative nature of the learning process. In [13], the authors designed a backstepping based controller for electromagnetic actuators which was robustified by an extremum seeking algorithm to estimation *some uncertain parameters* of the system. The effectiveness of the proposed scheme was illustrated numerically, however, no rigorous analysis was present concerning the stability of the combined model-based nominal controller and the model-free learning algorithm.
In this work we first use a nonlinear model of the electromagnetic actuator to design a nonlinear adaptive backstepping controller, based on classical adaptive technique. This first controller is proven to ensure bounded trajectory tracking errors as well as bounded uncertain parameters estimation errors. Next, we use the so-called Input-to-State (ISS) theory to develop a nonlinear ISS-adaptive controller, merged with gradient-descent estimation filters. This second controller ensures bounded tracking errors as well as bounded estimation errors, furthermore, due to the ISS result, we conclude that the tracking errors decrease with the estimation errors.

This paper is organized as follows: We first present in Section II some notations and preliminaries. In Section III, we recall the nonlinear model of electromagnetic actuators. Then, in Section IV, we report the adaptive nonlinear controllers, with stability analysis. Numerical validation of the proposed controller is given in Section V, and finally, concluding remarks are stated in Section VI.

2 Preliminaries

Throughout the paper we will use $\|.\|$ to denote the Euclidean norm; i.e., for $x \in \mathbb{R}^n$ we have $\|x\| = \sqrt{x^T x}$. Also, we will use the notations $diag\{m_1, ..., m_n\}$ for $n \times n$ diagonal matrix, and $\dot{(.)}$ for the short notation of time derivative. We denote by C^k functions that are k times differentiable.

Let us now introduce some definitions that will be used subsequently. For this purpose, we first consider the general dynamical time-varying system definition. Consider the nonlinear time-varying dynamical system

$$\dot{x}(t) = f(t, x(t)), \quad x(t_0) = x_0, \quad t \geq t_0 \tag{2.1}$$

where $x(t) \in \mathcal{D} \subseteq \mathbb{R}^n$ such that $0 \in \mathcal{D}$, $f : [t_0, t_1) \times \mathcal{D} \to \mathbb{R}^n$ is such that $f(\cdot, \cdot)$ is jointly continuous in t and x, and for every $t \in [t_0, t_1)$, $f(t, 0) = 0$ and $f(t, \cdot)$ is locally Lipschitz in x uniformly in t for all t in compact subsets of $[0, \infty)$. The above assumptions guarantee the existence and uniqueness of the solution $x(t)$ over the interval $[t_0, t_1)$. Without loss of generality, we assume $t_0 = 0$.

Definition 1. (LaSalle-Yoshizawa [14]) *Consider the time-varying system (2.1) and assume $[0, \infty) \times \mathcal{D}$ is a positively invariant set with respect to (2.1) where $f(t, \cdot)$ is Lipschitz in x, uniformly in t. Assume there exist a C^1 function $V : [0, \infty) \times \mathcal{D} \to \mathbb{R}$, continuous positive definite functions $W_1(\cdot)$ and $W_2(\cdot)$ and a continuous nonnegative function $W(\cdot)$, such that for all $(t, x) \in [0, \infty) \times \mathcal{D}$,*

$$\begin{aligned} W_1(x) \leq V(t, x) \leq W_2(x), \\ \dot{V}(t, x) \leq -W(x) \end{aligned} \tag{2.2}$$

hold. Then there exists $\mathcal{D}_0 \subseteq \mathcal{D}$ such that for all $(t_0, x_0) \in [0, \infty) \times \mathcal{D}_0$, $x(t) \to \mathcal{R} \triangleq \{x \in \mathcal{D} : W(x) = 0\}$ as $t \to \infty$. If, in addition, $\mathcal{D} = \mathbb{R}^n$ and $W_1(\cdot)$ is radially unbounded, then for all $(t_0, x_0) \in [0, \infty) \times \mathbb{R}^n$, $x(t) \to \mathcal{R} \triangleq \{x \in \mathbb{R}^n : W(x) = 0\}$ as $t \to \infty$.

Definition 2. ($\mathcal{K}$ function [15]) *A continuous function $\alpha : [0, a) \to [0, \infty)$ is said to belong to class $\mathcal{K}$ if it is strictly increasing and $\alpha(0) = 0$. It is said to belong to class $\mathcal{K}_\infty$ if $a = \infty$ and $\alpha(r) \to \infty$ as $r \to \infty$.*

Definition 3. ($\mathcal{KL}$ function [15]) *A continuous function $\beta : [0, a) \times [0, \infty) \to [0, \infty)$ is said to belong to class $\mathcal{KL}$ if, for each fixed s, the mapping $\beta(r, s)$ belongs to class $\mathcal{K}$ with respect to r and, for each fixed r, the mapping $\beta(r, s)$ is decreasing with respect to s and $\beta(r, s) \to 0$ as $s \to \infty$.*

Definition 4. (Integral Input-to-State Stability [16]) *Consider the system*

$$\dot{x} = f(t, x, u), \tag{2.3}$$

where $x \in \mathcal{D} \subseteq \mathbb{R}^n$ such that $0 \in \mathcal{D}$,and $f : [0, \infty) \times \mathcal{D} \times \mathcal{D}_u \to \mathbb{R}^n$ is piecewise continuous in t and locally Lipschitz in x and u, uniformly in t. The inputs are assumed to be measurable and locally essentially bounded functions $u : \mathbb{R}_{\geq 0} \to \mathcal{D}_u \subseteq \mathbb{R}^m$. Given any control $u \in \mathcal{D}_u$ and any $\xi \in \mathcal{D}_0 \subseteq \mathcal{D}$, there is a unique maximal solution of the initial value problem $\dot{x} = f(t, x, u)$,

$x(t_0) = \xi$. *Without loss of generality, assume* $t_0 = 0$. *The unique solution is defined on some maximal open interval, and it is denoted by* $x(\cdot, \xi, u)$. *System (2.3) is locally integral input-to-state stable (LiISS) if there exist functions* $\alpha, \gamma \in \mathcal{K}$ *and* $\beta \in \mathcal{KL}$ *such that, for all* $\xi \in \mathcal{D}_0$ *and all* $u \in \mathcal{D}_u$, *the solution* $x(t, \xi, u)$ *is defined for all* $t \geq 0$ *and*

$$\alpha(\|x(t,\xi,u)\|) \leq \beta(\|\xi\|, t) + \int_0^t \gamma(\|u(s)\|)ds \tag{2.4}$$

for all $t \geq 0$. *Equivalently, system (2.3) is LiISS if and only if there exist functions* $\beta \in \mathcal{KL}$ *and* $\gamma_1, \gamma_2 \in \mathcal{K}$ *such that*

$$\|x(t,\xi,u)\| \leq \beta(\|\xi\|, t) + \gamma_1\left(\int_0^t \gamma_2(\|u(s)\|)ds\right) \tag{2.5}$$

for all $t \geq 0$, *all* $\xi \in \mathcal{D}_0$ *and all* $u \in \mathcal{D}_u$.

