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A Theoretical Analysis of Authentication, Privacy and Reusability Across Secure Biometric Systems

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Abstract—We present a theoretical framework for the analysis of privacy and security tradeoffs in secure biometric authentication systems. We use this framework to conduct a comparative information-theoretic analysis of two biometric systems that are based on linear error correction codes, namely fuzzy commitment and secure sketches. We derive upper bounds for the probability of false rejection (P_{FR}) and false acceptance (P_{FA}) for these systems. We use mutual information to quantify the information leaked about a user’s biometric identity, in the scenario where one or multiple biometric enrollments of the user are fully or partially compromised. We also quantify the probability of successful attack (P_{SA}) based on the compromised information. Our analysis reveals that fuzzy commitment and secure sketch systems have identical P_{FR} , P_{FA} , P_{SA} and information leakage, but secure sketch systems have lower storage requirements. We analyze both single-factor (keyless) and two-factor (key-based) variants of secure biometrics, and consider the most general scenarios in which a single user may provide noisy biometric enrollments at several access control devices, some of which may be subsequently compromised by an attacker. Our analysis highlights the revocability and reusability properties of key-based systems and exposes a subtle design tradeoff between reducing information leakage from compromised systems and preventing successful attacks on systems whose data have not been compromised.

Index Terms—Biometrics, Fuzzy Commitment, Secure Sketch, Revocability, Reusability, Information Leakage, Privacy, Security

I. INTRODUCTION

Human biometric measurements such as fingerprints, iris scans, face images and ECG signals are attractive tools for identifying and authenticating users in access control situations. Unlike conventional identifying documents, biometrics are difficult to forge. Unlike passwords traditionally used for access control, they do not have to be remembered. However, biometrics also present some new challenges that are not encountered in traditional methods. Noise is a characteristic feature of all biometric measurements; every measurement is slightly different from all others. In access control systems, the issue of noise in biometric measurements is currently tackled through pattern recognition. Specifically, a measurement of the biometric is taken at the time of enrollment and stored in a database of enrolled identities. During authentication, the

person in question provides a “test” or a “probe” biometric for comparison with the stored enrollment biometric. If the probe and enrollment biometric are sufficiently close according to a similarity metric defined by the pattern recognition algorithm, then access is allowed.

Unfortunately, the standard method described above has a serious drawback: an adversary who compromises the device gains access to the enrollment biometric. This is a major security hazard; the attacker can subsequently use the enrollment biometric to gain repeated access to the system, and to any other biometric-based systems in which the user has enrolled. This is also a privacy hazard; the attacker has gained access to the user’s identifying information and can henceforth impersonate the user illegally. The seriousness of this hazard is greatly increased by the fact that biometrics are inherent properties of the human body and cannot be revoked and then re-issued like new credit card numbers. To mitigate growing concerns about security hazards and identity theft, new approaches to biometrics have been studied with a three-fold goal. First, the data stored on the access control device should provide little or no information about the actual biometric. Second, the stored data should not allow an attacker to gain unauthorized access to the system or to impersonate the identity of a legitimate user successfully. Third, if the user’s stored data is known to have been compromised, then it should be possible to revoke it and issue new stored data. This should prevent the adversary from gaining access or stealing the user’s identity in the future.

Secure biometric schemes proposed to fulfill the above desiderata fall under one of two related categories, viz., *fuzzy commitment* [1], [2], [3], [4], [5] and *secure sketch* schemes [5], [6], [7], [8], [9]. In fuzzy commitment a secret vector is combined with the user’s enrollment biometric via a *commitment function*. The output of the commitment function is stored on the access control device. Access control is accomplished by means of a *decommitment* function. The decommitment function takes as its inputs the stored data and the user’s probe biometric and attempts to recover the secret vector. If recovery is successful, access is allowed. In contrast, in secure sketch the user provides his/her biometric at enrollment and a “sketch” signal is derived and stored on the access control device. When combined with a probe biometric from the legitimate user, the enrollment biometric can be recovered. If the enrollment biometric is recovered successfully, then access is allowed. We later discuss how to verify the correctness of this recovery or of successful decommitment in fuzzy commitment. Linear error correcting codes (ECC) are the most widely used tool for constructing

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both fuzzy commitment schemes [2], [3], [10], [11] and secure sketch-based schemes [8], [9].

The relationship between secure sketches and fuzzy extractors was examined in [5] where it was shown that a secure sketch implies the existence of a fuzzy extractor. In the present paper, we analyze explicit ECC-based constructions of fuzzy commitment and secure sketch. We study both the security and privacy hazards mentioned above. Regarding the former, we derive upper bounds on the false rejection rate (FRR) and false acceptance rate (FAR) for both types of systems. Regarding the latter, we characterize the privacy leakage as the mutual information between the compromised stored data and the user’s biometric. Further, smart adversaries may be able to increase their likelihood of gaining access to a system above the FAR if they have access to some partial compromise of stored data and condition their attack on that knowledge. We term this the probability of a “successful attack” (P_{SA}) and quantify it in some situations. Our analysis establishes a strong statement of equivalence: secure sketches and fuzzy commitment schemes are equivalent in terms of the FRR, FAR, information leakage, and P_{SA} .

There have been many insightful studies of the information leakage that occurs when data stored on the access control device is compromised [12], [5], [13], [14]. An important insight is that a useful sketch, i.e., one that correctly authenticates noisy samples from a legitimate user, must leak some information about the underlying biometric [5]. Extending this idea, [12] considers a generalized challenge-response setting in which a strong adversary examines sketches from several chosen perturbations of the challenger’s biometric, until the biometric has been guessed completely. We consider a different scenario in which an adversary compromises a chosen subset of the *available* access control devices and, knowing the error correcting codes associated with each, attempts to attack the user’s system. We think that this problem formulation is more reflective of emerging networks of biometric systems. Further, it raises many interesting challenges, e.g., we may ask how to choose the perturbations or error correcting codes so as to leak the least information about the user’s biometric. In this sense, our work is related to the privacy analysis of [13], where the authors consider a sketch indistinguishability game and sketch irreversibility game and give conditions on the ECC design that minimizes the adversary’s advantage. We note that, in the analyses of [12], [5], [13], the emphasis is on information leakage about the user’s biometric as the adversary’s prime objective. In practice, however, the adversary may have a second objective, namely to compromise some devices and use the information gained to login to other devices. It may not be necessary to discover the user’s biometric. Our analysis reveals a subtle conflict between reducing information leakage from compromised systems and preventing successful attacks on systems whose data have not been compromised.

A different, but related, line of work focuses on the problem of secret key agreement via public discussion [15], [16], [17], [18], [19]. In this problem two parties hold correlated pieces of information and desire to generate matching secret keys through a public discussion. However, an eavesdropper who taps into the public discussion should learn nothing about

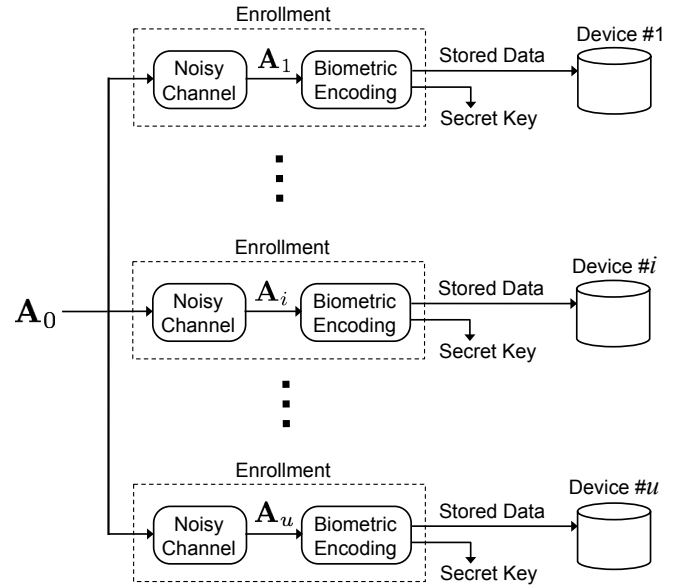


Fig. 1. Noisy measurements A_1, \dots, A_u of a user’s underlying biometric A_0 are encoded at each access control device to generate authentication data, which is stored in the device, and a secret key. Our goal is to analyze the tradeoffs between authentication performance and information leakage from compromised stored authentication data and secret keys.

the keys. Of interest in this line of work is the fundamental *asymptotic* tradeoff between the secret key rate (security) and biometric information leakage (privacy). Secret key agreement by itself does not form a biometric authentication system but it can be used to construct one. In contrast, we explicitly analyze the fundamental *non-asymptotic* privacy-security tradeoff in biometric systems that are based on linear ECCs and explicitly relate them to ECC-design parameters.

The remainder of this paper is organized as follows: Section II describes a general framework for analyzing secure biometrics and defines the metrics by which security and privacy¹ are evaluated. In Section III, we describe how to realize fuzzy commitment and secure sketch schemes using linear ECCs. We show the equivalence between the realizations of fuzzy commitment and secure sketch in terms of their security and privacy metrics. In Section IV, we expand our attention to include multiple devices. We derive the information leakage for attack scenarios in which an adversary compromises the stored data and/or secret keys of multiple devices. We show how the information leakage depends on the ECCs used at the devices. We characterize how the selection of the ECCs affects the probability that the adversary can use information gained from the compromised devices to successfully attack (i.e., gain access to) uncompromised devices, and how this objective conflicts with the aim of minimizing information leaked about the user’s biometric. Section V concludes the paper.

¹In this work, compromising privacy refers to leaking information about the user’s biometric, while compromising security refers to gaining access to the system.

II. A GENERALIZED SECURE BIOMETRICS FRAMEWORK

Consider the scenario with several access control devices shown in Figure 1. A user has a biometric \mathbf{A}_0 given by nature. He enrolls at several access control devices using noisy measurements \mathbf{A}_i of the underlying biometric \mathbf{A}_0 . From each measurement \mathbf{A}_i , encoded data is extracted and stored on the respective device to aid in authentication. Optionally, a secret key or password is provided to the user. A legitimate user should be able to gain access to any of the devices by providing a probe biometric that is again a noisy measurement of the underlying \mathbf{A}_0 . Any analysis of the privacy and security tradeoffs in secure biometrics must take into account not only the authentication performance but also the information leakage when the stored data and/or keys for one or more devices are compromised.

With the above motivation, we start by presenting an abstract model of a secure biometric system for a single access control device in Section II-A. We then describe design objectives in terms of the system’s performance metrics in Section II-B.

A. Model of a Secure Biometric System

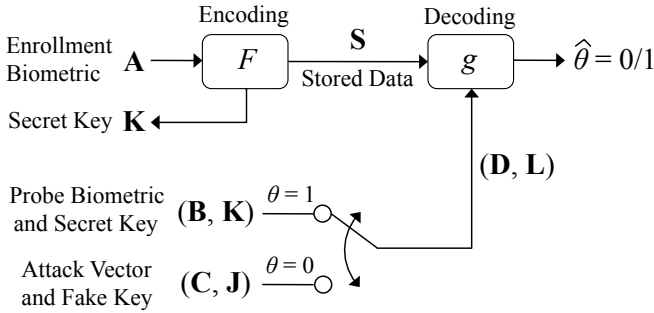


Fig. 2. Generalized model of a secure biometric system for a single access control device. This model encompasses both fuzzy commitment-based and secure sketch-based realizations that are described and analyzed in Section III. For keyless realizations, \mathbf{K} is null. For two-factor realizations, \mathbf{K} is a secret key output by the randomized encoding function. Given the probe biometric and, in two-factor realizations, a secret key, the decoder solves a hypothesis testing problem to determine if the user is genuine or an impostor.

