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A Block Diagonal Jacket Matrices for MIMO Broadcast Channels

Md. Hashem Ali Khan, Jun Li, Moon Ho Lee, Kyeong Jin Kim

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Index Terms— **MIMO systems, Degrees of freedom, Interference Alignment, Interfering broadcast channel (IFBC), Diagonal Jacket matrix.**

I. Introduction

HADAMARD matrices and Hadamard transforms have been of a great deal of interest and have been applied to communications signaling, image processing, signal representation and error correction coding theory. It is well known that any algorithm requiring eigenvalue decomposition (EVD) suffers from high computational cost. In mobile wireless communication systems, in which MIMO technique is utilized, the channel characteristics may vary faster than the computation process of the precoding/decoding algorithm that is based on EVD of the channel matrix that is changing instantaneously. In paper [1], the authors proposed MIMO channel precoding/decoding based on Jacket matrix decomposition where we believe that the required computation complexity in obtaining diagonal-similar matrices is smaller than that required in the conventional EVD.

Definition 1: Let $J_N \triangleq \{a_{i,j}\}$ be a $N \times N$ matrix, then it is called a Jacket matrix when $J_N^{-1} = \frac{1}{N}\left\{\left(a_{i,j}\right)^{-1}\right\}^T$. That is, the inverse of the Jacket matrix can be determined by its element-wise inverse [2]-[3].

Definition 2: Let A be an $n \times n$ matrix. If there exists a Jacket matrix J such that $A = J\Sigma J^{-1}$, where Σ is a diagonal matrix, then we say that A is Jacket similar to the diagonal matrix Σ. Moreover, we say that A is Jacket diagonalizable [4].

Theorem 1: A 4×4 matrix $\mathbb{J}$ is Jacket similar to the diagonal matrix if and only if $\mathbb{J}$ has the following form

$$\mathbb{J}_4 = \begin{pmatrix} [\mathbf{A}]_2 & [\mathbf{B}]_2 \\ [\mathbf{C}]_2 & [\mathbf{A}]_2 \end{pmatrix}, \quad (1)$$

i.e., the entries of the main diagonal of a matrix are equal.

Proof: Refer to [4] for the proof.

In [5]-[6], it was shown that the maximum sum rate in the multiuser MIMO broadcast channels (BC) can be achieved by dirty paper coding (DPC). However, the DPC is difficult to implement in practical systems due to its high computational complexity. A suboptimal strategy of the DPC [7], the Tomlinson-Harashima precoding is still impractical due to its complexity, since this algorithm is based on nonlinear modulo operations. In linear processing systems, several practical precoding techniques have been proposed, typically as the channel inversion method [8], and the BD method [9].

Interference mitigation techniques have become an important part of wireless network design. An interference alignment (IA) technique has been proposed recently in [12] and [16] as an efficient capacity achieving scheme at high signal-to-noise ratio (SNR) regime. The fundamental concept of IA is to align the interference signals in a particular subspace at each receiver so that an interference free orthogonal subspace can be solely allocated for data transmission [13]-[15]. Multi-cell and multi-user downlink transmission schemes have been actively discussed for future generation cellular networks in [17].

In this paper, we focus on the performance in the high SNR region and investigate the case where the noise also clearly affects the transmission. This topic is important because

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coordinated transmission is typically applied to the cell edge users who receive not only strong interference from the possible cooperating cells but also a considerable amount of interference from cells that are impossible to cooperate with. The proposed method combines IA with an advanced multi-user MIMO (MU-MIMO) beamforming method. The IA method eliminates the inter-cell interference and the MU-MIMO technology not only manages the intra-cell inter-user interference but also preserves the strength of the desired signal. The performance gain over the existing methods is shown based on numerical simulation results. The main contributions of this paper are summarized as follows:

- We propose a new IA algorithm in the multi-cell two-user MIMO-IFBC consisting of two parts: the receive beamforming vector design for effective inter-cell-interference (ICI) channel alignment, and transmit beamforming vector design for removing ICI and inter-user-interference (IUI).
- The MIMO precoding/decoding is divided into two steps. The first step utilizes the IA principle to eliminate ICI while expending the least amount of resources. The second step utilizes the existing powerful MU-MIMO technologies to implement efficient co-channel multiuser transmission. The IA based methods orthogonalize all sources of interference, i.e., ICI and IUI to the desired signal and use the block diagonalization (BD) to eliminate IUI.

The rest of this paper is organized as follows. In Section II, we describe system model. Section III, we analyze block diagonal Jacket decomposition of an equivalent channel matrix. In Section IV, we analyze IA and MU-MIMO based three-cell coordinated beamforming scheme. In Section V, we discus IA scheme and simulation results in Section VI. Finally, we conclude the paper in Section VII.

II. System Model

In this section, we describe a system model for the multi-cell MIMO-IFBC as shown in Figure 1. The system of the L-cell K-user MIMO-IFBC consists of L base stations (BSs) with M antennas per BS and K users with N receive antennas per user in each cell.