Definition 5. (Weakly Zero-Detectability [17]) *Let an output for the system (2.3) be a continuous map* $h : \mathcal{D} \to \mathbb{R}^p$, *with* $h(0) = 0$. *For each initial state* $\xi \in \mathcal{D}_0$, *and each input* $u \in \mathcal{D}_u$, *let* $y(t, \xi, u)$ *be the corresponding output function; i.e.,* $y(t, \xi, u) = h(x(t, \xi, u))$, *defined on some maximal interval* $[0, T_{\xi,u})$. *The system (2.3) with output* h *is said to be weakly zero-detectable if, for each* ξ *such that* $T_{\xi,0} = \infty$ *and* $y(t, \xi, 0) \equiv 0$, *it must be the case that* $x(t, \xi, 0) \to 0$ *as* $t \to \infty$.

3 System modelling

Following [11, 10, 3], we consider the nonlinear electromagnetic actuator model

$$\begin{aligned} &m\frac{d^2x}{dt^2} = k(x_0 - x) + \eta\frac{dx}{dt} - \frac{ai^2}{2(b+x)^2} + f_d \\ &u = Ri + \frac{a}{b+x}\frac{di}{dt} - \frac{ai}{(b+x)^2}\frac{dx}{dt},\ 0 \leq x \leq x_f, \end{aligned} \tag{3.6}$$

where, x represents the armature position physically constrained between the initial position of the armature 0, and the maximal position of the armature x_f, $\frac{dx}{dt}$ represents the armature velocity, m is the armature mass, k the spring constant, x_0 is the initial length of the spring, η the damping coefficient (assumed to be constant), $\frac{ai^2}{2(b+x)^2}$ represents the electromagnetic force (EMF) generated by the coil, a, b being constant parameters of the coil, f_d a constant term modelling disturbance forces, e.g. static friction, R the resistance of the coil, $L = \frac{a}{b+x}$ the coil inductance (assumed to be armature-position dependent), $\frac{ai}{(b+x)^2}\frac{dx}{dt}$ represents the back EMF. Finally, i denotes the coil current, $\frac{di}{dt}$ its time derivative and u represents the control voltage applied to the coil. In this model we do not consider the saturation region of the flux linkage in the magnetic field generated by the coil, since we assume a current and armature motion ranges within the linear region of the flux.

4 Adaptive Nonlinear Backstepping Control

4.1 Classical Backstepping Adaptive Controller Consider the dynamical system (3.6). Defining the state vector $\mathbf{z} := [z_1 \ \ z_2 \ \ z_3]^T = [x \ \ \dot{x} \ \ i]^T$, the objective of the control is to make the variables (z_1, z_2) track a sufficiently smooth (at least C^2) time-varying position and velocity trajectories $z_1^{ref}(t)$, $z_2^{ref}(t) = \frac{dz_1^{ref}(t)}{dt}$ that satisfy the following constraints:

$$\begin{aligned} &z_1^{ref}(t_0) = z_{1_{int}}, \quad z_1^{ref}(t_f) = z_{1_f}, \\ &\dot{z}_1^{ref}(t_0) = \dot{z}_1^{ref}(t_f) = 0, \\ &\ddot{z}_1^{ref}(t_0) = \ddot{z}_1^{ref}(t_f) = 0, \end{aligned} \tag{4.7}$$

where t_0 is the starting time of the trajectory, t_f is the ending time, $z_{1_{int}}$ is the initial position and z_{1_f} is the final position. To start, let us first write the system (3.6) in the following way:

$$\begin{aligned} &\dot{z}_1 = z_2 \\ &\dot{z}_2 = \frac{k}{m}(x_0 - z_1) + \frac{\eta}{m}z_2 - \frac{a}{2m(b+z_1)^2}z_3^2 + \frac{f_d}{m} \\ &\dot{z}_3 = -\frac{R}{\frac{a}{b+z_1}}z_3 + \frac{z_3}{b+z_1}z_2 + \frac{u}{\frac{a}{b+z_1}}. \end{aligned} \tag{4.8}$$

Consider the system in (4.8) with constant uncertainty in spring constant k, damping coefficient η and additive disturbance f_d. Since the parameters are unknown, we will use the certainty equivalence [18] and define the virtual input $\tilde{u}$ where the parameters k, η, f_d are replaced by their estimates $\hat{k}$, $\hat{\eta}$ and $\hat{f}_d$:

$$\begin{aligned} \tilde{u} = &\frac{2m(b+z_1)^2}{a}(\frac{\hat{k}}{m}(x_0 - z_1) + \frac{\hat{\eta}}{m}z_2 + \frac{\hat{f}_d}{m} - \dot{z}_2^{ref} \\ &+c_3(z_1 - z_1^{ref}) + c_1(z_2 - z_2^{ref})). \end{aligned} \tag{4.9}$$

Together with the control input

(4.10)

$$\begin{aligned} u = &\frac{a}{b+z_1}(\frac{R(b+z_1)}{a}z_3 - \frac{z_2 z_3}{b+z_1} + \frac{1}{2z_3}(-c_2(z_3^2 - \tilde{u}) + \frac{a}{2m(b+z_1)^2}(z_2 - z_2^{ref}))) \\ &- \frac{m(b+z_1)}{z_3}\frac{\hat{k}}{m}z_2 + \frac{2mz_2}{z_3}(\frac{\hat{k}}{m}(x_0 - z_1) + \frac{\hat{\eta}}{m}z_2 + \frac{\hat{f}_d}{m} - \dot{z}_2^{ref} + c_1(z_2 - z_2^{ref}) \\ &+ c_3(z_1 - z_1^{ref})) + (\frac{m(b+z_1)}{z_3})(\frac{\dot{\hat{k}}}{m}(x_0 - z_1) + \frac{\dot{\hat{\eta}}}{m}z_2 + \frac{\dot{\hat{f}}_d}{m} + \frac{\hat{\eta}}{m}(\frac{\hat{k}}{m}(x_0 - z_1) \\ &+ \frac{\hat{\eta}}{m}z_2 - \frac{pz_3^2}{mz_1^2} + \frac{\hat{f}_d}{m})) + (\frac{m(b+z_1)}{z_3})(c_1(\frac{\hat{k}}{m}(x_0 - z_1) + \frac{\hat{\eta}}{m}z_2 - \frac{pz_3^2}{mz_1^2} + \frac{\hat{f}_d}{m} \\ &- \dot{z}_2^{ref}) - \ddot{z}_2^{ref} + c_3(z_2 - z_2^{ref})). \end{aligned}$$

In addition, we will make use of the following equations for the parameter estimation dynamics:

$$\begin{aligned} &\dot{\hat{k}} = \sigma_1(x_0 - z_1)((z_2 - z_2^{ref}) - (\hat{\eta} + mc_1)(\frac{z_1^2(z_3^2 - \tilde{u})}{p})) \\ &\dot{\hat{\eta}} = \sigma_2 z_2((z_2 - z_2^{ref}) - (\hat{\eta} + mc_1)(\frac{z_1^2(z_3^2 - \tilde{u})}{p})) \\ &\dot{\hat{f}}_d = \sigma_3((z_2 - z_2^{ref}) - (\hat{\eta} + mc_1)(\frac{z_1^2(z_3^2 - \tilde{u})}{p})), \end{aligned} \tag{4.11}$$

with $\sigma_1, \sigma_2, \sigma_3 > 0$ design parameters. The controller (4.9), (4.10) and (4.11) is obtained by the constructive proof of the next lemma.