Figure 2 depicts a generalized model of a secure biometric system for a single access control device. The system consists of encoding and decoding modules that manipulate features extracted from measurements of human biometrics. In biometrics parlance the terms “biometric”, “biometric measurement”, and “biometric feature vector”, have different meanings. A fingerprint, iris, or a face is a *biometric*, the *measurement* of which produces a digitized image from which *features* are extracted for authentication or recognition. However, for brevity of exposition, we interchangeably use the terms “biometric” and “biometric measurement” to denote a biometric feature vector denoted by \mathbf{A} , \mathbf{A}_i , \mathbf{B} , \mathbf{B}_i , etc. We make the additional simplifying assumption that all feature vectors and secret keys are length- n sequences of binary numbers. The generalization to non-binary finite alphabets is straightforward.

We note that, in some settings, an attacker may be interested in recovering the original biometric signal itself, e.g., the

digitized image of a fingerprint or iris, rather than the binary biometric feature vector and that this will involve tackling a (likely) ill-posed inverse problem. Making *privacy* guarantees in such settings will be quite system-specific (the inverse problem for irises or different feature sets of fingerprints being quite different) and we choose to leave this outside the scope of our work. Our goal here is not to analyze specific implementations, but rather to make system independent guarantees. Further, since the original biometric signal impacts the system *security* only through the binary biometric feature vector, if a system is secure with respect to the feature vector, it will also be secure with respect to the original biometric signal.² In that sense our security analysis should be interpreted as more general and more pessimistic than one based on recovering the original biometric signal: it is a worst-case analysis. For the above reasons, we only deal with binary biometric feature vectors and not the original biometric signal itself.

Biometric Measurement Model: The process of measuring a biometric, extracting suitable feature vectors, and converting them to length- n binary sequences is inherently prone to sensing uncertainty, e.g., in the orientation, size, and illumination of an iris or a face, as well as noise in the sensing elements. Since we are interested in scenarios where a user can enroll the same biometric at multiple access control devices (see Section IV-A), we posit an underlying “ground truth” length- n binary biometric feature vector $\mathbf{A}_0 := (A_{0,1}, \dots, A_{0,n})$ whose components have an i.i.d. Bernoulli(0.5) distribution.³ We need to model the combined effect of a measurement followed by the extraction of a length- n binarized feature vector (or, for brevity, the biometric measurement). We model this as component-wise modulo-two addition of \mathbf{A}_0 with a length- n i.i.d. Bernoulli “noise” sequence. The noise sequence is assumed to be independent of the ground truth and any previous and future measurement noise sequences. In the language of information theory, the biometric measurement is the output of a “binary symmetric channel” (BSC), where the channel input is \mathbf{A}_0 . Thus, at enrollment, the user provides an enrollment biometric measurement $\mathbf{A} := (A_1, \dots, A_n)$ which is the output of a BSC with crossover probability p_1 and channel input \mathbf{A}_0 . Similarly, at authentication, the user provides a probe biometric measurement $\mathbf{B} := (B_1, \dots, B_n)$, which is the output of a BSC with crossover probability α and channel input \mathbf{A}_0 . This second probe measurement is used by the decoding module of the access control device to verify the user’s identity. We further assume that $p_1 \in [0, 0.5)$ and $\alpha \in [0, 0.5)$, i.e., it is more likely that coordinates of the biometric measurement and probe measurement match than that they do not. To see the statistical dependency between \mathbf{A} and \mathbf{B} , observe that \mathbf{A}_0 , \mathbf{A} and \mathbf{B} are all i.i.d Bernoulli-0.5 sequences. This, along with the BSC channel dependency explained above, means that \mathbf{A} and \mathbf{B} are, in turn, related by a BSC

²This can be formalized by appealing to the Data Processing Inequality [20].

³Binarized feature vectors extracted from biometric measurements are, in general, neither independent nor identically distributed. It is, however, possible to design feature transformation algorithms that can convert them into binary feature vectors the statistics of which are quite close to those of i.i.d. Bernoulli(0.5) bits [9].

with crossover probability $p = p_1 * \alpha = p_1(1 - \alpha) + (1 - p_1)\alpha$.

Enrollment: The (potentially randomized) encoding function $F(\cdot)$ takes the enrollment biometric \mathbf{A} as input and produces as outputs $\mathbf{S} \in \mathcal{S}$, $|\mathcal{S}| < \infty$, which is stored on the access control device. Optionally, a key vector $\mathbf{K} \in \mathcal{K}$, $|\mathcal{K}| < \infty$, which is returned to the user, is also produced. Thus, $(\mathbf{S}, \mathbf{K}) = F(\mathbf{A})$. The encoding function is governed by the conditional distribution $P_{\mathbf{S}, \mathbf{K} | \mathbf{A}}$. Depending upon the physical realization of the system, the user may be required to carry the key \mathbf{K} on a smart card. Such systems are called *two-factor* systems because both the “factors”, namely the biometric and the key, are needed for authentication and are typically independent of each other. Systems where \mathbf{K} is null are called *keyless* systems; they do not require the use of a smart card.

In most practical implementations of secure biometrics, enrollment also involves computing a cryptographic hash of the biometric feature vector or of the generated keys. This hash is stored on the device and used as a final step during authentication. Cryptographic hashes are computationally secure but not information-theoretically secure. Since our aim is to provide information-theoretic security guarantees, we do not consider cryptographic hashes in our system model, i.e., cryptographic hashes are *not* part of the stored data or the second factor (key).

Authentication: To perform biometric authentication, a legitimate user provides the probe biometric \mathbf{B} and the key \mathbf{K} . An adversary, on the other hand, provides a stolen or artificially synthesized biometric \mathbf{C} and a stolen or artificially synthesized key \mathbf{J} . The presence of the legitimate user or the adversary is indicated by the unknown binary parameter θ . Let (\mathbf{D}, \mathbf{L}) denote the (biometric, key) pair that is provided during the authentication step. We write

$$(\mathbf{D}, \mathbf{L}) := \begin{cases} (\mathbf{B}, \mathbf{K}), & \text{if } \theta = 1, \\ (\mathbf{C}, \mathbf{J}), & \text{if } \theta = 0. \end{cases}$$

The authentication decision is computed by the decoding function as $\hat{\theta} = g(\mathbf{D}, \mathbf{L}, \mathbf{S})$. In keyless systems, the procedure is similar with \mathbf{K} , \mathbf{J} , and \mathbf{L} removed from the above description.

B. Performance Metrics

We now define metrics used to evaluate the performance of the secure biometric system of Figure 2. For example, it is necessary to quantify how reliably the system authenticates a genuine user and rejects an impostor, to quantify how much information is leaked about the underlying biometric when the stored data and/or the secret key are compromised, and so on.

- 1) **Probability of Missed Detection:** This quantity is also called the False Rejection Rate (FRR), defined as

$$P_{FR} := \Pr[\hat{\theta} = 0 | \theta = 1] = \Pr[g(\mathbf{B}, \mathbf{K}, \mathbf{S}) = 0].$$

The P_{FR} depends only on the known statistics of $(\mathbf{A}, \mathbf{B}, \mathbf{K})$ and the specification of the system, $F(\cdot)$ and $g(\cdot)$. A low value of P_{FR} indicates that the system reliably authenticates a genuine user. Thus P_{FR} quantifies the accuracy of the biometric system.

- 2) **Probability of False Detection:** A *baseline* probability of false detection, also called the False Acceptance Rate (FAR) is the worst-case probability of false detection across all attack vectors and keys that can be generated without any knowledge of the ground truth or of any measurements, keys, or stored data. It is defined as

$$\begin{aligned} P_{FA} &:= \max_{\mathbf{C}, \mathbf{J}} \Pr[\hat{\theta} = 1 | \theta = 0] \\ &= \max_{\mathbf{C}, \mathbf{J}} \Pr[g(\mathbf{C}, \mathbf{J}, \mathbf{S}) = 1], \end{aligned}$$

where (\mathbf{C}, \mathbf{J}) is independent of $(\mathbf{A}_0, \mathbf{A}, \mathbf{B}, \mathbf{K}, \mathbf{S})$. A low value of P_{FA} indicates that the system reliably prevents impostors from gaining access to the system by pure chance. Thus P_{FA} quantifies one aspect of the security of the biometric system. Typically, a system designer is faced with choosing an appropriate tradeoff between P_{FA} and P_{FR} .

- 3) **Privacy Leakage:** We measure the information leaked about the enrollment biometric \mathbf{A} (respectively the ground truth \mathbf{A}_0) in various scenarios of data exposure. These include scenarios in which either the stored data \mathbf{S} , the secret key \mathbf{K} , or both are compromised. We measure the privacy leakage in various scenarios using the following mutual information quantities: $I(\mathbf{A}; \mathbf{S})$, $I(\mathbf{A}; \mathbf{K})$, and $I(\mathbf{A}; \mathbf{S}, \mathbf{K})$ (respectively $I(\mathbf{A}_0; \mathbf{S})$, $I(\mathbf{A}_0; \mathbf{K})$, and $I(\mathbf{A}_0; \mathbf{S}, \mathbf{K})$). Mutual information is an information-theoretic measure of independence that has been used extensively in statistics, machine learning, and signal processing.⁴ If two quantities are statistically independent (e.g., the original/noisy biometric template and the stored data/key) then there is no privacy leakage.
- 4) **Probability of Successful Attack:** In the event of data exposure, the probability of false detection could increase beyond the nominal value of P_{FA} . In addition to exposure of the stored data \mathbf{S} and the secret key \mathbf{K} , mentioned above, we may also need to consider scenarios where an adversary coercively gains access to \mathbf{A} as well. We need to capture the possibility that the attacker’s biometric-key pair (\mathbf{C}, \mathbf{J}) is generated using knowledge of the compromised data $\mathcal{V} \subseteq \{\mathbf{A}, \mathbf{S}, \mathbf{K}\}$. We denote by $P_{SA}(\mathcal{V})$ the probability of false detection in such situations, defined as

$$\begin{aligned} P_{SA}(\mathcal{V}) &:= \max_{\mathbf{C}, \mathbf{J} | \mathcal{V}} \Pr[\hat{\theta} = 1 | \theta = 0] \\ &= \max_{\mathbf{C}, \mathbf{J} | \mathcal{V}} \Pr[g(\mathbf{C}, \mathbf{J}, \mathbf{S}) = 1]. \end{aligned}$$

We refer to $P_{SA}(\mathcal{V})$ as the “Successful Attack Rate” (SAR) to distinguish it from P_{FA} . The SAR captures the probability of false detection when an adversary’s

⁴Mutual information between two sets of quantities is a (pseudo) measure of distance: it is always non-negative and is equal to zero if, and only if, the two sets are statistically independent [20]. Furthermore, we can always write the mutual information between two random quantities \mathbf{X} and \mathbf{Y} as $I(\mathbf{X}; \mathbf{Y}) = H(\mathbf{X}) - H(\mathbf{X} | \mathbf{Y})$ where $H(\cdot)$ and $H(\cdot | \cdot)$ are, respectively, the entropy and conditional entropy of the argument(s). Thus, mutual information characterizes the *reduction in uncertainty* about one random quantity, \mathbf{X} , when given knowledge of another, \mathbf{Y} .

attack is aided by knowledge of \mathcal{V} . We note, in passing, that in any keyless or two-factor system, knowledge of the stored data \mathbf{S} can drastically improve the ability of the adversary to gain access, thus compromising the security of the system. We will characterize this effect in Theorem 2. Ideally, in two-factor systems, attackers that have knowledge of only one factor — i.e., either the enrollment biometric \mathbf{A} or the key \mathbf{K} , but not both — will not be able to use that information to improve their ability to authenticate falsely. This motivates the following definition. We say that a system is *two-factor secure* if $P_{SA}(\mathbf{A}) = P_{SA}(\mathbf{K}) = P_{FA}$.