We assume each BS tries to convey one data stream per user to its corresponding users, i.e., $S \le \min(M,N) = N$. The transmitted symbol vector $\mathrm{x}_i \in \mathbb{C}^M$ from BS i is generated as

$$\mathrm{x}_i = \mathbf{W}_i \mathbf{s}_i \quad (1)$$

where $\mathbf{W}_i \in \mathbb{C}^{M\times K}$ is the transmit precoding matrix of the i-th BS and $\mathbf{s}_i \in \mathbb{C}^K$ represents the data symbols for user k in cell i. The transmit beamforming matrix can be written as $\mathbf{W}_i = \left[\mathbf{w}_{i,1},\cdots,\mathbf{w}_{i,K}\right]$ and the data symbol vector can be written as $\mathbf{s}_i = \left[\mathbf{s}_{i,1}^T,\cdots,\mathbf{s}_{i,K}^T\right]^T$, where $\mathbf{V}_{i,k}$ and $\mathbf{s}_{i,k}$ represent the transmit beamforming vector and data symbol corresponding to the k-th user in cell i, respectively.

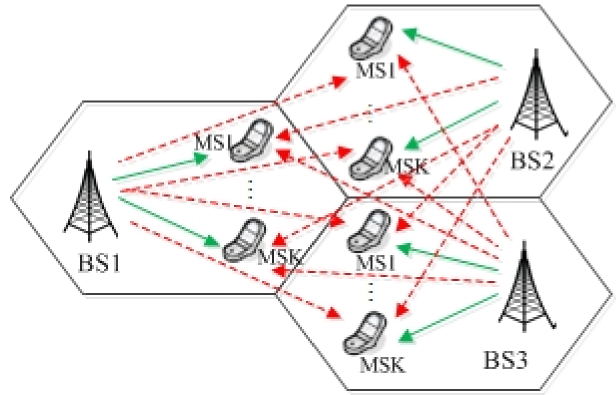

Figure 1. Multi-cell MIMO interfering broadcast channel with three BSs.

The total transmitted power at BS i is given by

$$P_i = tr\left(E\left[\mathrm{x}_i \mathrm{x}_i^H\right]\right). \quad (2)$$

The received signal at the k-th user is represented as

$$\mathbf{y}_{i,k} = \mathbf{H}_{i,k}\mathbf{W}_{i,k}\mathbf{s}_{i,k} + \sum^{3}\sum_{l=1,l\neq k}^{K} \mathbf{H}_{i,k}\mathbf{W}_{i,l}\mathbf{s}_{i,l} + n_{i,k} \quad (3)$$

where $\mathbf{H}_{i,k} \in \mathbb{C}^{N\times M}$ represents the MIMO channel between the BS and user k in cell i; $\mathbf{n}_{i,k} \in \mathbb{C}^{N\times 1}$ represents the noise and residual interference from non-cooperating cells at the receiver of user k in cell i. In (3), the first term represents the desired signal, the second term represents IUI and the total ICI. By properly designing the transmitter and receiver beamforming matrices, interference is efficiently suppressed and the desired signals are well protected. As a result, the system can achieve a better sum rate performance. Linear receive beamforming is applied in the receiver side to reconstruct the desired signal as:

$$\begin{aligned}\hat{\mathbf{s}}_{i,k} &= \mathbf{U}_{i,k}^H \mathbf{y}_{i,k} \\ &= \mathbf{U}_{i,k}^H\left(\mathbf{H}_{i,k}\mathbf{W}_{i,k}\mathbf{s}_{i,k} + \sum^{3}\sum_{l=1,l\neq k}^{K}\mathbf{H}_{i,k}\mathbf{W}_{i,l}\mathbf{s}_{i,l} + n_{i,k}\right) \\ &= \mathbf{U}_{i,k}^H\mathbf{H}_{i,k}\mathbf{W}_{i,k}\mathbf{s}_{i,k} + \mathbf{U}_{i,k}^H\sum^{3}\sum_{l=1,l\neq k}^{K}\mathbf{H}_{i,k}\mathbf{W}_{i,l}\mathbf{s}_{i,l} + \mathbf{U}_{i,k}^H n_{i,k}\end{aligned} \quad (4)$$

where $\mathbf{U}_{i,k} \in \mathbb{C}^{S\times N}$ is the receiver beamforming matrix and $\mathbf{U}_{i,k}^H n_{i,k}$ is the effective noise vector. Also, $\mathbf{H}_{i,k}$ is a block diagonal matrix, given by

$$\mathbf{H}_{i,k} = \begin{bmatrix} [D_1\Sigma_1 D_1^{-1}] & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & [D_K\Sigma_K D_K^{-1}] \end{bmatrix}. \quad (5)$$

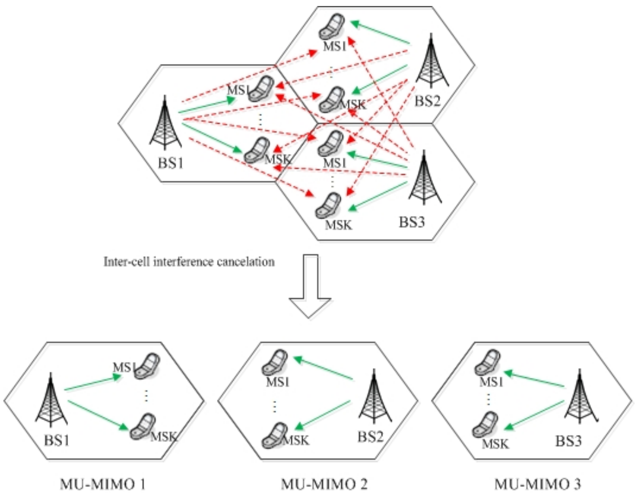

Figure 2. Two-step interference management.