Lemma 4.1. *Consider the closed-loop dynamics given by (4.8), (4.9), (4.10) and the parameter update laws* *given by (4.11). Then, there exist positive gains c_i, σ_i, $i = 1, 2, 3$ such that $z_1(t), z_2(t), z_3(t), \hat{k}(t), \hat{\eta}(t), \hat{f}_d(t)$ are globally bounded, and satisfy $\lim_{t\to\infty} z_2(t) = z_2^{ref}(t)$, $\lim_{t\to\infty} z_3^2(t) = \tilde{u}_{ref}(t)$.*

Proof. Primarily, we consider the mechanical subsystem, described by the dynamics (4.8), with the virtual control input $\tilde{u} := z_3^2$. Consider the Lyapunov function $V_{sub_{ad}} = \frac{c_3}{2}(z_1 - z_1^{ref})^2 + \frac{1}{2}(z_2 - z_2^{ref})^2 + \frac{1}{2m\sigma_1}(k - \hat{k})^2 + \frac{1}{2m\sigma_2}(\eta - \hat{\eta})^2 + \frac{1}{2m\sigma_3}(f_d - \hat{f}_d)^2$, where z_1^{ref} and z_2^{ref} are known C^k functions, $c_3, \sigma_1, \sigma_2, \sigma_3 > 0$ are design parameters, and $\hat{k}, \hat{\eta}, \hat{f}_d$ are estimates of the actual parameters k, η and f_d. Taking the derivative of $V_{sub_{ad}}$ along the first two equations of (4.8), we get

(4.12)
$$\dot{V}_{sub_{ad}} = (z_2 - z_2^{ref})[c_3(z_1 - z_1^{ref}) + \tfrac{k}{m}(x_0 - z_1) + \tfrac{\eta}{m} z_2 + \tfrac{f_d}{m} - \tfrac{a}{2m(b+z_1)^2}\tilde{u} - \dot{z}_2^{ref}] - \tfrac{1}{m\sigma_1}(k - \hat{k})(\dot{\hat{k}}) - \tfrac{1}{m\sigma_2}(\eta - \hat{\eta})(\dot{\hat{\eta}}) - \tfrac{1}{m\sigma_3}(f_d - \hat{f}_d)(\dot{\hat{f}}_d).$$

Substituting (4.9) into (4.12) , and defining $e_k := k - \hat{k}, e_\eta := \eta - \hat{\eta}, e_{f_d} := f_d - \hat{f}_d$ we have

(4.13)
$$\dot{V}_{sub_{ad}} = -c_1(z_2 - z_2^{ref})^2 + e_k \underbrace{\left(\tfrac{(x_0 - z_1)(z_2 - z_2^{ref})}{m} - \tfrac{\dot{\hat{k}}}{m\sigma_1}\right)}_{\Delta_1} + e_\eta \underbrace{\left(\tfrac{z_2(z_2 - z_2^{ref})}{m} - \tfrac{\dot{\hat{\eta}}}{m\sigma_2}\right)}_{\Delta_2} + e_{f_d} \underbrace{\left(\tfrac{(z_2 - z_2^{ref})}{m} - \tfrac{\dot{\hat{f}}_d}{m\sigma_3}\right)}_{\Delta_3}.$$

Next, we define the augmented Lyapunov function for the full system, $V_{aug_{ad}} = V_{sub_{ad}} + \frac{e^2}{2}$ with $e := z_3^2 - \tilde{u}$. Taking the derivative along the trajectories of the whole system and utilizing (4.9), we obtain

(4.14)
$$\begin{aligned}\dot{V}_{aug_{ad}} &= (z_2 - z_2^{ref})[c_3(z_1 - z_1^{ref}) + \tfrac{k}{m}(x_0 - z_1) + \tfrac{\eta}{m} z_2 + \tfrac{f_d}{m} - \tfrac{a}{2m(b+z_1)^2} z_3^2 - \dot{z}_2^{ref}] - \tfrac{1}{m\sigma_1}(k - \hat{k})(\dot{\hat{k}}) - \tfrac{1}{m\sigma_2}(\eta - \hat{\eta})(\dot{\hat{\eta}}) - \tfrac{1}{m\sigma_3}(f_d - \hat{f}_d)(\dot{\hat{f}}_d) \\ &\quad + (z_3^2 - \tilde{u}) \; 2z_3\left(-\tfrac{R(b+z_1)}{a} z_3 + \tfrac{z_2 z_3}{(b+z_1)} + \tfrac{b+z_1}{a} u\right) - \dot{\tilde{u}}\Bigg) \\ &= -c_1(z_2 - z_2^{ref})^2 + e_k\Delta_1 + e_\eta\Delta_2 + e_{f_d}\Delta_3 \\ &\quad + e \; \tfrac{a}{2m(b+z_1)^2}(z_2 - z_2^{ref}) - 2z_3\left(-\tfrac{R(b+z_1)}{a} z_3 + \tfrac{z_2 z_3}{(b+z_1)} + \tfrac{b+z_1}{a} u\right) - \dot{\tilde{u}} \;,\end{aligned}$$

(4.15)
$$\begin{aligned}\dot{\tilde{u}} = {}& \tfrac{4m(b+z_1)z_2}{a} \; (\tfrac{\hat{k}}{m}(x_0 - z_1) + \tfrac{\hat{\eta}}{m} z_2 + \tfrac{\hat{f}_d}{m} - \dot{z}_2^{ref} + c_1(z_2 - z_2^{ref}) + c_3(z_1 - z_1^{ref})) + \; \tfrac{2m(b+z_1)^2}{a} \; -\tfrac{\hat{k}}{m} z_2 \\ & + \; \tfrac{2m(b+z_1)^2}{a} \; \left(\tfrac{\dot{\hat{k}}}{m}(x_0 - z_1) + \tfrac{\dot{\hat{\eta}}}{m} z_2 + \tfrac{\dot{\hat{f}}_d}{m} + \tfrac{\hat{\eta}}{m}\left(\tfrac{e_k + \hat{k}}{m}(x_0 - z_1) + \tfrac{e_\eta + \hat{\eta}}{m} z_2 + \tfrac{e_{f_d} + \hat{f}_d}{m} - \tfrac{a}{2m(b+z_1)^2} z_3^2\right)\right) \\ & + \; \tfrac{2m(b+z_1)^2}{a} \; \left(c_1\left(\tfrac{e_k + \hat{k}}{m}(x_0 - z_1) + \tfrac{e_\eta + \hat{\eta}}{m} z_2 + \tfrac{e_{f_d} + \hat{f}_d}{m} - \tfrac{a}{2m(b+z_1)^2} z_3^2 - \dot{z}_2^{ref}\right) - \ddot{z}_2^{ref} + c_3(z_2 - z_2^{ref})\right).\end{aligned}$$