- 5) **Storage Requirements:** Lastly, the system data storage requirement is given by the minimum number of bits needed to represent \mathbf{S} . This is not more than $\log_2 |\mathcal{S}|$ bits. The key length requirement is given by the minimum number of bits needed to represent \mathbf{K} , which is not more than $\log_2 |\mathcal{K}|$ bits.

III. SYSTEM CONSTRUCTIONS

In this section, we focus on a single access control device *in isolation*, and analyze system privacy and security properties. The encoding and decoding functions in the generalized framework of Figure 2 accommodate many different instantiations. For instance, the decoding function g could involve recovering an estimate of the enrollment biometric (or a secret key bound to the enrollment biometric), then ascertaining whether the recovered estimate is accurate, and finally generating a binary authentication decision. One way to accomplish this is by means of an error-correcting code. We describe two types of systems which are both widely used special cases of the general model of Figure 2. The first is a fuzzy commitment system and the second is a secure sketch system. Both systems make use of error-correction coding which, in implementation, is dominated by linear codes. Hence, for both systems, we assume an implementation based on linear error correcting codes. We detail both keyless and keyed (two-factor) variants. The linear error correcting code construction allows us to demonstrate a number of performance-equivalence properties between fuzzy commitment and secure sketch systems. Considerations of privacy and security for a network of access control devices is deferred to Section IV.

A. Fuzzy Commitment Systems based on ECC

A *fuzzy commitment* scheme binds a random vector to an enrollment biometric \mathbf{A} to produce a length- n stored data vector \mathbf{S} . This is diagrammed in Figure 3 for the case of a two-factor (keyed) system. The keyless variant, shown in Figure 4, is the special case where the smart card key \mathbf{K} and decoding key \mathbf{L} are both the all-zero sequence. Note that both systems fit within the general framework of Figure 2.

We exclusively consider fuzzy commitment schemes wherein the random vector corresponds to a uniformly selected codeword of a binary $[n, k]$ linear error correcting code. We use \mathbf{G} to denote the code's $k \times n$ generator matrix and \mathbf{H} to denote the code's $m \times n$ parity check matrix with $m = n - k$.

Enrollment: The enrollment procedure first generates two independent i.i.d. Bernoulli(0.5) sequences, the key sequence $\mathbf{K} := (K_1, \dots, K_n)$ and the auxiliary sequence, $\mathbf{Z} := (Z_1, \dots, Z_k)$. The auxiliary sequence \mathbf{Z} selects a codeword $\mathbf{G}^T \mathbf{Z}$ uniformly from the set of all codewords of the linear error correction code with generator matrix \mathbf{G} . The codeword is then additively perturbed by the enrollment biometric \mathbf{A} and the result is additively masked by the randomly generated key sequence \mathbf{K} to produce the stored data \mathbf{S} :

$$\mathbf{S} = \mathbf{A} \oplus \mathbf{G}^T \mathbf{Z} \oplus \mathbf{K}.$$

Authentication: At authentication, the system has access to the stored data \mathbf{S} and is presented with the pair (\mathbf{D}, \mathbf{L}) . The authentication procedure consists of two steps. First, syndrome decoding is performed to produce an estimate $\hat{\mathbf{W}}$ of the error vector $\mathbf{A} \oplus \mathbf{D}$ as follows:

$$\hat{\mathbf{W}} = \underset{\mathbf{W}: \mathbf{H}\mathbf{W} = \mathbf{H}(\mathbf{D} \oplus \mathbf{L} \oplus \mathbf{S})}{\text{arg min}} d(\mathbf{W}),$$

where $d(\cdot)$ is the Hamming weight. If $\mathbf{L} = \mathbf{K}$, the masking effect of the key is canceled out and the syndrome decoding procedure is then operationally equivalent to the optimal channel decoding of the codeword $\mathbf{G}^T \mathbf{Z}$ when corrupted by $\mathbf{A} \oplus \mathbf{D}$. Second, given $\hat{\mathbf{W}}$, an estimate $\hat{\theta}$ of θ is made as

$$d(\hat{\mathbf{W}}) \underset{\hat{\theta}=0}{\overset{\hat{\theta}=1}{\leq}} \tau n s \quad (1)$$

If $\hat{\theta} = 1$ the decision is made that the biometric \mathbf{A} and the probe \mathbf{D} are close enough (the estimate of this distance is the weight of $\hat{\mathbf{W}}$) that access should be granted.

We make the following assumptions about system operating parameters. Recall that if $\mathbf{L} = \mathbf{K}$ the decoding process is the same as optimal channel decoding. This implies that if the rate of the error correcting code (specified by the choice of \mathbf{H}) is below the *channel capacity* of the binary symmetric channel (BSC) with crossover probability τ , BSC(τ), then the estimate $\hat{\mathbf{W}}$ will equal $\mathbf{A} \oplus \mathbf{D}$ with high probability. Our first assumption is thus that the rate $R = k/n$ of the code \mathbf{G} satisfies

$$R = k/n < 1 - h_b(\tau),$$

where $1 - h_b(\tau)$ is the BSC(τ) channel capacity and $h_b(p) := -p \log_2 p - (1-p) \log_2 (1-p)$ is the binary entropy function. Second, we require τ to be larger than p but smaller than 0.5, i.e., $0.5 > \tau > p$. Recall that p is the noise parameter of the probe channel, the BSC(p). With this relation between p and τ we write

$$R = k/n < 1 - h_b(\tau) < 1 - h_b(p),$$

or, equivalently,

$$\frac{m}{n} > h_b(\tau) > h_b(p).$$

In many practical realizations of fuzzy commitment the threshold test (1) is replaced with a hash check. Namely, in order to verify whether the random vector $\mathbf{G}^T \mathbf{Z}$ has been recovered exactly, a cryptographic hash of $\mathbf{G}^T \mathbf{Z}$ (alternately of \mathbf{Z}) is also stored at the access control device. This stored

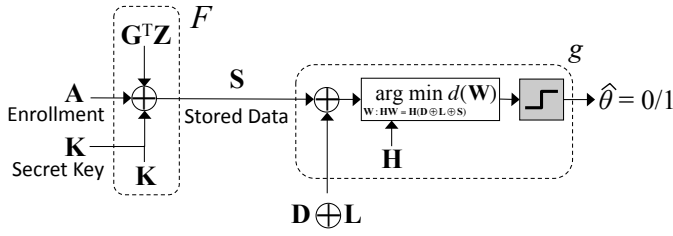


Fig. 3. A two-factor fuzzy commitment system stores the bitwise XOR of a randomly generated codeword of a linear error correcting code, the enrollment biometric, and a randomly generated secret key.

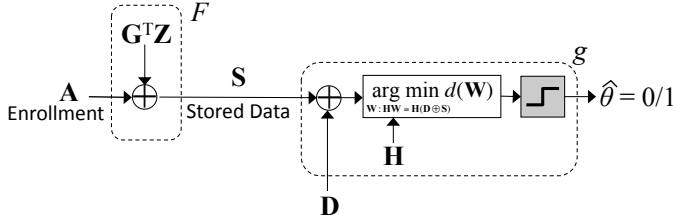


Fig. 4. A keyless fuzzy commitment system stores the bitwise XOR of a randomly generated codeword of a linear error correcting code and the enrollment biometric.

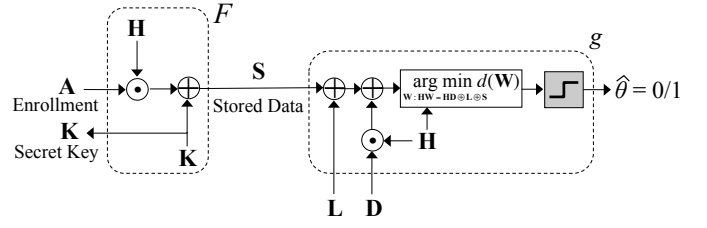


Fig. 5. A two-factor secure sketch system stores the bitwise XOR of the syndrome vector of a linear error correcting code generated by the enrollment biometric and a randomly generated secret key.

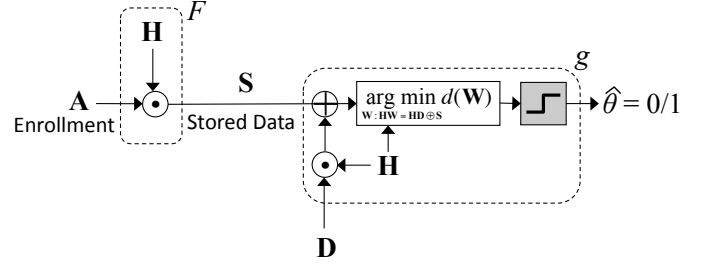


Fig. 6. A keyless secure sketch system stores the syndrome vector of a linear error correcting code generated by the enrollment biometric.

hash would need to match the hash of the $\mathbf{D} \oplus \mathbf{L} \oplus \mathbf{S} \oplus \hat{\mathbf{W}}$ for access to be granted. However, cryptographic hashes are not equivalent to the threshold test (1) since they are not information theoretically secure as noted in Section II. Thus, in the systems analyzed in this work, we do not use cryptographic hashes and, instead, rely on the threshold test (1).

B. Secure Sketch Systems based on ECC

We now introduce the second family of biometric storage systems studied, called *secure sketch* systems. While, as was the case for fuzzy commitment, there are other ways to develop a secure sketch, we concentrate on secure sketches implemented using linear error correcting codes. The baseline two-factor secure sketch scheme is diagrammed in Figure 5 and the keyless variant in Figure 6. Following the notation of Section III-A we denote by \mathbf{H} the $m \times n$ parity check matrix of a binary $[n, k]$ linear error correcting code with $m = n - k$.

Enrollment: The enrollment procedure first generates the key sequence $\mathbf{K} := (K_1, \dots, K_m)$ as an independent i.i.d. Bernoulli(0.5) sequence. The stored data \mathbf{S} is the length- m syndrome $\mathbf{H}\mathbf{A}$ of enrollment biometric feature vector masked by the key,

$$\mathbf{S} = \mathbf{H}\mathbf{A} \oplus \mathbf{K}.$$

Authentication: The authentication procedure performs syndrome decoding to produce an estimate $\hat{\mathbf{W}}$ of $\mathbf{A} \oplus \mathbf{D}$ as

$$\hat{\mathbf{W}} = \arg \min_{\mathbf{W}: \mathbf{H}\mathbf{W} = \mathbf{H}\mathbf{D} \oplus \mathbf{L} \oplus \mathbf{S}} d(\mathbf{W}).$$

The authentication decision is made using threshold test

$$d(\hat{\mathbf{W}}) \underset{\hat{\theta}=0}{\overset{\hat{\theta}=1}{\leq}} \tau n.$$

The assumptions on the values of τ and the coding rate R of the ECC are identical to those made in Section III-A. In

practical implementations, cryptographic hashes are often also stored and used to verify the correctness of the syndrome decoding procedure. However, for the reasons already discussed in the context of fuzzy commitment, we do not employ cryptographic hashes in our systems.

C. Equivalence of Fuzzy Commitment and Secure Sketch

We now develop an equivalence between the properties of the fuzzy commitment and secure sketch schemes presented in the previous two subsections. We show the conceptual equivalence between the two architectures and derive expressions for the performance metrics defined in Section II-B, showing the performance equivalence.