Further, we define the degrees of freedom (DoF) which is the pre-log factor of the sum rate. This is one of the key metrics for assessing the performance of the system in the multiple antenna systems at the high SNR regime. DoF is defined as

$$d \triangleq \lim_{\rho\to\infty} \frac{R(\rho)}{\log(\rho)} \tag{6}$$

where $R(\rho)$ denotes the sum rate at the SNR, $\rho = P/\sigma^2$. The sum rate is given by

$$R(\rho) = \log_2\left(1 + \frac{\left|\mathbf{U}_{i,k}^H \mathbf{H}_{i,k} \mathbf{W}_{i,k}\right|^2}{\sigma^2 + \sum_{l=1,l\neq k}^{L}\sum^{K}\left|\mathbf{U}_{i,l}^H \mathbf{H}_{i,k} \mathbf{W}_{i,l}\right|^2}\right). \tag{7}$$

III. Block Diagonal Jacket Decomposition of an Equivalent Channel Matrix

From the channel matrix given in (5), we discuss block diagonal Jacket decomposition. A special $M\times M$ Jacket matrix named a diagonal Jacket matrix can be defined as follows.

$$[J]_M = \begin{bmatrix} J_{1,1} & & 0 \\ & J_{1,2} & \\ 0 & & J_{M,M} \end{bmatrix}, \text{ and} \tag{8}$$

its inverse matrix as

$$[J]_M^{-1} = \begin{bmatrix} 1/J_{1,1} & & 0 \\ & 1/J_{1,2} & \\ 0 & & 1/J_{M,M} \end{bmatrix}. \tag{9}$$

Obviously, the unitary matrices can be also considered as the diagonal Jacket matrices.

Let $\mathbf{B}_2$ denote the 2×2 block matrix in the main diagonal of $\mathbf{H}_{i,k}$ [19]. Then, Eq. (5) can be written as

$$\mathbf{H}_{i,k} = \mathbf{I}_M \otimes \mathbf{B}_2 \tag{10}$$

where

$$\mathbf{B}_2 = D\Sigma D^{-1}, \tag{11}$$

$\mathbf{I}_M$ is an $M\times M$ identity matrix, and $\otimes$ is the Kronecker product. It is worthwhile to note that each block in the diagonal of the matrix in Eq. (5) is a 2×2 matrix that satisfies the condition specified in *theorem 1*, and hence we say that $\mathbf{B}_2$ can be eigenvalue decomposed by using Jacket matrices. In other words, we can write $\mathbf{B}_2$ as

$$\mathbf{B}_2 = \mathbf{J}_2\mathbf{\Sigma}_2\mathbf{J}_2^{-1}. \tag{12}$$

We show that $\mathbf{H}_{i,k}$ can be diagonally decomposed as

$$\begin{aligned}\mathbf{H}_{i,k} &= \mathbf{I}_M \otimes \mathbf{B}_2 = \mathbf{I}_M \otimes \left(\mathbf{J}_2\mathbf{\Sigma}_2\mathbf{J}_2^{-1}\right) \\ &= \left(\mathbf{I}_M \otimes \mathbf{J}_2\right) diag\left(\lambda_1,\lambda_2\cdots\lambda_k\right)\left(\mathbf{I}_M \otimes \mathbf{J}_2^{-1}\right) \\ &= \mathbf{J\Sigma J}^{-1}.\end{aligned} \tag{13}$$

Thus, we can write

$$\mathbf{H}_{i,k} = \mathbf{J\Sigma J}^{-1} \tag{14}$$

where

$$\mathbf{J} = \mathbf{I}_M \otimes \mathbf{J}_2 = \begin{bmatrix} \mathbf{J}_2 & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & \mathbf{J}_2 \end{bmatrix}_{M\times M}, \tag{15}$$

$$\mathbf{\Sigma} = \mathbf{I}_M \otimes \mathbf{\Sigma}_2 = \begin{bmatrix} \mathbf{\Sigma}_2 & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & \mathbf{\Sigma}_2 \end{bmatrix}_{M\times M}, \text{ and} \tag{16}$$

$$\mathbf{J}^{-1} = \mathbf{I}_M \otimes \mathbf{J}_2^{-1} = \begin{bmatrix} \mathbf{J}_2^{-1} & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & \mathbf{J}_2^{-1} \end{bmatrix}_{M\times M}. \tag{17}$$

The Kronecker channel is proved in Appendix I.

IV. IA and MU-MIMO Based Three-Cell Coordinated Beamforming Scheme

From the transmission model, we can see that the impairment in the desired signal is caused by ICI, IUI, and noise. In this section, we discuss the methods that suppress the adverse effects of these factors. To simplify the design, we divide the interference management into two consecutive steps

- Inter-cell interference cancellation
- Inter-user interference treatment

Although, the interference management procedure may cause some performance loss, it can simplify the coordinated beamforming design. Figure 2 illustrates the two-step interference management scheme.