Rewriting (4.14) by grouping the terms involving e_k, e_η and e_{f_d}, we get the following inequality:

(4.16)
$$\begin{aligned}\dot{V}_{aug_{ad}} = {}& -c_1(z_2 - z_2^{ref})^2 + e_k(\Delta_1 - (\tfrac{2m(b+z_1)^2}{a})(z_3^2 - \tilde{u})(\tfrac{x_0 - z_1}{m})(\tfrac{\hat{\eta}}{m} + c_1)) + e_\eta(\Delta_2 - (\tfrac{2m(b+z_1)^2}{a})(z_3^2 - \tilde{u})(\tfrac{z_2}{m})(\tfrac{\hat{\eta}}{m} + c_1)) \\ & + e_{f_d}(\Delta_3 - (\tfrac{2m(b+z_1)^2}{a})(z_3^2 - \tilde{u})(\tfrac{1}{m})(\tfrac{\hat{\eta}}{m} + c_1)) + (z_3^2 - \tilde{u})(T),\end{aligned}$$

with Δ_i, $i = 1, 2, 3$ defined as given in (4.13) and

(4.17)
$$\begin{aligned}T = {}& -\tfrac{a}{2m(b+z_1)^2}(z_2 - z_2^{ref}) + 2z_3(-\tfrac{R(b+z1)}{a} z_3 + \tfrac{z_2 z_3}{(b+z_1)} + \tfrac{b+z_1}{a} u) + (\tfrac{2m(b+z_1)^2}{a})(\tfrac{\hat{k}}{m} z_2) - (\tfrac{4m(b+z_1)z_2}{a})(\tfrac{\hat{k}}{m}(x_0 - z_1) + \tfrac{\hat{\eta}}{m} z_2 + \tfrac{\hat{f}_d}{m} - \dot{z}_2^{ref} + \\ & c_1(z_2 - z_2^{ref}) + c_3(z_1 - z_1^{ref})) - (\tfrac{2m(b+z_1)^2}{a})(\tfrac{\dot{\hat{k}}}{m}(x_0 - z_1) + \tfrac{\dot{\hat{\eta}}}{m} z_2 + \tfrac{\dot{\hat{f}}_d}{m} + \tfrac{\hat{\eta}}{m}(\tfrac{\hat{k}}{m}(x_0 - z_1) + \tfrac{\hat{\eta}}{m} z_2 - \tfrac{a}{2m(b+z_1)^2} z_3^2 + \tfrac{\hat{f}_d}{m})) - (\tfrac{2m(b+z_1)^2}{a}) \\ & (c_1(\tfrac{\hat{k}}{m}(x_0 - z_1) + \tfrac{\hat{\eta}}{m} z_2 - \tfrac{a}{2m(b+z_1)^2} z_3^2 + \tfrac{\hat{f}_d}{m} - \dot{z}_2^{ref}) - \ddot{z}_2^{ref} + c_3(z_2 - z_2^{ref})).\end{aligned}$$

In order to render $\dot{V}_{aug_{ad}}$ equal to $-c_1(z_2 - z_2^{ref})^2 - c_2(z_3^2 - \tilde{u})^2$, we have to eliminate the terms multiplying the estimation errors e_k, e_η and e_{f_d} and set T equal to $-c_2(z_3^2 - \tilde{u})$. We achieve this by using the parameter dynamics (4.11) and the control input (4.10). Based on the negativeness of $\dot{V}_{aug_{ad}}$ and the definitions of $V_{aug_{ad}}$, $V_{sub_{ad}}$, we conclude about the global boundedness of $z_1, z_2, e, \hat{k}, \hat{\eta}$ and $\hat{f}_d$. Finally, LaSalle-Yoshizawa Theorem implies the regulation of z_2 to z_2^{ref} and z_3^2 to $\tilde{u}$.

Remark 1. *Notice that $\hat{k}, \hat{\eta}$ and $\hat{f}_d$ do not necessarily converge to k, η and f_d when the control scheme discussed in Lemma 4.1 is utilized. This prevents us from proving convergence of z_1 to z_1^{ref} by analyzing the zero dynamics of the mechanical subsystem. Hence, by analyzing the zero dynamics of (4.8), (4.9), (4.10) and (4.11) with the output $(z_2 - z_2^{ref}, z_3^2 - \tilde{u})$, we can only conclude about the boundedness of $\|z_1 - z_1^{ref}\|$.*

4.2 ISS Adaptive Backstepping Controller We now address the control problem of the adaptive trajectory tracking with asymptotic convergence of the estimation errors e_k, e_η and e_{f_d}. First, the backstepping controller is modified as follows:

(4.18)
$$\begin{aligned}u = {}& \tfrac{a}{b+z_1}(\tfrac{R(b+z_1)}{a} z_3 - \tfrac{z_2 z_3}{(b+z_1)} + \tfrac{1}{2z_3}(\tfrac{a}{2m(b+z_1)^2}(z_2 - z_2^{ref}) \\ & - c_2(z_3^2 - \tilde{u}))) \tfrac{2mz_2}{z_3}(\tfrac{\hat{k}}{m}(x_0 - z_1) + \tfrac{\hat{\eta}}{m} z_2 + \tfrac{\hat{f}_d}{m} + c_3(z_1 - z_1^{ref}) \\ & + c_1(z_2 - z_2^{ref}) + \kappa_1(z_2 - z_2^{ref})\|\psi\|_2^2 - \dot{z}_2^{ref}) \\ & + \tfrac{m(b+z_1)}{z_3}((\tfrac{\hat{k}}{m}(x_0 - z_1) + \tfrac{\hat{\eta}}{m} z_2 + \tfrac{\hat{f}_d}{m} - \tfrac{a}{2m(b+z_1)^2} z_3^2 - \dot{z}_2^{ref}) \\ & (c_1 + \kappa_1\|\psi\|_2^2 + \tfrac{\hat{\eta}}{m}) + \tfrac{\hat{\eta}}{m}\dot{z}_2^{ref}) \\ & + \tfrac{m(b+z_1)}{z_3}(2\kappa_1(z_2 - z_2^{ref})(\tfrac{(x_0 - z_1)(-z_2)}{m^2} + \\ & \tfrac{z_2(\tfrac{\hat{k}}{m}(x_0 - z_1) + \tfrac{\hat{\eta}}{m} z_2 + \tfrac{\hat{f}_d}{m} - \tfrac{a z_3^2}{2m(b+z_1)^2})}{m^2})) \\ & - \kappa_2(z_3^2 - \tilde{u})\left|\tfrac{m(b+z_1)}{z_3}\right|^2[\left|c_1 + \kappa_1\|\psi\|_2^2 + \tfrac{\hat{\eta}}{m}\right|^2 \\ & + \left|2\kappa_1(z_2 - z_2^{ref})\right|^2\left|\tfrac{z_2}{m^2}\right|^2]\|\psi\|_2^2 - \kappa_3(z_3^2 - \tilde{u})\left|\tfrac{m(b+z_1)}{z_3}\right|^2\|\psi\|_2^2 \\ & + \tfrac{m(b+z_1)}{z_3}(-\tfrac{\hat{k}}{m} z_2 - \ddot{z}_2^{ref} + c_3(z_2 - z_2^{ref})),\end{aligned}$$