The encoding and decoding similarities and differences between the keyed and keyless variants of fuzzy commitment and secure sketch systems become apparent by examining Figs. 3–6. Both systems use a linear error correction code to generate the stored data from the biometric template (with or without a key), and the decoders of both systems involve finding the minimum weight codeword. Reviewing the decoding procedures of fuzzy commitment and secure sketch one sees that the procedures are nearly identical. The authentication decision is determined by whether or not $\hat{\mathbf{W}}$, the lowest Hamming weight sequence in a given coset, has Hamming weight greater or less than τn . The coset is specified by its syndrome and the only difference between the systems is how this syndrome is computed.

In the two-factor secure sketch system, the syndrome is specified as

$$\begin{aligned} q_{SS}(\mathbf{D}, \mathbf{L}, \mathbf{S}) &= \mathbf{H}\mathbf{D} \oplus \mathbf{L} \oplus \mathbf{S} \\ &= \mathbf{H}(\mathbf{A} \oplus \mathbf{D}) \oplus \mathbf{K} \oplus \mathbf{L} \end{aligned} \quad (2)$$

In the two-factor fuzzy commitment system, the syndrome is

specified as

$$\begin{aligned} q_{FC}(\mathbf{D}, \mathbf{L}, \mathbf{S}) &= \mathbf{H}(\mathbf{D} \oplus \mathbf{L} \oplus \mathbf{S}) \\ &= \mathbf{H}(\mathbf{A} \oplus \mathbf{D}) \oplus \mathbf{H}\mathbf{G}^T \mathbf{Z} \oplus \mathbf{H}(\mathbf{K} \oplus \mathbf{L}) \\ &= \mathbf{H}(\mathbf{A} \oplus \mathbf{D}) \oplus \mathbf{H}(\mathbf{K} \oplus \mathbf{L}) \end{aligned} \quad (3)$$

The decision for $\hat{\theta}$ is a deterministic function of the syndrome, defined identically for both systems.

In both systems, during the authentication of the legitimate user, where $\mathbf{D} = \mathbf{B}$ and $\mathbf{L} = \mathbf{K}$, the computed syndrome is identical and equal to $\mathbf{H}(\mathbf{A} \oplus \mathbf{B})$. Note that this is true of both keyed and keyless variants of the systems. Thus, the distribution of $\hat{\theta}$ given $\theta = 1$ is identical for both types of systems and thus the FRR is identical.

In determining the FAR – the case of an attack by an uninformed adversary – the input vectors $(\mathbf{D}, \mathbf{L}) = (\mathbf{C}, \mathbf{J})$ can have an arbitrary joint distribution, but must be independent of the pair (\mathbf{A}, \mathbf{K}) . Regardless of the distribution of (\mathbf{C}, \mathbf{J}) , the syndrome in both systems is i.i.d. Bernoulli(0.5), since \mathbf{A} is assumed to be an independent i.i.d. Bernoulli(0.5) sequence and \mathbf{H} has full row rank (cf. Lemma 1 below). Since the syndromes are equal in distribution for both systems, the authentication decisions $\hat{\theta}$ are also equal in distribution for both systems, and hence the FAR performance is the same.

Determining the SAR of these systems requires consideration of scenarios when the adversary has access to \mathbf{A} , \mathbf{S} , and/or \mathbf{K} . In contrast to the scenario considered for the FAR analysis, the availability of this additional information may allow the adversary to alter the distribution of the decoding syndrome. However, as we will see in Theorem 1 below, the SAR for secure sketch and fuzzy commitment is also the same.

Before we proceed, consider the following result that will be useful in understanding and proving some of the theorems that follow:

Lemma 1 *Let \mathbf{A} be a length- n i.i.d. Bernoulli-(0.5) random vector and let \mathbf{H} and $\tilde{\mathbf{H}}$ be, respectively, $m \times n$ and $\tilde{m} \times n$ full row-rank binary matrices whose rows are linearly independent of each other. Then, for any pair of binary vectors, \mathbf{s} and $\tilde{\mathbf{s}}$, of lengths m and \tilde{m} respectively, $\Pr[\mathbf{H}\mathbf{A} = \mathbf{s} | \tilde{\mathbf{H}}\mathbf{A} = \tilde{\mathbf{s}}] = \Pr[\mathbf{H}\mathbf{A} = \mathbf{s}] = 2^{-m}$.*

The proof of this lemma appears in Appendix A. Note that, since the channel codes are assumed to operate at a rate $R = k/n = 1 - m/n$ which is below capacity they have a positive error exponent $E(R) > 0$. This means that the probability of decoding error when using these codes on a BSC- p is bounded as

$$P_e \leq 2^{-nE(R)},$$

where

$$E(R) = \max_{\rho: 0 \leq \rho \leq 1} -\log \left\{ 2 \left[\frac{1}{2} \left((1-p)^{\frac{1}{1+\rho}} + p^{\frac{1}{1+\rho}} \right) \right]^{1+\rho} \right\} - \rho R.$$

It is well known that codes exist that satisfy these properties [20]. In anticipation of Theorem 1, note that the KL divergence between two Bernoulli distributions, Bernoulli(q) and

Bernoulli(p) is defined as

$$D(q||p) := q \log_2 \frac{q}{p} + (1-q) \log_2 \frac{1-q}{1-p}.$$

Theorem 1 *The FRR and FAR of both keyed and keyless variants of fuzzy commitment and secure sketch is the same and is bounded as*

- (i) $P_{FR} \leq 2^{-nD(\tau||p)} + 2^{-nE(R)}$,
- (ii) $P_{FA} \leq 2^{-n(1-R-h_b(\tau))}$.

The SAR of the two-factor (keyed) fuzzy commitment and secure sketch schemes for various cases of data exposure are identical and given by

- (iii) $P_{SA}(\mathbf{K}) = P_{FA}$,
- (iv) $P_{SA}(\mathbf{A}) = P_{FA}$,
- (v) $P_{SA}(\mathbf{S}) = P_{SA}(\mathbf{A}, \mathbf{K}) = P_{SA}(\mathbf{A}, \mathbf{S}) = P_{SA}(\mathbf{S}, \mathbf{K}) = P_{SA}(\mathbf{A}, \mathbf{S}, \mathbf{K}) = 1$.

The SAR of the keyless fuzzy commitment and secure sketch schemes for various cases of data exposure are identical and given by

- (vi) $P_{SA}(\mathbf{S}) = P_{SA}(\mathbf{A}) = P_{SA}(\mathbf{A}, \mathbf{S}) = 1$.

Please refer to Appendix B for the proof of the theorem. In parts (i) and (ii) the theorem characterizes exponentially decaying upper bounds on the FRR and FAR, and hence also lower bounds on the exponents. In order to obtain these exponentially decaying bounds, the operating parameters must satisfy the previously listed assumptions, that is, $0.5 > \tau > p$ and $m/n > h_b(\tau)$. Note that for all of our systems, knowledge of the stored data \mathbf{S} drastically improves the ability of the adversary to gain access. For all of our systems, the SAR is equal to one for an adversary enhanced with the knowledge of \mathbf{S} , cf. parts (v) and (vi) above. This is because, as is formalized in the proof, an adversary with knowledge of \mathbf{S} can gain access by choosing (\mathbf{C}, \mathbf{J}) based on knowledge of \mathbf{S} so that the decoding coset contains a low-weight error sequence with probability one. In fact, this limitation is not unique to ECC-based systems as the following theorem shows.

Theorem 2 *For any two-factor system,*

- (i) $P_{SA}(\mathbf{S}) \geq 1 - P_{FR}$.

If for every $\mathbf{S} \in \mathcal{S}$, there exist \mathbf{D}, \mathbf{L} such that $g(\mathbf{D}, \mathbf{L}, \mathbf{S}) = 1$, then

- (ii) $P_{SA}(\mathbf{S}) = 1$.

The proof appears in Appendix C.

Fuzzy commitment and secure sketch also have identical privacy leakage as demonstrated by the following theorem.

Theorem 3 *In the two-factor fuzzy commitment and secure sketch systems, the privacy leakage of \mathbf{A} from \mathbf{S} , from \mathbf{K} , or from (\mathbf{S}, \mathbf{K}) is, respectively,*

- (i) $I(\mathbf{A}; \mathbf{K}) = 0$,
- (ii) $I(\mathbf{A}; \mathbf{S}) = 0$,
- (iii) $I(\mathbf{A}; \mathbf{S}, \mathbf{K}) = m = n(1-R) > 0$.

In the keyless variant of fuzzy commitment and secure sketch the privacy leakage of \mathbf{A} from \mathbf{S} is

- (iv) $I(\mathbf{A}; \mathbf{S}) = m = n(1-R) > 0$.

The proof of this theorem is given in Appendix D. From an authentication perspective, it is interesting that the additional independent source of randomness \mathbf{Z} in fuzzy commitment based systems does not improve the privacy leakage properties in comparison to secure sketch based systems where such randomness is unavailable. It is also worth noting here that leakage of the stored data allows the adversary to authenticate with probability one, but reveals at most m out of n biometric bits. Thus, a security compromise does not imply a privacy compromise.

The fuzzy commitment and secure sketch systems are equivalent in terms of many performance metrics but they differ in terms of storage and key length requirements. The fuzzy commitment system requires n bits to store the data since $H(\mathbf{S}) = n$. It also uses an n -bit key to mask the stored data in the two-factor variant. On the other hand, secure sketch system requires only m bits for storage since $H(\mathbf{S}) = m$ due to the fact that only the syndrome of \mathbf{A} is being stored. Similarly, it also uses only an m -bit key to mask the stored data in the two-factor variant.

IV. LINKAGE RESISTANCE AND REVOCABILITY PROPERTIES

In this section we consider two desirable properties for secure biometrics – revocability and resistance to linkage attacks – and study them in the context of noisy enrollments at multiple access control devices. We will only consider two-factor systems in this section. Although the results to be presented apply equally to both secure sketch and fuzzy commitment based systems, proofs will be provided only for secure-sketch based systems since the two types of systems are performance-equivalent, as was discussed in Section III-C.

Revocability is the ability to tolerate partial compromises of data. By *partial* compromise we mean that, in a two-factor access control system, either the key or the stored data has been revealed to the adversary, but *not* both. On the other hand, we say that a two-factor system is *fully* compromised if both the key and the stored data have been revealed to the adversary. A secure biometric is said to be revocable if, given knowledge of a partial compromise, the user or a system administrator can delete certain data and establish a new enrollment based on the same biometric without any loss in privacy or authentication performance.

Linkage attacks can occur in situations where the same biometric is used to enroll in multiple biometric systems, e.g., on several access control devices. If an adversary compromises a subset of the devices, the compromised data can be used to attack the remaining devices. The compromised data can both leak information about the underlying biometric and can be exploited to mount a successful attack, i.e., gaining unauthorized access, on one of the remaining devices.