A. ICI management

We impose a cascaded structure for the MIMO linear precoding and decoding matrices [18]. More specifically, we define MIMO precoding matrix $\mathbf{W}_{i,k}$ to be the product of two matrices, i.e.,

$$\mathbf{W}_{i,k} = \mathbf{W}_i^1 \mathbf{W}_{i,k}^2 \tag{18}$$

where matrix $\mathbf{W}_i^1$ is a common precoding matrix for all mobile stations within cell i, and matrix $\mathbf{W}_{i,k}^2$ is a MS specific precoding matrix. We use $\mathbf{W}_i^1$ to cancel ICI and use $\mathbf{W}_{i,k}^2$ to control the K-user transmission within a cell.

In the receiver side, we define MIMO receiver beamforming matrix $\mathbf{U}_{i,k}$ to be

$$\mathbf{U}_{i,k} = \mathbf{U}_{i,k}^2 \mathbf{U}_{i,k}^1 \tag{19}$$

where matrix $\mathbf{U}_{i,k}^1$ is applied to cancel ICI, and $\mathbf{U}_{i,k}^2$ is used to reconstruct user data.

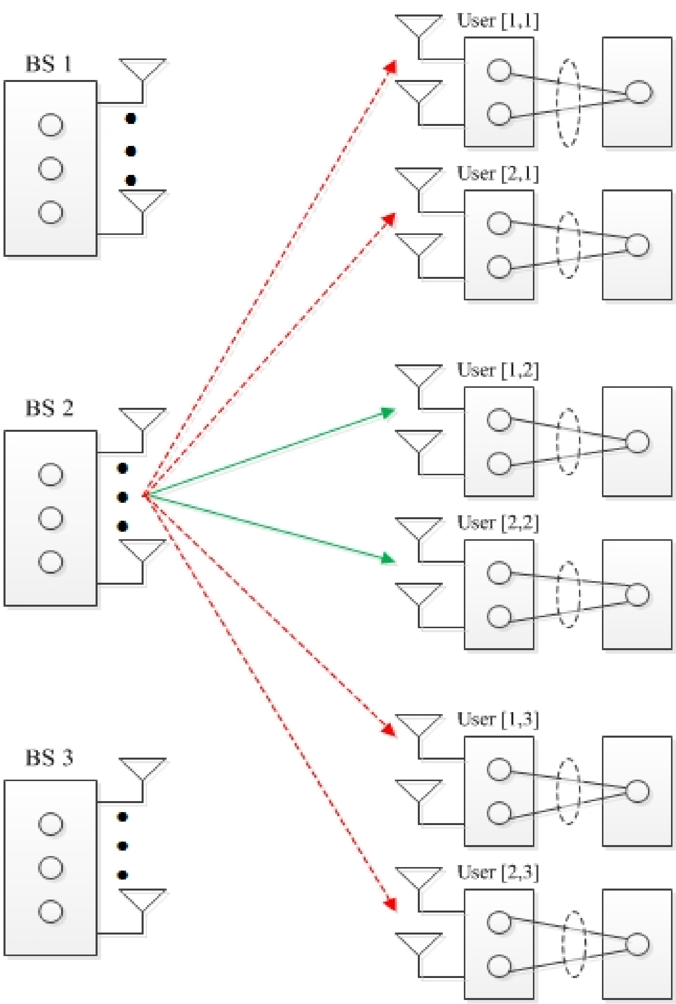

Figure 3. IA for the $(L,K,M,N) = (3, 2, 3, 2)$ MIMO-IFBC.

Combining (3), (4), (18), and (19), we obtain,

$$\begin{aligned}\hat{\mathbf{s}}_{i,k} &= \left(\mathbf{U}_{i,k}^2 \mathbf{U}_{i,k}^1\right)^H \mathbf{H}_{i,k} \mathbf{W}_i^1 \mathbf{W}_{i,k}^2 \mathbf{s}_{i,k} \\ &+ \sum_{l=1, l\neq k}^{3} \sum^{K} \left(\mathbf{U}_{i,k}^2 \mathbf{U}_{i,k}^1\right)^H \mathbf{H}_{i,k} \mathbf{W}_i^1 \mathbf{W}_{i,k}^2 \mathbf{s}_{i,l} + \mathbf{U}_{i,k}^H n_{i,k}.\end{aligned} \tag{20}$$

In order to decode the useful signal efficiently, both ICI and IUI should be aligned into the interference space at the receiver. Both ICI and IUI are aligned into the subspace which is orthogonal to $\mathbf{U}_{i,k}$. Therefore, the following condition must be satisfied for the k-*th* user in the i-*th* cell.

$$\left(\mathbf{U}_{j,1}^2\right)^H \mathbf{H}_{i,j,1} \mathbf{W}_i^1 = 0, \forall i \neq l, j \in \{1, 2, \cdots, K\}. \tag{21}$$

B. MU-MIMO Beamforming

In order to satisfy the condition given by (9), we first define the channel matrix for all users except the user k as:

$$\tilde{\mathbf{H}}_{i,k} = \left[\mathbf{H}_{i,1}^T, \cdots, \mathbf{H}_{i,k-1}^T, \mathbf{H}_{i,k+1}^T, \cdots, \mathbf{H}_{i,K}^T\right]^T. \tag{22}$$

Now, we perform the singular value decomposition (SVD) of $\tilde{\mathbf{H}}_{i,k}$ as:

$$\tilde{\mathbf{H}}_{i,k} = \tilde{\mathbf{U}}_{i,k} \left[\mathbf{\Lambda}_{i,k} \;\; \mathbf{0}\right] \left[\mathbf{V}_{i,k}^{(1)} \;\; \mathbf{V}_{i,k}^{(0)}\right]^H \tag{23}$$

where $\mathbf{\Lambda}_{i,k} = diag(\lambda_1, \lambda_2, \cdots, \lambda_K)$ with all non-negative singular values $\tilde{\mathbf{H}}_{i,k}$ to be its diagonal elements with a dimension equals to the rank of $\tilde{\mathbf{H}}_{i,k}$.