$$\begin{aligned}\tilde{u} = {}& \tfrac{2m(b+z_1)^2}{a}(\tfrac{\hat{k}}{m}(x_0 - z_1) + \tfrac{\hat{\eta}}{m} z_2 + \tfrac{\hat{f}_d}{m} + c_3(z_1 - z_1^{ref}) \\ & + c_1(z_2 - z_2^{ref}) - \dot{z}_2^{ref}) + \tfrac{2m(b+z_1)^2}{a}(\kappa_1(z_2 - z_2^{ref})\|\psi\|_2^2),\end{aligned} \tag{4.19}$$

where the uncertain parameters $k,\ \eta,\ f_d$ have been replaced by their estimated parameters $\hat{k},\ \hat{\eta},\ \hat{f}_d$, with $\psi \triangleq \begin{bmatrix} \frac{x_0-z_1}{m} & \frac{z_2}{m} & \frac{1}{m} \end{bmatrix}^T$. We can now state the following lemma.

LEMMA 4.2. *Consider the closed-loop dynamics given by (4.8), (4.18) and (4.19), with constant but unknown parameters k, η, f_d and the parameter error vector $\Delta \triangleq \begin{bmatrix} k-\hat{k} & \eta-\hat{\eta} & f_d-\hat{f}_d \end{bmatrix}^T$. Then, there exist positive gains $c_1,\ c_2,\ c_3,\ \kappa_1,\ \kappa_2$ and κ_3 such that $(z_1(t), z_2(t))$ are uniformly bounded and the system (4.8) is locally integral input-to state stable (LiISS) with respect to $(\Delta, \dot{\Delta})$.*

Proof. Consider the full mechanical subsystem that consists of only the first two equations with the virtual control input $\tilde{u} := z_3^2$:

$$
\begin{aligned}
\dot{z}_1 &= z_2 \\
\dot{z}_2 &= \frac{k}{m}(x_0-z_1) + \frac{\eta}{m}z_2 + \frac{f_d}{m} - \frac{a}{2m(b+z_1)^2}\tilde{u}.
\end{aligned} \tag{4.20}
$$

Defining the Lyapunov function $V_{sub} = \frac{c_3}{2}(z_1 - z_1^{ref})^2 + \frac{1}{2}(z_2 - z_2^{ref})^2$, with $c_3 > 0$, we would like to design $\tilde{u}$ so that $\dot{V}_{sub} = -c_1(z_2 - z_2^{ref})^2$ along the trajectories of (4.20), but since the system parameters k, η and f_d are unknown, we design the virtual input to be $\tilde{u}$ given by (4.19). Inserting $\tilde{u}$ from (4.19) into $\dot{V}_{sub}$, we have the following derivation:

$$
\begin{aligned}
&\dot{V}_{sub} = c_3(z_1-z_1^{ref})(\dot{z}_1-\dot{z}_1^{ref}) + (z_2-z_2^{ref})(\dot{z}_2-\dot{z}_2^{ref}) \\
&= (z_2-z_2^{ref})(c_3(z_1-z_1^{ref}) + \tfrac{k}{m}(x_0-z_1) + \tfrac{\eta}{m}z_2 \\
&-\dot{z}_2^{ref} - \tfrac{a}{2m(b+z_1)^2}\tilde{u}) \\
&= -c_1(z_2-z_2^{ref})^2 + (z_2-z_2^{ref})(\tfrac{(k-\hat{k})(x_0-z_1)}{m} + \tfrac{(\eta-\hat{\eta})z_2}{m} \\
&+ \frac{f_d-\hat{f}_d}{m}) - \kappa_1(z_2 - z_2^{ref})^2\|\psi\|_2^2.
\end{aligned} \tag{4.21}
$$

Using the definitions of the vectors ψ and Δ, we have

$$
\begin{aligned}
&\dot{V}_{sub} \le c_1(z_2-z_2^{ref})^2 + |z_2-z_2^{ref}|\|\psi^T\|_2\|\Delta\|_2 \\
&-\kappa_1(z_2 - z_2^{ref})^2\|\psi\|_2^2 \\
&\le -c_1(z_2-z_2^{ref})^2 - \kappa_1\left[|z_2-z_2^{ref}|\|\psi\|_2 - \tfrac{\|\Delta\|_2}{2\kappa_1}\right]^2 + \tfrac{\|\Delta\|_2^2}{4\kappa_1} \\
&\le -c_1(z_2-z_2^{ref})^2 + \tfrac{\|\Delta\|_2^2}{4\kappa_1},
\end{aligned} \tag{4.22}
$$

where $\Delta = \begin{bmatrix} k-\hat{k} & \eta-\hat{\eta} & f_d-\hat{f}_d \end{bmatrix}^T$ is the vector holding the discrepancy between actual system parameters and estimated parameters. Note that we have made use of the nonlinear damping term $-\kappa_1(z_2 - z_2^{ref})^2\|\psi\|_2^2$ to attain a negative quadratic term of ψ and Δ $\left(\text{i.e.,} -\kappa_1\left[|z_2 - z_2^{ref}|\|\psi\|_2 - \frac{\|\Delta\|_2}{2\kappa_1}\right]^2\right)$ and a positive term that is a function of Δ only $\left(\frac{\|\Delta\|_2^2}{4\kappa_1}\right)$, hence rendering V_{sub} an iISS-Lyapunov function for the mechanical subsystem. Next, we define the Lyapunov function for the full system:$V_{aug} = V_{sub} + \frac{(z_3^2-\tilde{u})^2}{2}$. Taking the derivative of V_{aug} along the trajectories of the full system, leads to the following inequality:

$$
\begin{aligned}
&\dot{V}_{aug} \le -c_1(z_2-z_2^{ref})^2 + \tfrac{\|\Delta\|_2^2}{4\kappa_1} + (z_3^2-\tilde{u}) \\
&\quad -\tfrac{a(z_2-z_2^{ref})}{2m(b+z_1)^2} - \dot{\tilde{u}} \quad + (z_3^2-\tilde{u})(2z_3(-\tfrac{R(b+z_1)}{a}z_3 \\
&+ \frac{z_2z_3}{(b+z_1)} + \frac{b+z_1}{a}u)),
\end{aligned} \tag{4.23}
$$