A. Performance Measures for Multiple Biometric Systems

We now present our model for parallel enrollment across multiple biometric systems. We assume that the biometric in question has been enrolled in u systems. Each of the u biometric systems has an enrollment vector. These vectors,

\mathbf{A}_i , $i \in \{1, \dots, u\}$, are related in a conditionally independent manner to a common underlying biometric \mathbf{A}_0 according to the measurement model in Section II. In other words,

$$P_{\mathbf{A}_i|\mathbf{A}_0}(\mathbf{a}_i|\mathbf{a}) = (1 - p_i)^{n-d_H(\mathbf{a}_i,\mathbf{a})} p_i^{d_H(\mathbf{a}_i,\mathbf{a})}$$

where $p_i \in [0, 0.5)$, all vectors are binary, and $d_H(\cdot, \cdot)$ is the Hamming distance between its arguments. For convenience, we define $p_0 = 0$. Encoding and decoding functions $\{F_i(\cdot), g_i(\cdot)\}_{i=1}^u$ are paired and need not be identical for all systems. At enrollment, each system $i \in \{1, \dots, u\}$ observes \mathbf{A}_i , and the stored data and key for system i are generated as $(\mathbf{S}_i, \mathbf{K}_i) = F_i(\mathbf{A}_i)$. The joint distribution across the u systems is given by

$$P_{\mathbf{S}^u, \mathbf{K}^u, \mathbf{A}_0}(\mathbf{s}^u, \mathbf{k}^u, \mathbf{a}) = P_{\mathbf{A}_0}(\mathbf{a}) \prod_{i=1}^u P_{\mathbf{S}_i, \mathbf{K}_i|\mathbf{A}_0}(\mathbf{s}_i, \mathbf{k}_i|\mathbf{a}), \quad (4)$$

where

$$P_{\mathbf{S}_i, \mathbf{K}_i|\mathbf{A}_0}(\mathbf{s}_i, \mathbf{k}_i|\mathbf{a}) = \sum_{\mathbf{a}_i} \Pr [F_i(\mathbf{a}_i) = (\mathbf{s}_i, \mathbf{k}_i)] P_{\mathbf{A}_i|\mathbf{A}_0}(\mathbf{a}_i|\mathbf{a}),$$

and \mathbf{S}^u and \mathbf{K}^u are, respectively, the u -tuples of stored data and key vectors.

Recall from the discussion of Section II (cf. Figure 2) that the legitimate user of system i will try to authenticate using $(\mathbf{B}, \mathbf{K}_i)$ while an adversary will use some (\mathbf{C}, \mathbf{J}) . The crossover probability of system- j 's probe channel will be denoted by $\alpha_j \in [0, 0.5)$. The FRR and FAR are, respectively, given by

$$\begin{aligned} P_{FR}(i) &:= \Pr [g_i(\mathbf{B}, \mathbf{K}_i, \mathbf{S}_i) = 0], \\ P_{FA}(i) &:= \max_{\mathbf{C}, \mathbf{J}} \Pr [g_i(\mathbf{C}, \mathbf{J}, \mathbf{S}_i) = 1], \end{aligned}$$

which are the same as the definitions for a single system in isolation.

In contrast, the existence of multiple systems necessitates the generalization of the definition of SAR, in order to account for compromises across multiple biometric systems. Expanding upon the framework of Section II, we define \mathcal{V} to be a subset of $\{\mathbf{S}_1, \mathbf{K}_1, \mathbf{S}_2, \mathbf{K}_2, \dots, \mathbf{S}_u, \mathbf{K}_u\}$. Equivalently we write $\mathcal{V} = \cup_{i=1}^u \mathcal{V}_i$ where $\mathcal{V}_i \subseteq \{\mathbf{S}_i, \mathbf{K}_i\}$, possibly the empty set. Also, to be able to study the effect of compromised enrollment biometrics, we define the set \mathcal{A} to be a subset of $\{\mathbf{A}_0, \mathbf{A}_1, \mathbf{A}_2, \dots, \mathbf{A}_u\}$.

Given knowledge of \mathcal{V} and \mathcal{A} by an adversary, the SAR against system i is

$$P_{SA}(i, \mathcal{V}, \mathcal{A}) = \max_{\mathbf{C}, \mathbf{J}|\mathcal{V}, \mathcal{A}} \Pr [g_i(\mathbf{C}, \mathbf{J}, \mathbf{S}_i) = 1].$$

B. Privacy Leakage Across Multiple Systems

In this section we give a tight characterization of privacy leakage, i.e., the amount of information leaked about the user's biometric when some subset of the stored data is compromised. In the analysis that follows, we assume that all u biometric systems are secure sketch-based systems with parity check matrices $\mathbf{H}_1, \dots, \mathbf{H}_u$ which may have different row-sizes but the same column-size. As we have already proved the equivalence between secure sketch and fuzzy commitment in

System	Keyless		Two-factor	
	Fuzzy Commitment	Secure Sketch	Fuzzy Commitment	Secure Sketch
False Rejection Rate	$P_{FR} \leq 2^{-nD(\tau p)} + 2^{-nE(R)}$			
False Acceptance Rate	$P_{FA} \leq 2^{-n(1-R-h_b(\tau))}$			
Successful Attack Rate	$P_{SA}(\mathbf{A}) = P_{SA}(\mathbf{S}) = 1$		$P_{SA}(\mathbf{A}) = P_{SA}(\mathbf{K}) = P_{FA}$ $P_{SA}(\mathbf{S}) = P_{SA}(\mathbf{A}, \mathbf{K}) = 1$	
Privacy Leakage	$I(\mathbf{A}; \mathbf{S}) = m$		$I(\mathbf{A}; \mathbf{K}) = 0, I(\mathbf{A}; \mathbf{S}, \mathbf{K}) = m$ $I(\mathbf{A}; \mathbf{S}) = 0$	
Storage Requirements	$H(\mathbf{S}) = n$	$H(\mathbf{S}) = m$	$H(\mathbf{S}) = H(\mathbf{K}) = n$	$H(\mathbf{S}) = H(\mathbf{K}) = m$

TABLE I
SUMMARY AND COMPARISON OF SYSTEM PERFORMANCE

Section III, the results derived for multiple secure sketch-based systems immediately extend to multiple fuzzy commitment-based biometric systems. In other words, statements about the parity check matrices \mathbf{H}_i can be appropriately modified into similar statements about the generator matrices \mathbf{G}_i used in fuzzy commitment-based systems.

While deriving the privacy leakage, we also state simplifications for a number of interesting special cases. In particular we consider both the “noiseless” enrollment case where $\mathbf{A}_0 = \mathbf{A}_1 = \dots = \mathbf{A}_u$ and the “identical” enrollment function case where all systems use the same ECC, i.e., $\mathbf{H}_1 = \dots = \mathbf{H}_u$. We also write $\text{rank}(\mathbf{H}_1, \dots, \mathbf{H}_j)$ to denote the rank of $[\mathbf{H}_1^T, \dots, \mathbf{H}_j^T]$.

Our main result connects the amount of information leakage with an easily-characterized rank property of the parity check matrices of the compromised systems.

Theorem 4 *Given the enrollment model of (4), assume, without loss of generality, an ordering of the systems such that for some index l , $0 \leq l \leq u$, $\mathcal{V}_i = \{\mathbf{S}_i, \mathbf{K}_i\}$ for all $i \in [1, l]$ and $\mathcal{V}_i \subset \{\mathbf{S}_i, \mathbf{K}_i\}$ for all $i > l$. Then, the information about \mathbf{A}_0 leaked by $\mathcal{V} = \cup_{i=1}^u \mathcal{V}_i$ is*

$$I(\mathbf{A}_0; \mathcal{V}) = \begin{cases} 0 & \text{if } l = 0 \\ I(\mathbf{A}_0; \mathbf{H}_1 \mathbf{A}_1, \dots, \mathbf{H}_l \mathbf{A}_l) & \text{else} \end{cases}.$$

Additionally:

(i) *In general,*

$$I(\mathbf{A}_0; \mathcal{V}) \leq \text{rank}(\mathbf{H}_1, \dots, \mathbf{H}_l).$$

(ii) *For noiseless, non-identical enrollment functions,*

$$I(\mathbf{A}_0; \mathcal{V}) = \text{rank}(\mathbf{H}_1, \dots, \mathbf{H}_l).$$

while for the identical enrollment function case with $l \geq 1$, we have

$$I(\mathbf{A}_0; \mathcal{V}) = \text{rank}(\mathbf{H}_1).$$

The proof of this theorem is given in Appendix E. Importantly, this theorem tells us that information about the underlying biometric is leaked only if there is at least one fully compromised system (i.e., $l > 0$). Hence, unless both

the key and stored data of a particular system have been compromised, that system can be revoked by erasing the uncompromised data (e.g., the key if the stored data has been leaked). The theorem indicates that biometric measurement noise can only help mask the private data. To see this, consider the case $l > 0$ and note that if the enrollment noise is high enough, the information between \mathbf{A}_0 and $\mathbf{H}_1 \mathbf{A}_1, \dots, \mathbf{H}_l \mathbf{A}_l$ can be quite small, certainly smaller than when there is no enrollment noise. This last statement follows from the information processing inequality which tells us that the privacy leakage when enrollments are noisy is upper bounded by the privacy leakage when enrollments are noiseless.

Part (i) also tells us that the privacy leakage depends on the rank of the matrix formed by stacking the parity check matrices of the fully compromised systems. We term this the “collective” rank of the set in question. The collective rank is at most equal to the sum of the ranks of the individual parity check matrices and will be strictly less if there is linear dependence between the rows of the matrices. Further, as part (ii) tells us, in the special case of noiseless enrollments we can make an exact statement about privacy leakage in terms of collective rank. Finally, in the special case of noiseless enrollments and identical enrollment functions, the first fully compromised system leaks all the information there is to be leaked about the underlying biometric.

We can sketch a candidate design rule arising from these results. To obtain a set of systems that minimize the privacy leakage in the face of the compromise of some subset of the stored data and keys, the collection of parity check matrices should be designed to maximize the linear dependencies across the matrices. Of course, at the same time the matrices must individually specify good error correcting codes, else the false rejection rate would be too high. But, such minimal privacy leakage comes at a cost. To achieve minimum collective rank, one should simply use the same parity check matrix for each system. However, as we discuss in the next subsection, this choice makes the remaining uncompromised systems more vulnerable to false authentications. Thus, if we design the multiple systems to minimize privacy leakage, we pay a price

in terms of the security of the individual systems.

C. Authentication Attacks with Multiple Systems

In situations where some subset of systems based on the same biometric have been compromised, an attacker may be able to use the compromised data to enhance his ability to authenticate falsely. The following theorem states results on the successful attack rates for our two-factor secure biometric systems. The theorem is proved in Appendix F.

Theorem 5 *Let u noisy, non-identical enrollments be generated for a secure two-factor biometric system (fuzzy commitment or secure sketch). Consider any system $j \in \{1, \dots, u\}$.*

- (i) *If either $\mathbf{S}_j \in \mathcal{V}_j$ or both $\mathbf{A}_j \in \mathcal{A}$ and $\mathbf{K}_j \in \mathcal{V}_j$, then*

$$P_{SA}(j, \mathcal{V}, \mathcal{A}) = 1.$$

- (ii) *If $\mathbf{K}_j \in \mathcal{V}_j$ and for some $i \neq j$, $\mathbf{A}_i \in \mathcal{A}$ and $p_i \leq p_j$ then*

$$P_{SA}(j, \mathcal{V}, \mathcal{A}) \geq 1 - P_{FR}(j).$$

- (iii) *If $\mathcal{V}_j = \{\}$, the null set, then*

$$P_{SA}(j, \mathcal{V}, \mathcal{A}) = P_{FA}(j).$$

- (iv) *If $\mathbf{S}_j \notin \mathcal{V}_j$, $\mathcal{A} = \{\}$, and $\mathcal{V}_i \subset \{\mathbf{S}_i, \mathbf{K}_i\}$ for each $i \neq j$ then*

$$P_{SA}(j, \mathcal{V}, \mathcal{A}) = P_{FA}(j).$$

In part (i), an adversary who has access to the stored data of the target system can easily find a low-weight element of the coset corresponding to \mathbf{S}_j , yielding access with probability one as per Theorem 1(v).⁵

In part (ii), the adversary has access to the key of the system to be attacked and at least one enrollment biometric of some other system \mathbf{A}_i or the ground truth biometric \mathbf{A}_0 . In these settings we show that the adversary can use this data to imitate a probe biometric of the legitimate user and launch an authentication attack with a high probability of success.