The equivalent channel matrix is defined as follows

$$\tilde{\mathbf{H}}_{eff,i,k} = \left(\mathbf{U}_{i,k}^1\right)^H \mathbf{H}_{i,k} \mathbf{W}_i^1 \tag{24}$$

and

$$\tilde{n}_{i,k} = \mathbf{U}_{i,k}^H n_{i,k}. \tag{25}$$

Since ICI is totally removed, we can rewrite (20) as

$$\begin{aligned}\hat{\mathbf{s}}_{i,k} &= \left(\mathbf{U}_{i,k}^2\right)^H \tilde{\mathbf{H}}_{eff,i,k} \mathbf{W}_{i,k}^2 \mathbf{s}_{i,k} \\ &+ \sum_{l=1, l\neq k}^{3} \sum^{K} \left(\mathbf{U}_{i,k}^2\right)^H \tilde{\mathbf{H}}_{eff,i,k} \mathbf{W}_{i,k}^2 \mathbf{s}_{i,l} + \tilde{n}_{i,k}\end{aligned} \tag{26}$$

which represents the channel input-output relationship of a standard single-cell MU-MIMO system.

V. Interference Alignment Scheme

In this section, we introduce IA for L-cell K-user MIMO-IFBC networks which cancel both ICI and IUI at the same time with the help of user cooperation, and investigate the benefits of user cooperation in terms of DoF.

A. Motivating example for $(L,K,M,N) = (3,2,3,2)$

Figure 3 illustrates our scheme clearly. Consider a simple case of MIMO-IFBC consisting of three BSs equipped with three transmits antennas per BS and two users with two receive antennas per user in each cell, referred to $(L,K,M,N)=(3,2,3,2)$ for notational convenience. The BS 2 wants to deliver two symbols, $s^{[1,2]}$ and $s^{[2,2]}$ to the user [1, 2] and user [2, 2] using the transmit beamforming vectors $\mathbf{W}^{[1,2]}$ and $\mathbf{W}^{[2,2]}$, respectively. In general, for given receive beamforming vectors, the minimum number of transmit antennas is four so that the transmit beamforming vectors cancel all ICI and IUI [10]-[11]. For example, in order to transmit the symbol $s^{[1,2]}$ without causing any interference to the other users, the beamforming vector $\mathbf{W}^{[1,2]}$ should satisfy the following condition,

$$\begin{aligned}\mathbf{W}^{[1,2]} \subset \mathrm{null}([&\underbrace{\left(\mathbf{U}^{[2,2]H}\mathbf{H}_2^{[2,2]}\right)^H}_{IUI\ channel}\\ &\underbrace{\left(\mathbf{U}^{[1,1]H}\mathbf{H}_2^{[1,1]}\right)^H\left(\mathbf{U}^{[2,1]H}\mathbf{H}_2^{[2,1]}\right)^H}_{ICI\ channel\,(Cell\ 2)}\\ &\underbrace{\left(\mathbf{U}^{[1,3]H}\mathbf{H}_2^{[1,3]}\right)^H\left(\mathbf{U}^{[2,3]H}\mathbf{H}_2^{[2,3]}\right)^H}_{ICI\ channel\,(Cell\ 3)}]^H)\end{aligned} \tag{27}$$

where null(·) denotes the null space of a matrix. However, our proposed scheme can remove both ICI and IUI with three transmit antennas by performance ICI channel alignment [13]-[14].

Step 1:Receive beamforming vectors

The user [1,2] and user [2,2] design the receive beamforming vectors $\mathbf{U}^{[1,2]}$ and $\mathbf{U}^{[2,2]}$, respectively, so that the ICI channels from each interfering BS are aligned as follows:

Case I: BS 1

$$\mathrm{span}\left(\mathbf{H}_2^{[1,1]H}\mathbf{U}^{[1,1]}\right)=\mathrm{span}\left(\mathbf{H}_2^{[2,1]H}\mathbf{U}^{[2,1]}\right), \tag{28}$$

$$\mathrm{span}\left(\mathbf{H}_3^{[1,1]H}\mathbf{U}^{[1,1]}\right)=\mathrm{span}\left(\mathbf{H}_3^{[2,1]H}\mathbf{U}^{[2,1]}\right), \tag{29}$$

Case II: BS 2

$$\mathrm{span}\left(\mathbf{H}_1^{[1,2]H}\mathbf{U}^{[1,2]}\right)=\mathrm{span}\left(\mathbf{H}_1^{[2,2]H}\mathbf{U}^{[2,2]}\right), \tag{30}$$

$$\mathrm{span}\left(\mathbf{H}_3^{[1,2]H}\mathbf{U}^{[1,2]}\right)=\mathrm{span}\left(\mathbf{H}_3^{[2,2]H}\mathbf{U}^{[2,2]}\right), \tag{31}$$

Case III: BS 3

$$\mathrm{span}\left(\mathbf{H}_1^{[1,3]H}\mathbf{U}^{[1,3]}\right)=\mathrm{span}\left(\mathbf{H}_1^{[2,3]H}\mathbf{U}^{[2,3]}\right), \tag{32}$$

$$\mathrm{span}\left(\mathbf{H}_2^{[1,3]H}\mathbf{U}^{[1,3]}\right)=\mathrm{span}\left(\mathbf{H}_2^{[2,3]H}\mathbf{U}^{[2,3]}\right), \tag{33}$$

where span (·) denotes the space spanned by the column vectors of a matrix. We can find the intersection subspace satisfying Eqs. (28)-(33) by solving the following matrix equations.