where $\dot{\tilde{u}}$ writes as

$$
\begin{aligned}
\dot{\tilde{u}} &= \tfrac{4m(b+z_1)z_2}{a}(\tfrac{\hat{k}}{m}(x_0-z_1) + \tfrac{\hat{\eta}}{m}z_2 + \tfrac{\hat{f}_d}{m} + c_3(z_1-z_1^{ref}) \\
&+ c_1(z_2-z_2^{ref})) + \tfrac{4m(b+z_1)z_2}{a}(\kappa_1(z_2-z_2^{ref})\|\psi\|_2^2 - \dot{z}_2^{ref}) \\
&+ \tfrac{2m(b+z_1)^2}{a}(\tfrac{\dot{\hat{k}}}{m}(x_0-z_1) + \tfrac{\dot{\hat{\eta}}}{m}z_2 + \tfrac{\dot{\hat{f}}_d}{m}) + \tfrac{2m(b+z_1)^2}{a}((\tfrac{k}{m}(x_0-z_1) \\
&+ \tfrac{\eta}{m}z_2 + \tfrac{f_d}{m} - \tfrac{a}{2m(b+z_1)^2}z_3^2 - \dot{z}_2^{ref})(c_1 + \kappa_1\|\psi\|_2^2 + \tfrac{\hat{\eta}}{m}) + \tfrac{\hat{\eta}}{m}\dot{z}_2^{ref}) \\
&+ \tfrac{2m(b+z_1)^2}{a}(2\kappa_1(z_2-z_2^{ref})(\tfrac{(x_0-z_1)-z_2}{m^2} \\
&+ \frac{z_2(\frac{k}{m}(x_0-z_1) + \frac{\eta}{m}z_2 + \frac{f_d}{m} - \frac{az_3^2}{2m(b+z_1)^2})}{m^2})) \\
&+ \tfrac{2m(b+z_1)^2}{a}(-\tfrac{\hat{k}}{m}z_2 - \ddot{z}_2^{ref} + c_3(z_2-z_2^{ref})).
\end{aligned} \tag{4.24}
$$

By substituting the control input given in (4.18) into (4.23), we attain the following inequality:

$$
\begin{aligned}
&\dot{V}_{aug} \le -c_1(z_2-z_2^{ref})^2 + \tfrac{\|\Delta\|_2^2}{4\kappa_1} - c_2(z_3^2-\tilde{u})^2 \\
&-(z_3^2-\tilde{u})(\tfrac{m(b+z_1)}{z_3}(\tfrac{(k-\hat{k})(x_0-z_1)}{m} + \tfrac{(\eta-\hat{\eta})z_2}{m} + \tfrac{f_d-\hat{f}_d}{m}) \\
&(c_1 + \kappa_1\|\psi\|_2^2 + \tfrac{\hat{\eta}}{m})) - (z_3^2-\tilde{u})(2\kappa_1(z_2-z_2^{ref})(\tfrac{z_2(b+z_1)}{mz_3}) \\
&(\tfrac{(k-\hat{k})(x_0-z_1)}{m} + \tfrac{(\eta-\hat{\eta})z_2}{m} + \tfrac{f_d-\hat{f}_d}{m})) \\
&-(z_3^2-\tilde{u})(\tfrac{m(b+z_1)}{z_3})(\tfrac{\dot{\hat{k}}}{m}(x_0-z_1) + \tfrac{\dot{\hat{\eta}}}{m}z_2 + \tfrac{\dot{\hat{f}}_d}{m}) \\
&-\kappa_3(z_3^2-\tilde{u})^2\left|\tfrac{m(b+z_1)}{z_3}\right|^2\|\psi\|_2^2 - \kappa_2(z_3^2-\tilde{u})^2 \\
&[\left|\tfrac{m(b+z_1)}{z_3}\right|^2\Big|c_1 \\
&+\kappa_1\|\psi\|_2^2 + \tfrac{\hat{\eta}}{m}\Big|^2 + \left|2\kappa_1(z_2-z_2^{ref})\right|^2\left|\tfrac{z_2(b+z_1)}{mz_3}\right|^2]\|\psi\|_2^2.
\end{aligned} \tag{4.25}
$$

Using the aforementioned definitions of the vectors ψ and Δ, and noting that $\dot{\Delta} = \begin{bmatrix} -\dot{\hat{k}} & -\dot{\hat{\eta}} & -\dot{\hat{f}}_d \end{bmatrix}^T$, we can further bound $\dot{V}_{aug}$ in the following way:

$$
\begin{aligned}
&\dot{V}_{aug} \le -c_1(z_2-z_2^{ref})^2 + \tfrac{\|\Delta\|_2^2}{4\kappa_1} - c_2(z_3^2-\tilde{u})^2 \\
&+\left|z_3^2-\tilde{u}\right|\left|\tfrac{m(b+z_1)}{z_3}\right|\left|c_1 + \kappa_1\|\psi\|_2^2 + \tfrac{\hat{\eta}}{m}\right|\|\psi^T\|_2\|\Delta\|_2 \\
&+\left|z_3^2-\tilde{u}\right|\left|2\kappa_1(z_2-z_2^{ref})\right|\left|\tfrac{z_2(b+z_1)}{mz_3}\right|\|\psi^T\|_2\|\Delta\|_2 \\
&+\left|z_3^2-\tilde{u}\right|\left|\tfrac{m(b+z_1)}{z_3}\right|\|\psi^T\|_2\|\dot{\Delta}\|_2 - \kappa_3(z_3^2-\tilde{u})^2\left|\tfrac{m(b+z_1)}{z_3}\right|^2\|\psi\|_2^2 \\
&-\kappa_2(z_3^2-\tilde{u})^2\left[\left|\tfrac{m(b+z_1)}{z_3}\right|^2\left|c_1 + \kappa_1\|\psi\|_2^2 + \tfrac{\hat{\eta}}{m}\right|^2\right. \\
&\left.+\left|2\kappa_1(z_2-z_2^{ref})\right|^2\left|\tfrac{z_2(b+z_1)}{mz_3}\right|^2\right]\|\psi\|_2^2.
\end{aligned} \tag{4.26}
$$

By making use of the nonlinear damping terms the same way as they have been utilized in deriving (4.22), we get

$$
\begin{aligned}
&\dot{V}_{aug} \le -c_1(z_2-z_2^{ref})^2 + \tfrac{\|\Delta\|_2^2}{4\kappa_1} - c_2(z_3^2-\tilde{u})^2 \\
&-\kappa_2\left[\left|z_3^2-\tilde{u}\right|\left|\tfrac{m(b+z_1)}{z_3}\right|\left|c_1 + \kappa_1\|\psi\|_2^2 + \tfrac{\hat{\eta}}{m}\right|\|\psi\|_2 - \tfrac{\|\Delta\|_2}{2\kappa_2}\right]^2 \\
&+\tfrac{\|\Delta\|_2^2}{4\kappa_2} - \kappa_2\left[\left|z_3^2-\tilde{u}\right|\left|2\kappa_1(z_2-z_2^{ref})\right|\left|\tfrac{z_2(b+z_1)}{mz_3}\right|\|\psi\|_2\right. \\
&\left.-\tfrac{\|\Delta\|_2}{2\kappa_2}\right]^2 + \tfrac{\|\Delta\|_2^2}{4\kappa_2} - \kappa_3\left[\left|z_3^2-\tilde{u}\right|\left|\tfrac{m(b+z_1)}{z_3}\right|\|\psi\|_2 - \tfrac{\|\dot{\Delta}\|_2}{2\kappa_3}\right] \\
&+\frac{\|\dot{\Delta}\|_2^2}{4\kappa_3}.
\end{aligned} \tag{4.27}
$$