In contrast, in parts (iii)–(iv), the adversary cannot do better than the nominal false acceptance rate. In part (iii), neither the key nor the stored data of the target system are compromised, but the \mathcal{V}_i for $i \neq j$ can be arbitrary. Then, because \mathbf{K}_j is independent of all other parts of the system, the attacker cannot improve his probability of success over that of random guessing. In part (iv), the key of the target system may be compromised, but in all other systems only a strict subset of the data is compromised (either just the stored data, just the key, or neither) and, further, no enrollment biometrics are compromised. In this situation the adversary is again not able to authenticate with probability higher than the FAR.

The following theorem considers the effect of the joint structure of the parity check matrices employed on different access control devices on the probability of successful attack. It establishes that if certain joint structure is present, the

⁵Note that in this part if only \mathbf{S}_j is leaked, but not \mathbf{K}_j , then this is a revocable scenario, i.e., the old \mathbf{K}_j can be revoked and a new key assigned. Until this is done, however, the probability of successful attack is one, as given above. Once \mathbf{K}_j is revoked, the probability of successful attack becomes $P_{FA}(j)$.

adversary can leverage this structure to improve dramatically the likelihood of being able to falsely authenticate on uncompromised systems. The theorem is proved in Appendix G.

Theorem 6 *Given the enrollment model of (4), assume the two-factor systems are ordered such that there is some index l , $1 \leq l \leq u$ such that $\mathcal{V}_i = \{\mathbf{S}_i, \mathbf{K}_i\}$ for all $i \leq l$ and $\mathcal{V}_i = \mathbf{K}_i$ for all $i > l$. Let $\mathcal{A} = \{\}$. Now, consider any system index $j \geq l + 1$.*

- (i) *For noiseless enrollments if $\text{rank}(\mathbf{H}_1, \dots, \mathbf{H}_l, \mathbf{H}_j) = \text{rank}(\mathbf{H}_1, \dots, \mathbf{H}_l)$ then*

$$P_{SA}(j, \mathcal{V}, \mathcal{A}) = 1.$$

- (ii) *For noisy enrollments if $\text{rank}(\mathbf{H}_1, \dots, \mathbf{H}_l, \mathbf{H}_j) = \text{rank}(\mathbf{H}_1, \dots, \mathbf{H}_l)$ and for all $0 \leq i \leq u$, $p_i \leq \alpha_j$, where α_j is the crossover probability of system- j 's probe channel, then*

$$P_{SA}(j, \mathcal{V}, \mathcal{A}) \geq 1 - P_{FR}(j).$$

- (iii) *If $\text{rank}(\mathbf{H}_1, \dots, \mathbf{H}_l, \mathbf{H}_j) = \text{rank}(\mathbf{H}_1, \dots, \mathbf{H}_l) + \text{rank}(\mathbf{H}_j)$ then (in either the noisy or noiseless case)*

$$P_{SA}(j, \mathcal{V}, \mathcal{A}) = P_{FA}(j).$$

The conditions in the first two parts of Theorem 6 mean that the row space of \mathbf{H}_j lies within the span of the rows of $\mathbf{H}_1, \dots, \mathbf{H}_l$. In this situation, an attacker can gain access with high probability. In contrast, if the parity check matrix \mathbf{H}_j used to define the stored data in the system under attack is linearly independent of the matrices defining the compromised systems, then the compromised data is useless in attempts to improve the successful attack rate beyond the nominal false acceptance rate of the system.

To build intuition, we study the implications of Theorem 6 through a sequence of examples. In keeping with our previous development, we consider secure sketch-based biometric systems, though the results translate to fuzzy commitment-based access control devices as well. In each example we consider three biometric systems, $u = 3$. The three enrollment matrices $\mathbf{H}_1, \mathbf{H}_2, \mathbf{H}_3$, are each of size $m \times n$ and full rank m where $n = 3m$. We consider an adversary that is trying to authenticate with respect to system #3, having gained access to all data *except* \mathbf{S}_3 , i.e., $\mathcal{V} = \{\mathbf{S}_1, \mathbf{K}_1, \mathbf{S}_2, \mathbf{K}_2, \mathbf{K}_3\}$. In some of the examples, we will find it useful to refer back to Lemma 1, which relates linear independence between the rows of the parity check matrices to statistical independence of the syndromes $\mathbf{H}_i \mathbf{A}_i$.

Example 1 (noiseless enrollments) Consider noiseless enrollments, $\mathbf{A}_0 = \mathbf{A}_1 = \mathbf{A}_2 = \mathbf{A}_3$ and $\mathbf{H}_3 = \mathbf{H}_1 \oplus \mathbf{H}_2$. In this setting, using the elements of \mathcal{V} , the adversary can calculate the stored data of the third system as $\mathbf{S}_3 = \mathbf{S}_1 \oplus \mathbf{S}_2 \oplus \mathbf{K}_1 \oplus \mathbf{K}_2 \oplus \mathbf{K}_3$. Picking $\mathbf{C} (= \mathbf{D})$ such that $\mathbf{H}_3 \mathbf{C} = \mathbf{S}_3$ and setting $\mathbf{J} = \mathbf{L} = \mathbf{0}$, the all-zeros syndrome, the adversary can force the decoder to the coset containing the all-zeros vector. Recall that the decoder looks for the lowest weight vector in the set $\mathbf{H}_3 \mathbf{D} \oplus \mathbf{S}_3 \oplus \mathbf{L}$. The probability of success of this attack is one.

Example 2 (identical enrollment functions) Consider the setting where $\mathbf{H}_1 = \mathbf{H}_2 = \mathbf{H}_3$. If enrollments are noiseless then, e.g., $\mathbf{S}_3 = \mathbf{S}_1 \oplus \mathbf{K}_1 \oplus \mathbf{K}_3$ and the attack of Example 1 works, allowing the adversary to access system #3 with probability one. In fact, compromising the stored data and key of any single system will allow an attacker to access any other system whose key is compromised with probability one. If enrollments are noisy but $p_1 = p_2 = p_3$ then $\mathbf{S}_1 \oplus \mathbf{K}_1$ specifies a coset that contains a vector close to \mathbf{A}_0 . Pick *any* element of this coset as \mathbf{D} and use \mathbf{K}_3 for \mathbf{L} . These choices will yield the same probability of successful attack as a legitimate probe generated from \mathbf{A}_0 , i.e., at least $1 - P_{FR}$.

Example 3 (linearly independent enrollment functions) Now consider the case when the rows of \mathbf{H}_1 , the rows of \mathbf{H}_2 , and the rows of \mathbf{H}_3 are all linearly independent of one another. Then, by Lemma 1, whether or not enrollments are noisy, the information about the biometric leaked by the compromised data is independent from \mathbf{S}_3 . Hence, the compromised data does not enhance the adversary's ability to authenticate falsely.

Example 3 suggests that a cross-system design of the codes, i.e., $\mathbf{H}_1, \dots, \mathbf{H}_u$, that minimizes the linear dependence between parity check matrices can obviate the danger of linkage attacks. However, it is not always possible to design fully independent parity check matrices while maintaining the desired full rank of each. This is due to dimensionality restrictions. In the examples, $m = n/3$. Thus, if we added another biometric system, i.e., $u = 4$, maintaining full linear independence is not possible.

Example 4 (partially linearly dependent enrollment functions) Theorem 6 considers the two extreme cases of linear dependence between the parity check matrix \mathbf{H}_j of the system under attack and those of the compromised systems, $\mathbf{H}_1, \dots, \mathbf{H}_l$. Full linear dependence is considered in parts (i) and (ii) of the theorem, and full linear independence in part (iii). In this example we consider an intermediate scenario of *partial* linear dependence.

In particular, let $\mathbf{H}_a, \mathbf{H}_b, \mathbf{H}_c, \mathbf{H}_d$ be full-rank $m/2 \times n$ matrices where all of the rows are linearly independent. Let $\mathbf{H}_1^T = [\mathbf{H}_a^T \mathbf{H}_b^T]$, $\mathbf{H}_2^T = [\mathbf{H}_a^T \mathbf{H}_c^T]$, and $\mathbf{H}_3^T = [\mathbf{H}_a^T \mathbf{H}_d^T]$. Again let $\mathcal{V} = \{\mathbf{S}_1, \mathbf{K}_1, \mathbf{S}_2, \mathbf{K}_2, \mathbf{K}_3\}$. The first half of the vector $\mathbf{S}_3 \oplus \mathbf{K}_3$ equals $\mathbf{H}_a \mathbf{A}$, which, for noiseless enrollments, is the same as the first half of the $\mathbf{S}_1 \oplus \mathbf{K}_1$ and $\mathbf{S}_2 \oplus \mathbf{K}_2$ vectors, both of which can be calculated from the stored information. However, by Lemma 1 the second half of the $\mathbf{S}_3 \oplus \mathbf{K}_3$ vector is statistically independent of all compromised data.

We now describe a natural attack on the system described in Example 4. First note that, in the same manner as in the earlier examples, the attacker can set the first half of the syndrome arbitrarily. One attack would be to pick these $m/2$ constraints to eliminate as *few* low-weight sequences as possible. Ideally, these constraints would be picked so that, regardless of the remaining $m/2$ bits of the syndrome, each possible coset (after all m syndrome bits are set) would contain at least one low-weight sequence (i.e., a sequence with fewer than τn ones). Whether such an attack is possible depends on the

specific \mathbf{H}_a and \mathbf{H}_d matrices. One should note that low-weight sequences are not uniformly distributed over the cosets of \mathbf{H}_a . This means that even determining whether such an attack is possible for specific \mathbf{H}_a and \mathbf{H}_d matrices is likely quite computationally challenging. These considerations illustrate the difficulty of determining the SAR in these settings. At a minimum, we can say that the SAR must be at least as large as the FAR. This follows since the attacker can make the SAR equal to the FAR simply by ignoring the compromised data and setting all m syndrome bits at random.

D. Formulation of ECC Design Problem for Multiple Systems

In the previous two subsections, we analyzed information leakage and authentication attacks when an adversary has compromised multiple enrollments based on the same underlying biometric. Theorems 4 and 5 tell us that unless there are fully compromised systems, no information is leaked about the underlying biometric and there is no way to improve the probability of successful attack beyond the nominal false acceptance rate. Thus, in cases of only partial data compromise two-factor designs are secure to linkage attacks and can be revoked.

One way to view these results is from the perspective of reusability. A set of access-control systems can be thought of as a series of re-enrollments established after successive data compromise. If any one element – but not both – of the stored data \mathbf{S}_i and key \mathbf{K}_i are lost, the user can simply destroy the other and regenerate a fresh $(\mathbf{S}_{i+1}, \mathbf{K}_{i+1})$ pair. The previous, partially compromised, enrollments do not cause any privacy leakage nor do they enhance the adversary's ability to attack the newly enrolled system.

Furthermore, from Theorem 6 we learn that, in general, the effectiveness of linkage attacks depends on the joint structure of the error-correcting codes deployed. Furthermore, Examples 2–4 in particular give hints as to how the collection of systems can be jointly designed to mitigate the amount of privacy leakage or minimize the successful attack rate when some systems have been fully compromised. We observe from the examples that there is a natural tradeoff between robustness to privacy leakage and robustness to authentication attacks. Linear dependence between parity check matrices results in an increased probability of successful attack while linear independence results in increased privacy leakage. We now present a design formulation that formalizes this tradeoff.