Case I: BS 1

$$\begin{bmatrix}\mathbf{I}_M & & -\mathbf{H}_2^{[1,1]H} & \\ \mathbf{I}_M & & & -\mathbf{H}_2^{[2,1]H}\\ & \mathbf{I}_M & -\mathbf{H}_3^{[1,1]H} & \\ & \mathbf{I}_M & & -\mathbf{H}_3^{[2,1]H}\end{bmatrix}\begin{bmatrix}\mathbf{h}_{[1,2]}^{ICI}\\ \mathbf{h}_{[1,3]}^{ICI}\\ \mathbf{U}^{[1,1]}\\ \mathbf{U}^{[2,1]}\end{bmatrix}=\mathbf{F}_1\mathbf{x}_1=0\,, \tag{34}$$

Case II: BS 2

$$\begin{bmatrix}\mathbf{I}_M & & -\mathbf{H}_1^{[1,2]H} & \\ \mathbf{I}_M & & & -\mathbf{H}_1^{[2,2]H}\\ & \mathbf{I}_M & -\mathbf{H}_3^{[1,2]H} & \\ & \mathbf{I}_M & & -\mathbf{H}_3^{[2,2]H}\end{bmatrix}\begin{bmatrix}\mathbf{h}_{[2,1]}^{ICI}\\ \mathbf{h}_{[2,3]}^{ICI}\\ \mathbf{U}^{[1,2]}\\ \mathbf{U}^{[2,2]}\end{bmatrix}=\mathbf{F}_2\mathbf{x}_2=0\,, \tag{35}$$

Case III: BS 3

$$\begin{bmatrix}\mathbf{I}_M & & -\mathbf{H}_1^{[1,3]H} & \\ \mathbf{I}_M & & & -\mathbf{H}_1^{[2,3]H}\\ & \mathbf{I}_M & -\mathbf{H}_2^{[1,3]H} & \\ & \mathbf{I}_M & & -\mathbf{H}_2^{[2,3]H}\end{bmatrix}\begin{bmatrix}\mathbf{h}_{[3,1]}^{ICI}\\ \mathbf{h}_{[3,2]}^{ICI}\\ \mathbf{U}^{[1,3]}\\ \mathbf{U}^{[2,3]}\end{bmatrix}=\mathbf{F}_3\mathbf{x}_3=0\,, \tag{36}$$

where $\mathbf{h}_{[i,j]}^{ICI}$ implies the direction of aligned effective interference channels. Therefore, the receive beamforming vectors for ICI channel alignment can be obtained explicitly with probability one.

Step 2: Transmit beamforming vectors

Since the effective ICI channels from each interfering BS are aligned, the transmit beamforming vectors are designed with the effective channels as

Case I: BS 1

$$\mathbf{W}^{[1,1]}\subset\mathrm{null}\left(\left[\mathbf{U}^{[2,1]H}\mathbf{H}_1^{[2,1]}\ \mathbf{h}_{[2,1]}^{ICI}\ \mathbf{h}_{[3,1]}^{ICI}\right]\right), \tag{37}$$

$$\mathbf{W}^{[2,1]}\subset\mathrm{null}\left(\left[\mathbf{U}^{[1,1]H}\mathbf{H}_1^{[1,1]}\ \mathbf{h}_{[2,1]}^{ICI}\ \mathbf{h}_{[3,1]}^{ICI}\right]\right), \tag{38}$$

Case II: BS 2

$$\mathbf{W}^{[1,2]}\subset\mathrm{null}\left(\left[\mathbf{U}^{[2,2]H}\mathbf{H}_2^{[2,2]}\ \mathbf{h}_{[1,2]}^{ICI}\ \mathbf{h}_{[3,2]}^{ICI}\right]\right), \tag{39}$$

$$\mathbf{W}^{[2,2]}\subset\mathrm{null}\left(\left[\mathbf{U}^{[1,2]H}\mathbf{H}_2^{[1,2]}\ \mathbf{h}_{[1,2]}^{ICI}\ \mathbf{h}_{[3,2]}^{ICI}\right]\right), \tag{40}$$

Case III: BS 3

$$\mathbf{W}^{[1,3]}\subset\mathrm{null}\left(\left[\mathbf{U}^{[2,3]H}\mathbf{H}_3^{[2,3]}\ \mathbf{h}_{[1,3]}^{ICI}\ \mathbf{h}_{[2,3]}^{ICI}\right]\right), \tag{41}$$

$$\mathbf{W}^{[2,3]}\subset\mathrm{null}\left(\left[\mathbf{U}^{[1,3]H}\mathbf{H}_3^{[1,3]}\ \mathbf{h}_{[1,3]}^{ICI}\ \mathbf{h}_{[2,3]}^{ICI}\right]\right). \tag{42}$$