Finally, using the inequality (4.27), we have

$$\dot{V}_{aug} \le -c_1(z_2-z_2^{ref})^2 - c_2(z_3^2-\tilde{u})^2 + \frac{1}{4\kappa_1} + \frac{1}{2\kappa_2} \quad \|\Delta\|_2^2 + \frac{\|\dot{\Delta}\|_2^2}{2\kappa_3}. \tag{4.28}$$

It is easy to see that the uncertain system can be expressed in the following nonlinear time-varying form:

$$\dot{e} = f(t, e, \tilde{\Delta}), \tag{4.29}$$

with $e \in \mathcal{D}_e$, $\tilde{\Delta} \in \mathcal{D}_{\tilde{\Delta}}$, where $e := [z_1 - z_1^{ref} \;\; z_2 - z_2^{ref} \;\; z_3^2 - \tilde{u}]^T$ and $\tilde{\Delta} = [\Delta \; \dot{\Delta}]^T$. Then, by considering the output map defined by $h = [z_2 - z_2^{ref} \;\; z_3^2 - \tilde{u}]^T$, we can show that the system (4.29) with h is weakly zero-detectable (i.e. using an analysis of the zero-dynamics of (4.29) with $h \equiv \tilde{\Delta} \equiv 0$). Next, using the weakly-zero-delectability property together with inequality (4.28), we can conclude (via some additional steps, which are not included here due to space limitations, but will be included in a journal version of this work) that system (4.29) is LiISS with respect to the input $\tilde{\Delta}$, implying that there exist functions $\alpha \in \mathcal{K}$, $\beta \in \mathcal{KL}$ and $\gamma \in \mathcal{K}$, such that, for all $e(0) \in \mathcal{D}_e$ and $\tilde{\Delta} \in \mathcal{D}_{\tilde{\Delta}}$, e is defined and

$$\|e(t)\| \le \beta(\|e(0)\|, t) + \alpha\left(\int_0^t \gamma(\|\tilde{\Delta}\|)\right)ds \tag{4.30}$$

for all $t \ge 0$.

4.2.1 Estimation Module The motivation behind proving that the system is LISS with respect to $(\Delta, \dot{\Delta})$ is, if by an estimation method, the vectors $\|\Delta\|_2$ and $\|\dot{\Delta}\|_2$ can be taken to 0, then we can claim via (4.30) that the system becomes stable. The advantage of using this method is that it provides modularity in the sense that the control law can be designed independently from the estimation law. Thus, it would be sufficient to design an estimation law that will take $\|\Delta\|_2$ and $\|\dot{\Delta}\|_2$ to 0 sufficiently fast. To this purpose, we use a gradient descent-based filters [18]. We have three parameters that are varying over time k, η, f_d. These parameters enter the dynamics through the following equation:

$$\dot{z}_2 = f(z,u) + F(z,u)^T\theta := -\frac{pz_3^2}{mz_1^2} + \begin{bmatrix} \frac{x_0-z_1}{m} & \frac{z_2}{m} & \frac{1}{m} \end{bmatrix}^T \begin{bmatrix} k \\ \eta \\ f_d \end{bmatrix}. \tag{4.31}$$

The main problem with estimation for the system at hand is there is only a single equation through which the uncertain parameters enter the dynamics (4.31). Hence, using the *x-Swapping Scheme* given in [18], we can only estimate one parameter at a time. To this purpose, we state the following assumption:

Assumption 1. *The uncertain parameters k, η and f_d vary slowly, and over a given period of time, only a single parameter can change while the others stay constant.*

We use the following equations for the estimation filters [18]:

$$\begin{aligned} \dot{\Omega}^T &= (A_0 - \lambda F(z,u)^T F(z,u) P)\Omega^T + F(z,u)^T \\ \dot{\Omega}_0 &= (A_0 - \lambda F(z,u)^T F(z,u) P)(\Omega_0 - z) + f(z,u) \\ \epsilon &= z - \Omega_0 - \Omega^T \hat{\theta}, \end{aligned} \tag{4.32}$$

with the gradient law for updating estimated parameters:

$$\dot{\hat{\theta}} = \Gamma \frac{\Omega\epsilon}{1+\nu\|\Omega\|_F^2}, \qquad \Gamma = \Gamma^T > 0, \quad \nu \ge 0. \tag{4.33}$$

In the filter equations given in (4.32), $A_0, P = P^T > 0$ are constant,design matrices that satisfy the Lyapunov equation, $PA_0 + A_0P = -I$, and λ is a design variable. Since we estimate only one parameter at a time, the equations become scalar for each parameter. The following equations are used for estimating k, η and f_d separately:

- For the parameter $\hat{\mathbf{k}}$:

$$\begin{aligned} \dot{\Omega} &= (A_0 - \lambda \tfrac{(x_0-z_1)^2}{m^2} P)\Omega + \tfrac{(x_0-z_1)}{m} \\ \dot{\Omega}_0 &= (A_0 - \lambda \tfrac{(x_0-z_1)^2}{m^2} P)(\Omega_0 - z_2) + \tfrac{\eta z_2}{m} + \tfrac{f_d}{m} - \tfrac{pz_3^2}{mz_1^2} \\ \epsilon &= z_2 - \Omega_0 - \Omega\hat{k} \\ \dot{\hat{k}} &= \Gamma \tfrac{\Omega\epsilon}{1+\nu\|\Omega\|_F^2}. \end{aligned} \tag{4.34}$$

- For the parameter $\hat{\eta}$:

$$\begin{aligned} \dot{\Omega} &= (A_0 - \lambda \tfrac{z_2^2}{m^2} P)\Omega + \tfrac{z_2}{m} \\ \dot{\Omega}_0 &= (A_0 - \lambda \tfrac{z_2^2}{m^2} P)(\Omega_0 - z_2) + \tfrac{(x_0-z_1)}{m} + \tfrac{f_d}{m} - \tfrac{pz_3^2}{mz_1^2} \\ \epsilon &= z_2 - \Omega_0 - \Omega\hat{\eta} \\ \dot{\hat{\eta}} &= \Gamma \tfrac{\Omega\epsilon}{1+\nu\|\Omega\|_F^2}. \end{aligned} \tag{4.35}$$

- For the parameter $\hat{\mathbf{f}}_{\mathbf{d}}$:

$$\begin{aligned} \dot{\Omega} &= (A_0 - \lambda \tfrac{1}{m^2} P)\Omega + \tfrac{1}{m} \\ \dot{\Omega}_0 &= (A_0 - \lambda \tfrac{1}{m^2} P)(\Omega_0 - z_2) + \tfrac{\eta z_2}{m} + \tfrac{(x_0-z_1)}{m} - \tfrac{pz_3^2}{mz_1^2} \\ \epsilon &= z_2 - \Omega_0 - \Omega\hat{f}_d \\ \dot{\hat{f}}_d &= \Gamma \tfrac{\Omega\epsilon}{1+\nu\|\Omega\|_F^2}. \end{aligned} \tag{4.36}$$

Lemma 4.3. *Consider the closed-loop dynamics given by (4.8), (4.18) and (4.19), with an unknown parameter k, η, or f_d. Then, under Assumption 1, there exist positive gains c_1, c_2, c_3, κ_1, κ_2 and κ_3 such that the closed-loop dynamics given by (4.8), (4.18), (4.19) and the filters (4.34), (4.35), (4.36) are stable, and that the unknown parameter is asymptotically estimated.*

Proof. The proof is straightforward from the result of Lemma 4.2, and the known convergence properties of the gradient descent-based filters [18].