Our objective is to design u parity-check matrices $\mathbf{H}_1, \dots, \mathbf{H}_u$, all full-rank $m \times n$ matrices to optimize certain properties. To define these properties we consider all $\binom{u}{L}$ cardinality- L subsets of the parity-check matrices. Denote the l^{th} such subset as \mathcal{S}_l for $1 \leq l \leq \binom{u}{L}$. The parameter L corresponds to the number of biometric systems that the adversary *can potentially compromise*, and the subset \mathcal{S}_l represents one set of systems that adversary may have compromised. For any subset \mathcal{S}_l , we define $\mathcal{S}_l[i]$ to be the index of the i^{th} parity check matrix in the subset. That is $1 \leq i \leq L$ and $1 \leq \mathcal{S}_l[i] \leq u$. Further (with some abuse of notation) we define $\mathbf{H}_{\mathcal{S}_l}$ to be the $Lm \times n$ matrix formed by “stacking” all matrices in the

subset into a single matrix, i.e.,

$$\mathbf{H}_{S_l} = [\mathbf{H}_{S_l[1]}^T \cdots \mathbf{H}_{S_l[L]}^T]^T, 1 \leq l \leq \binom{u}{L}.$$

We use r_l to denote the *collective rank* of the l^{th} stacked matrix defined as

$$r_l = \text{rank}(\mathbf{H}_{S_l}).$$

The collective rank is bounded by $1 \leq r_l \leq \min\{lm, n\}$ and is the amount of privacy leakage when the adversary has gained access both to the key and to the stored data of the L systems in S_l . Theorem 4 establishes that, for noiseless enrollments, the stacked rank is exactly equal to the privacy leakage, and that, for noisy enrollments, the stacked rank provides an upper bound on the privacy leakage.

Now, for each subset S_l and system $j \in \{1, \dots, u\}$, define the *residual rank* of matrix \mathbf{H}_j as

$$t_{l,j} = \text{rank}(\mathbf{H}_{S_l}, \mathbf{H}_j) - r_l.$$

Note that $t_{l,j} = 0$ if the row-space of \mathbf{H}_j is spanned by the rows of \mathbf{H}_{S_l} , which would happen automatically if $j \in S_l$. Also, $0 \leq t_{l,j} \leq m$, with equality to m if all rows of \mathbf{H}_j are linearly independent of the rows of \mathbf{H}_{S_l} . The residual rank provides a loose characterization of the systems' linkage attack resistance to authentication attacks. Consider an adversary that has compromised the keys and stored data of the enrollments of the systems in S_l . When $t_{l,j} = m$, the adversary does not benefit from a higher probability of successful attack for system j . On the other hand, when $t_{l,j} = 0$ and the key of system- j is compromised, the adversary will be able to falsely authenticate at system j with probability one if the enrollments are noiseless, and with high probability even if the enrollments are noisy. For intermediate values of $t_{l,j}$, and as was discussed in Example 4 of Section IV-C, determining the corresponding linkage resistance against authentication attacks is complicated. Thus the parameter $t_{l,j}$ is a rough measure of linkage attack resistance. However, for noiseless enrollments, $t_{l,j}$ provides a lower bound on the corresponding SAR given by

$$P_{SA}(j, \mathcal{V}) \geq 2^{-t_{l,j}},$$

where \mathcal{V} are the keys and stored data for the systems in S_l . This is because uniformly sampling from one of the $2^{t_{l,j}}$ cosets containing the enrolled biometric is always a strategy that is available to the attacker.

When designing a collection of systems, roughly speaking, minimizing r_l corresponds to reducing privacy leakage while maximizing $t_{l,j}$ corresponds to reducing the probability of successful attack. The system designer must not only choose matrices with desirable error-correcting properties but also consider the optimization of these parameters across different values of l , j , and L . One possible approach is to use the following pessimistic performance measures, r_{\max} and t_{\min} , which are respectively defined as

$$r_{\max} := \max_{1 \leq l \leq \binom{u}{L}} r_l,$$

$$t_{\min} := \min_{1 \leq l \leq \binom{u}{L}} \min_{j \in S_l^c} t_{l,j},$$

where we note that the optimizing l may not be the same for both measures. The design of a set of parity check matrices that yield low FRRs, while minimizing r_{\max} and maximizing t_{\min} appears to be a challenging avenue for future research.

V. CONCLUSIONS

In this paper, we presented a generalized framework for modeling secure biometric systems and characterizing their security and privacy properties. We conducted a detailed information-theoretic analysis of two related types of systems based on linear error correcting codes, namely secure sketch and fuzzy commitment. We also considered two variants of each scheme: keyless and keyed. The second is a two-factor scheme in which the biometric system is augmented by a secret key held on a smart card. We showed that secure sketch and fuzzy-commitment systems are equivalent in terms of the false rejection rate, false acceptance rate, successful attack rate, and privacy leakage during partial or full compromise of biometric templates and smart-card keys. We did, however, find a difference in their storage requirements with secure sketch requiring less storage.

In either keyless or two-factor schemes, compromising the stored data renders the biometric system vulnerable to attack. If the data stored on the device is lost, an adversary can gain access to the system with probability one. However, for a two-factor system the user's biometric sample remains protected (the information-theoretic privacy of the user is maintained) so long as the secret key is not compromised. In this scenario, the enrollment can be revoked and a new one established. If, however, both the stored data and the key are compromised, the two-factor scheme is no worse than a keyless scheme.

We also analyzed the information leakage and authentication performance when a user's biometric is enrolled at several access control devices. We studied the repercussions of data compromise in a subset of the systems. For two-factor schemes, the successful attack rate is no larger than the nominal false acceptance rate of the system so long as no single system suffers from a theft of *both* the stored data and smart card key. Furthermore, no information is leaked about the user's biometric in this case.

When some subset of systems is fully compromised, i.e., both the stored data and the secret key are compromised, we showed that the information leaked about the user's biometric depends on the rank of a matrix formed by stacking the parity check matrices of the compromised devices. The successful attack rate in this scenario depends on the design of the parity check matrices of the compromised devices, specifically on the number of independent rows in these matrices. We showed via examples that, while designing multiple biometric systems, there exists a fundamental tradeoff between the user's privacy (i.e., the information leaked about the underlying biometric) and the user's security (i.e., the probability that the adversary can falsely authenticate as a genuine user).

Many interesting problems remain open. Most importantly, in our opinion, is the situation of multiple fully-compromised systems. Providing the complete characterization of the tradeoff between privacy leakage and probability of successful

attack in this setting is elusive. Such a characterization would provide guidelines for the design of the parity check matrices for the constituent systems. Even with such a characterization, the joint design of parity check matrices to achieve a point on that optimum tradeoff curve will be a challenge.

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APPENDIX A PROOF OF LEMMA 1

By Bayes' Theorem

$$\begin{aligned} \Pr[\mathbf{H}\mathbf{A} = \mathbf{s} | \tilde{\mathbf{H}}\mathbf{A} = \tilde{\mathbf{s}}] &= \frac{\Pr[\mathbf{H}\mathbf{A} = \mathbf{s}, \tilde{\mathbf{H}}\mathbf{A} = \tilde{\mathbf{s}}]}{\Pr[\tilde{\mathbf{H}}\mathbf{A} = \tilde{\mathbf{s}}]} \\ &= \frac{\Pr\left[\begin{bmatrix} \mathbf{H} \\ \tilde{\mathbf{H}} \end{bmatrix} \mathbf{A} = \begin{bmatrix} \mathbf{s} \\ \tilde{\mathbf{s}} \end{bmatrix}\right]}{\Pr[\tilde{\mathbf{H}}\mathbf{A} = \tilde{\mathbf{s}}]}. \end{aligned}$$

Since $\tilde{\mathbf{H}}$ is full rank, all $2^{\tilde{m}}$ length- \tilde{m} possible syndrome vectors $\tilde{\mathbf{s}}$ are reachable by different choices of \mathbf{A} . Also, by the Theorem of Lagrange, all cosets are of equal size. Thus, since all realizations of \mathbf{A} are equally likely, $\tilde{\mathbf{H}}\mathbf{A}$ is uniformly distributed, i.e., $\Pr[\tilde{\mathbf{H}}\mathbf{A} = \tilde{\mathbf{s}}] = 2^{-\tilde{m}}$. Since \mathbf{H} and $\tilde{\mathbf{H}}$ are linearly independent, the matrix $[\mathbf{H}^T \tilde{\mathbf{H}}^T]$ has full rank. Therefore, by the same logic, the numerator is equal to $2^{-(m+\tilde{m})}$.

APPENDIX B PROOF OF THEOREM 1

In the paragraphs preceding the statement of Theorem 1, it was proved that the FRR and FAR of both keyed and keyless variants of fuzzy commitment and secure sketch are the same.

(i) The FRR is given by

$$P_{FR} = \Pr[d(\hat{\mathbf{W}}) > \tau n],$$

where, since for the legitimate user $\mathbf{D} = \mathbf{B}$ and $\mathbf{L} = \mathbf{K}$,

$$\hat{\mathbf{W}} = \arg \min_{\mathbf{W}: \mathbf{H}\mathbf{W} = \mathbf{H}(\mathbf{A} \oplus \mathbf{B})} d(\mathbf{W}).$$

The FRR can be bounded by

$$\begin{aligned} P_{FR} &= \Pr[d(\hat{\mathbf{W}}) > \tau n, \hat{\mathbf{W}} = \mathbf{A} \oplus \mathbf{B}] \\ &\quad + \Pr[d(\hat{\mathbf{W}}) > \tau n, \hat{\mathbf{W}} \neq \mathbf{A} \oplus \mathbf{B}] \\ &\leq \Pr[d(\mathbf{A} \oplus \mathbf{B}) > \tau n] + \Pr[\hat{\mathbf{W}} \neq \mathbf{A} \oplus \mathbf{B}]. \end{aligned}$$

The decoding procedure to produce $\hat{\mathbf{W}}$ is operationally equivalent to the optimal syndrome decoding of \mathbf{A} from the noisy version \mathbf{B} , since

$$\begin{aligned} \hat{\mathbf{W}} &= \arg \min_{\mathbf{W}: \mathbf{H}\mathbf{W} = \mathbf{H}(\mathbf{A} \oplus \mathbf{B})} d(\mathbf{W}) \\ &= \mathbf{B} \oplus \arg \min_{\mathbf{A}': \mathbf{H}\mathbf{A}' = \mathbf{H}\mathbf{A}} d(\mathbf{A}' \oplus \mathbf{B}). \end{aligned}$$

Thus, the probability that $\hat{\mathbf{W}}$ fails to recover $\mathbf{A} \oplus \mathbf{B}$ is equal to the probability of decoding error of the code, which is bounded as

$$\Pr[\hat{\mathbf{W}} \neq \mathbf{A} \oplus \mathbf{B}] \leq 2^{-nE(R)}.$$

The probability that $\mathbf{A} \oplus \mathbf{B}$ fails the threshold test can be bounded by the Chernoff-Hoeffding bound [21],

$$\Pr[d(\mathbf{A} \oplus \mathbf{B}) > \tau n] \leq 2^{-nD(\tau|p)}.$$

Combining these two bounds yields the bound on the FRR.