We find the intersection subspace satisfying the condition Eqs. (37)- (42) by solving the following matrix equation.

$$\begin{bmatrix}\mathbf{I}_M & -\mathbf{H}_i^{[1,i+1]H} & 0 & \cdots & 0\\ \mathbf{I}_M & 0 & -\mathbf{H}_i^{[2,i+1]H} & & 0\\ \vdots & \vdots & \vdots & \ddots & \vdots\\ \mathbf{I}_M & 0 & 0 & \cdots & -\mathbf{H}_i^{[K,i+1]H}\end{bmatrix}\begin{bmatrix}\mathbf{h}_{[i,j]}^{ICI}\\ \mathbf{U}^{[1,i+1]}\\ \vdots\\ \mathbf{U}^{[K,i+1]}\end{bmatrix}=\mathbf{F}_i\mathbf{x}_i=0\,. \tag{43}$$

VI. SIMULATIONS RESULTS

In this section, we compare the achievable DoF for different schemes. For simple comparisons, we focus only on the system configuration of $(L,K,M,N)=(3,2,3,2)$. The coordinated ZF scheme proposed by [10]-[11] can attain three DoF. The reason of the increase of DoF comes from that the proposed IA that can utilize the signal space efficiently rather than the coordinated ZF scheme.

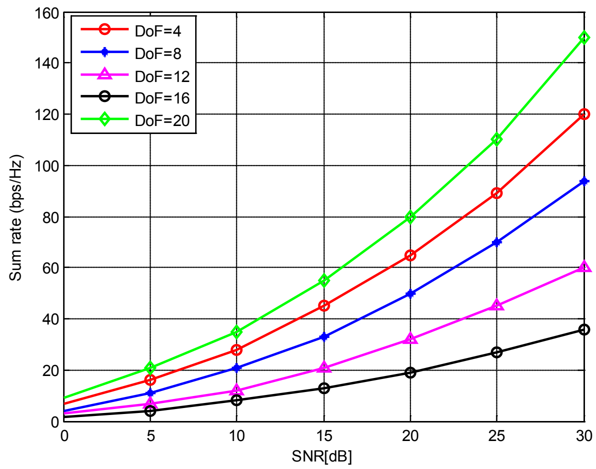

Figure 4. DoF for $(L,K,M,N)=(3,2,3,2)$ MIMO-IFBC.

Figure 4 illustrates the sum rate performance of the proposed IA scheme for various system configurations as functions of the SNR. As shown in this figure, the sum rate increases linearly with the slope of 4, 8, 12, 16 and 20 in (3, 2, 2), (6, 4, 2), (9, 6, 2), (12, 8, 2) and (12, 10, 2) MIMO-IFBCs, respectively. It confirms that the sum rate performances exactly coincide with the optimal DoF. We also observe that the sum rate of the proposed scheme grows linearly with the slope.

VII. CONCLUSION

In this paper, we proposed a general framework for multi-cell cooperative transmission. The treatment of interference is divided into two steps: Inter-cell interference elimination and intra-cell inter-user interference management. IA methods are applied in the first step to achieve a higher DoFs transmission and the second step utilizes the existing powerful MU-MIMO technologies to implement efficient co-channel multiuser transmission. The number of user antennas is smaller than that of BS antennas, and the sum rate slope of the MIMO-IFBC is equal to that of the IFBC.

REFERENCES

[1] M. H. Lee, M. M. Matalgah, W. Song, "Fast method for precoding and decoding of distributive multi-input multi-output channels in relay based decode-and-forward cooperative wireless networks", *IET Commun.*, vol. 4, no. 2, pp. 144–153, 2010.

[2] M. H. Lee, "A new reverse Jacket transform and its fast algorithm," *IEEE Trans. Circuits Syst. II*, vol. 47, no. 1, pp. 39–47, Jan. 2000.

[3] Z. Chen, M. H. Lee, and G. Zeng, "Fast cocyclic Jacket transform," *IEEE Trans. Signal Process.*, vol. 56, pp. 2143–2148, May 2008.

[4] M. H. Lee, N. L. Manev, X. D. Zhang "Jacket transform eigenvalue decomposition", *Appl. Math. Comput.*, 198, pp. 854–864, 2008.

[5] H. Weingarten, Y. Steinberg, and S. Shamai, "The capacity region of the Gussian MIMO broadcast channel," *IEEE Trans. Inform. Theory*, vol. 52, no. 9, pp. 3936-3964, Sept. 2006.

[6] M. H. M. Costa, "Writing on dirty paper," *IEEE Trans. Inform. Theory*, vol. 29, pp. 439-441, May 1983.

[7] C. Windpassinger, R. F. H. Fischer, T. Vencel, and J. B. Huber, "Precoding in multiantenna and multiuser communications", *IEEE Trans. Wireless Commun.*, vol. 3, no. 4, July 2004.

[8] C. B. Peel, B. M. Hochwald, and A. L. Swindelhurst, "A vectorperturbation technique for near-capacity multiantenna multiuser communication-part I: channel inversion and regularization," *IEEE Trans. Commun,* vol. 53, 195-202, Jan. 2005.