Parameter	Value
m	$0.27\ [kg]$
R	$6\ [\Omega]$
η	$-0.25\ [kg/sec]$
x_0	$8\ [mm]$
k	$75\ [N/mm]$
a	$14.96 \times 10^{-6}\ [Nm^2/A^2]$
b	$4 \times 10^{-5}\ [m]$

Table 1: Numerical values of the mechanical parameters

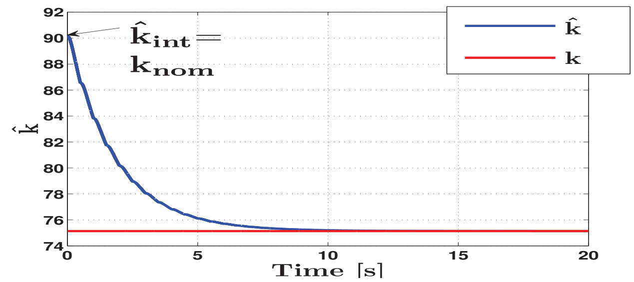

Figure 1: Estimation of k over time

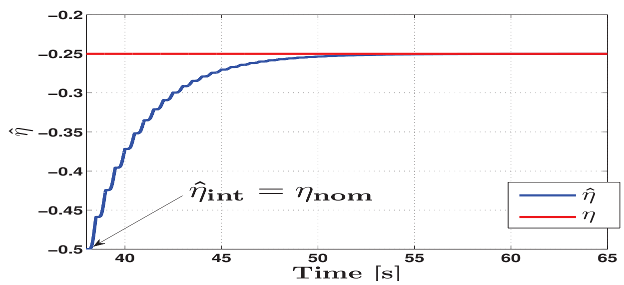

Figure 2: Estimation of η over time

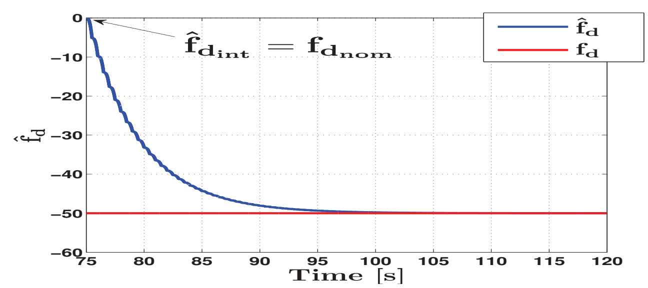

Figure 3: Estimation of f_d over time

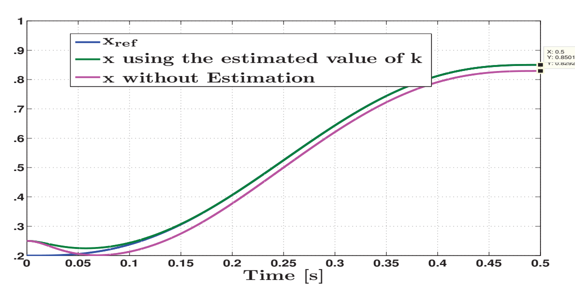

Figure 4: Moving element actual position vs. desired position

5 Simulations

We show here the behavior of the proposed approach on the example of electromagnetic actuator presented in [11], where the model (3.6) is used with the numerical values of Table 1. The desired trajectory has been selected as the $5th$ order polynomial $x^{ref}(t) = \sum_{i=0}^{5} a_i (t/t_f)^i$, where the a_is have been computed to satisfy the boundary constraints $x^{ref}(0) = 0.2, x^{ref}(t_f) = x_f, \dot{x}^{ref}(0) = \dot{x}^{ref}(t_f) = 0, \ddot{x}^{ref}(0) = \ddot{x}^{ref}(t_f) = 0$, with $t_f = 0.5\ sec$, $x_f = 0.85\ mm$. Due to space limitations, we only report hereafter the results of the ISS adaptive backstepping controller. However, we can underline here that the first adaptive controller leads to numerical results in concordance with the theoretical analysis, i.e. convergence of the armature velocity to the desired velocity, with bounded position tracking error and bounded uncertain parameters estimation errors. To test the ISS adaptive controller, we considered the following scenario: We considered uncertainties in the model appearing sequentially over time. First, at $t = 0\ sec$, we considered that the parameter k has an error of 16%. Next, we consider that at $t = 38\ sec$, the parameter η sustains an error of 50%, finally at $t = 75\ sec$, we assume a disturbance force f_d of $-50N$ (static friction force). We simulated the controller (4.18) and (4.19) with the gains $c_1 = 100$, $c_2 = 100$, $c_3 = 50$, $\kappa_1 = \kappa_2 = \kappa_3 = 1$. For the filters (4.34), (4.35), (4.36), we used the gains $A_0 = -0.5,\ P = 1,\ \lambda = 1,\ \Gamma = 100$. We underline here that, due to the structure of the model, we could estimate only one parameter at the time (see Section 4.2.1). We see clearly on Figures 1, 2, 3 that the numerical results are concordant with the theoretical analysis, since the estimated parameters converge all to their actual value. Furthermore, we see on Figures 4, 5 that we achieve very good tracking of both the desired position and the desired velocity trajectories.

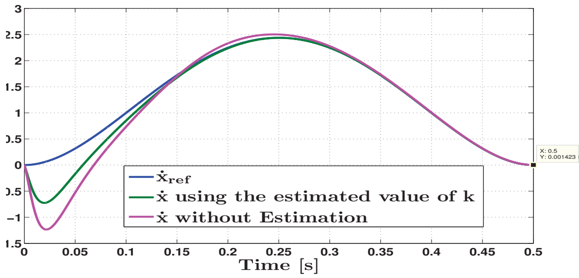

Figure 5: Moving element actual velocity vs. desired velocity

6 Conclusion

We have studied in this paper the problem of adaptive control for electromagnetic actuators. We have developed two trajectory tracking controller based on adaptive backstepping approaches. We have studied the stability properties of the proposed controller and shown the performance on a numerical example.

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