[9] Q. H. Spencer, A. L. Swindelhurst, and M. Haardt, "Zero-forcing method for downlink spatial multiplexing in multiuser MIMO channels," *IEEE Trans. Signal Processing*, vol. 52, no. 2, pp. 461-471, Feb. 2004.

[10] S.-H. Park and I. Lee, "Degrees of freedom and sum rate maximization for two mutually interfering broadcast channels," *Proc. IEEE Int. Conf. Commun. (ICC)*, Dresden, Germany, June 2009.

[11] J. Kim, S.-H. Park, H. Sung, and I. Lee, "Sum rate analysis of two-cell MIMO broadcast channels: spatial multiplexing gain," Proc. *IEEE Int. Conf. Commun. (ICC)*, Cape Town, South Africa, May 2010.

[12] V. R. Cadambe and S. A. Jafar, "Interference alignment and degrees of freedom of the K-user interference channel," *IEEE Trans. Inf. Theory*, vol. 54, no. 8, pp. 3425–3441, Aug. 2008.

[13] J. Tang, S. Lambotharan, "Interference alignment techniques for MIMO multi-cell interfering broadcast channels", *IEEE Trans. on Commun.,*vol. 61, no. 1,January 2013.

[14] W. Shin, N. Lee, J. B. Lim, C. Shin, K. Jang, "On the design of interference alignment scheme for two-cell MIMO interfering broadcast channels", *IEEE Trans. on Wireless Commun*.,vol.10, no. 2, Feb. 2013.

[15] R. K. Mungara, A. Tolli, M. Juntti, "Degrees of freedom and interference mitigation for MIMO interfering broadcast channels", *2011 IEEE GLOBECOM Workshops,* Dec. 5, 2011, Texas, USA.

[16] S. A. Jafar, M. J. Fakhereddin, "Degrees of Freedom for the MIMO Interference Channel", *IEEE Trans. Inf. Theory*, vol. 53, no. 7, July 2007.

[17] 3GPP TR 36.814 V9.0.0, "Further advancements for E-UTRA physical layer aspects (release 9)," Oct. 2006.

[18] C. Na, X. Hou, A. Harada, "Two-cell coordinated transmission scheme based on interference alignment and MU-MIMO beamforming", *75th Vehicular Technology Conference (VTC Spring)*, 6-9 May 2012, Yokohama, Japan.

[19] Moon Ho Lee, *Jacket Matrices-Construction and Its Application for Fast Cooperative Wireless Signal Processing*. LAP LAMBERT Academic publishing, Germany, November, 2012.

Appendix I:

The mobile communication diagonal channel matrix is given by Eq. (10)

$$\mathbf{H}_{i,k}=\mathbf{I}_M\otimes\mathbf{B}_2 \qquad \text{(A-1)}$$

where

$$\mathbf{B}_2=\begin{pmatrix}\cos 45^0 & -i\sin 45^0\\ \sin 45^0 & i\cos 45^0\end{pmatrix}=\frac{1}{\sqrt{2}}\begin{pmatrix}1 & -i\\ 1 & i\end{pmatrix}$$

$$=\begin{bmatrix} 0.8881 & -0.3251+0.3251i \\ 0.3251+0.3251i & 0.8881 \end{bmatrix} \begin{bmatrix} 0.9659-0.2588i & 0 \\ 0 & -0.2588+0.9659i \end{bmatrix} \begin{bmatrix} 0.8881 & 0.3251-0.3251i \\ -0.3251-0.3251i & 0.8881 \end{bmatrix} = \mathbf{Q}\mathbf{\Lambda}\mathbf{Q}^H. \quad \text{(A-2)}$$

The 4×4 block-wise Jacket matrix is

$$\mathbf{H}_4 = \begin{pmatrix} [\mathbf{B}_2] & 0 \\ 0 & [\mathbf{B}_2] \end{pmatrix} = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & -i & 0 & 0 \\ 1 & i & 0 & 0 \\ 0 & 0 & 1 & -i \\ 0 & 0 & 1 & i \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \otimes \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & -i \\ 1 & i \end{pmatrix} = [\mathbf{I}]_2 \otimes [\mathbf{B}_2]. \quad \text{(A-3)}$$

Now the channel matrix $\mathbf{H}_{i,k}$ is decomposed by the EVD

$$\mathbf{H}_{i,k} = \mathbf{Q}\mathbf{\Lambda}\mathbf{Q}^H. \quad \text{(A-4)}$$

Then we get the EVD as:

$$\mathbf{H}_{i,k}\mathbf{H}_{i,k}^H = \mathbf{Q}\mathbf{\Sigma}\mathbf{\Sigma}^H\mathbf{Q}^H = \mathbf{Q}\mathbf{\Lambda}\mathbf{Q}^H \quad \text{(A-5)}$$

where $\mathbf{Q}^H\mathbf{Q} = \mathbf{Q}\mathbf{Q}^H = \mathbf{I}_N$ and, $\mathbf{\Lambda} = dig(\lambda_1, \lambda_2, \cdots, \lambda_N)$ with its diagonal elements given as

$$\lambda_i = \begin{cases} \sigma_i^2, \ if\ i = 1, 2, \cdots, N_{\min} \triangleq (M, N) \\ 0, \ if\ i = N_{\min}+1, \cdots, N \end{cases}. \quad \text{(A-6)}$$

Therefore, EVD can be also applied to block diagonal Jacket matrices.