(ii) As discussed in the paragraphs preceding the statement of Theorem 1, in both the keyed and keyless variants of both fuzzy commitment and secure sketch systems, regardless of

the distribution of (\mathbf{C}, \mathbf{J}) , the syndrome is i.i.d. Bernoulli(0.5). Since \mathbf{H} has full row rank, this implies that all syndromes, or equivalently all cosets, are equally likely to be selected with probability 2^{-m} (there are 2^m cosets). Since P_{FA} is equal to the probability of selecting a coset whose coset-leader (the minimum Hamming weight word in the coset) has a Hamming weight not more than τn and the number of such cosets is not more than the total number of sequences in $\{0, 1\}^n$ with Hamming weight less than τn , it follows that

$$\begin{aligned} P_{FA} &\leq 2^{-m} |\{\mathbf{w} : d(\mathbf{w}) \leq \tau n\}| \\ &= 2^{-m} \sum_{i=0}^{\tau n} |\{\mathbf{w} : d(\mathbf{w}) = i\}| \\ &= 2^{-m} \sum_{i=0}^{\tau n} \binom{n}{i} \\ &\leq 2^{-m} 2^{nh_b(\tau)} \\ &= 2^{-n(\frac{m}{n} - h_b(\tau))} = 2^{-n(1-R-h_b(\tau))}, \end{aligned}$$

where second inequality above is due to [22, Lemma 8, Ch. 10] since $\tau < 0.5$.

(iii) In both of the two-factor systems, an adversary with knowledge of only \mathbf{K} submits attack vectors (\mathbf{C}, \mathbf{J}) that are independent of \mathbf{A} . Hence, the distribution of the syndrome is Bernoulli(0.5), as in the FAR analysis, and thus

$$P_{SA}(\mathbf{K}) = P_{FA}.$$

(iv) An adversary with knowledge of only \mathbf{A} , submits attack vectors (\mathbf{C}, \mathbf{J}) that are independent of \mathbf{K} . Hence again the distribution of the syndrome is Bernoulli(0.5), and thus

$$P_{SA}(\mathbf{A}) = P_{FA}.$$

(v) Recall that $q_{SS}(\mathbf{D}, \mathbf{L}, \mathbf{S}) = \mathbf{H}\mathbf{D} \oplus \mathbf{L} \oplus \mathbf{S}$ and $q_{FC}(\mathbf{D}, \mathbf{L}, \mathbf{S}) = \mathbf{H}(\mathbf{D} \oplus \mathbf{L} \oplus \mathbf{S})$. With knowledge of \mathbf{S} , an adversary can choose $\mathbf{C} = \mathbf{0}$ and $\mathbf{J} = \mathbf{S}$ to make $q_{SS}(\mathbf{D}, \mathbf{L}, \mathbf{S}) = q_{FC}(\mathbf{D}, \mathbf{L}, \mathbf{S}) = \mathbf{0}$ so that $\hat{\mathbf{W}} = \mathbf{0}$ and system authenticates the adversary. Thus,

$$P_{SA}(\mathbf{S}) = 1.$$

Since $P_{SA}(\mathcal{V}_1, \mathcal{V}_2) \geq P_{SA}(\mathcal{V}_1)$, we also have

$$P_{SA}(\mathbf{S}) = P_{SA}(\mathbf{A}, \mathbf{S}) = P_{SA}(\mathbf{K}, \mathbf{S}) = P_{SA}(\mathbf{A}, \mathbf{K}, \mathbf{S}) = 1.$$

In a similar manner, one can show that with knowledge of both \mathbf{A} and \mathbf{K} , an adversary can set the syndrome to any desired value and thus,

$$P_{SA}(\mathbf{A}, \mathbf{K}) = 1.$$

(vi) As in the proof of part (v), in the keyless versions of the fuzzy commitment and secure sketch systems, an adversary with knowledge of \mathbf{S} alone or \mathbf{A} alone can set the syndrome to a value that makes the decoder select a coset with a low-weight sequence with probability one. Hence,

$$P_{SA}(\mathbf{S}) = P_{SA}(\mathbf{A}) = 1.$$

Finally, since $P_{SA}(\mathcal{V}_1, \mathcal{V}_2) \geq P_{SA}(\mathcal{V}_1)$, we also have

$$P_{SA}(\mathbf{A}, \mathbf{S}) = 1.$$

APPENDIX C PROOF OF THEOREM 2

(i) Let $\mathcal{S}_a \subset \mathcal{S}$ denote the subset for which there exist \mathbf{D}, \mathbf{L} such that $g(\mathbf{D}, \mathbf{L}, \mathbf{S}) = 1$. If $\mathbf{S} \notin \mathcal{S}_a$, then $\hat{\theta} = 0$. Therefore, the FRR must be bounded by

$$P_{FR} \geq \Pr[\mathbf{S} \notin \mathcal{S}_a].$$

Since the adversary can gain access (with probability one) when $\mathbf{S} \in \mathcal{S}_a$, the SAR can be bounded as

$$P_{SA}(\mathbf{S}) \geq \Pr[\mathbf{S} \in \mathcal{S}_a] \geq 1 - P_{FR}.$$

(ii) If $\mathcal{S}_a = \mathcal{S}$, then the adversary can always choose \mathbf{C} and \mathbf{J} such that $\hat{\theta} = g(\mathbf{D}, \mathbf{L}, \mathbf{S}) = 1$ in order to gain access with probability one.

APPENDIX D PROOF OF THEOREM 3

In the two-factor fuzzy commitment scheme, \mathbf{A}, \mathbf{K} , and \mathbf{Z} are mutually independent Bernoulli(0.5) sequences and $\mathbf{S} = \mathbf{A} \oplus \mathbf{G}^T \mathbf{Z} \oplus \mathbf{K}$. In the two-factor secure sketch scheme, \mathbf{A} and \mathbf{K} are mutually independent Bernoulli(0.5) sequences and $\mathbf{S} = \mathbf{H}\mathbf{A} \oplus \mathbf{K}$. Thus for both two-factor schemes, \mathbf{A} and \mathbf{K} are mutually independent and so are \mathbf{A} and \mathbf{S} . This implies that

$$I(\mathbf{A}; \mathbf{S}) = I(\mathbf{A}; \mathbf{K}) = 0$$

for both two-factor fuzzy and two-factor secure sketch schemes.

For the two-factor fuzzy commitment scheme,

$$\begin{aligned} I(\mathbf{A}; \mathbf{S}, \mathbf{K}) &= H(\mathbf{S}, \mathbf{K}) - H(\mathbf{S}, \mathbf{K}|\mathbf{A}) \\ &= H(\mathbf{K}) + H(\mathbf{S}|\mathbf{K}) - H(\mathbf{K}|\mathbf{A}) - H(\mathbf{S}|\mathbf{K}, \mathbf{A}) \\ &= H(\mathbf{K}) + H(\mathbf{A} \oplus \mathbf{G}^T \mathbf{Z}) - H(\mathbf{K}) - H(\mathbf{G}^T \mathbf{Z}) \\ &= n - k = m. \end{aligned}$$

For the two-factor secure sketch scheme,

$$\begin{aligned} I(\mathbf{A}; \mathbf{S}, \mathbf{K}) &= H(\mathbf{S}, \mathbf{K}) - H(\mathbf{S}, \mathbf{K}|\mathbf{A}) \\ &= H(\mathbf{K}) + H(\mathbf{S}|\mathbf{K}) - H(\mathbf{K}|\mathbf{A}) - H(\mathbf{S}|\mathbf{A}, \mathbf{K}) \\ &= H(\mathbf{K}) + H(\mathbf{H}\mathbf{A}) - H(\mathbf{K}) - 0 \\ &= H(\mathbf{H}\mathbf{A}) = m. \end{aligned}$$

In the keyless fuzzy commitment scheme,

$$\begin{aligned} I(\mathbf{A}; \mathbf{S}) &= H(\mathbf{S}) - H(\mathbf{S}|\mathbf{A}) \\ &= H(\mathbf{A} \oplus \mathbf{G}^T \mathbf{Z}) - H(\mathbf{A} \oplus \mathbf{G}^T \mathbf{Z}|\mathbf{A}) \\ &= H(\mathbf{A}) - H(\mathbf{G}^T \mathbf{Z}) \\ &= n - k = m. \end{aligned}$$

And finally, in the keyless secure sketch scheme,

$$I(\mathbf{A}; \mathbf{S}) = H(\mathbf{S}) - H(\mathbf{S}|\mathbf{A}) = H(\mathbf{S}) = m.$$

APPENDIX E
PROOF OF THEOREM 4

To yield the main result, we show that

$$\begin{aligned} & I(\mathbf{A}_0; \mathcal{V}_1, \dots, \mathcal{V}_u) \\ & \stackrel{(a)}{=} I(\mathbf{A}_0; \mathcal{V}_1, \dots, \mathcal{V}_l) \\ & \stackrel{(b)}{=} I(\mathbf{A}_0; \mathbf{K}_1, \dots, \mathbf{K}_l, \mathbf{H}_1 \mathbf{A}_1, \dots, \mathbf{H}_l \mathbf{A}_l) \\ & \stackrel{(c)}{=} I(\mathbf{A}_0; \mathbf{H}_1 \mathbf{A}_1, \dots, \mathbf{H}_l \mathbf{A}_l). \end{aligned}$$

Each step is justified by the following arguments:

- (a) is due to the chain rule for mutual information since $(\mathcal{V}_{l+1}, \dots, \mathcal{V}_u) \perp\!\!\!\perp (\mathbf{A}_0, \mathcal{V}_1, \dots, \mathcal{V}_l)$.
- (b) since $\mathcal{V}_1, \dots, \mathcal{V}_l$ is informationally equivalent to $\mathbf{K}_1, \dots, \mathbf{K}_l, \mathbf{H}_1 \mathbf{A}_1, \dots, \mathbf{H}_l \mathbf{A}_l$. For the secure sketch system, the equivalence is immediate since for $i \in \{1, \dots, l\}$, $\mathcal{V}_i = (\mathbf{S}_i, \mathbf{K}_i) = (\mathbf{H}_i \mathbf{A}_i, \mathbf{K}_i)$. To show the information equivalence for the fuzzy commitment system, note that for $i \in \{1, \dots, l\}$, $\mathcal{V}_i = (\mathbf{A}_i \oplus \mathbf{G}_i^T \mathbf{Z}_i, \mathbf{K}_i)$. Since $(\mathbf{H}_i \mathbf{A}_i, \mathbf{K}_i)$ is a function of \mathcal{V}_i , the information processing inequality gives $I(\mathbf{A}_0; \mathcal{V}_1, \dots, \mathcal{V}_l) \geq I(\mathbf{A}_0; \mathbf{K}_1, \dots, \mathbf{K}_l, \mathbf{H}_1 \mathbf{A}_1, \dots, \mathbf{H}_l \mathbf{A}_l)$. But the information processing inequality also gives $I(\mathbf{A}_0; \mathcal{V}_1, \dots, \mathcal{V}_l) \leq I(\mathbf{A}_0; \mathbf{K}_1, \dots, \mathbf{K}_l, \mathbf{H}_1 \mathbf{A}_1, \dots, \mathbf{H}_l \mathbf{A}_l)$ since $\mathbf{A}_0 - (\mathbf{K}_1, \dots, \mathbf{K}_l, \mathbf{H}_1 \mathbf{A}_1, \dots, \mathbf{H}_l \mathbf{A}_l) - (\mathbf{K}_1, \dots, \mathbf{K}_l, \mathbf{A}_1 \oplus \mathbf{G}_1^T \mathbf{Z}_1, \dots, \mathbf{A}_l \oplus \mathbf{G}_l^T \mathbf{Z}_l)$ forms a Markov chain. This is because $\mathbf{A}_i \oplus \mathbf{G}_i^T \mathbf{Z}_i$ is a codeword that is independently chosen from the coset corresponding to $\mathbf{H}_i \mathbf{A}_i$.
- (c) is due to the chain rule for mutual information since $(\mathbf{K}_1, \dots, \mathbf{K}_l) \perp\!\!\!\perp (\mathbf{A}_0, \mathbf{H}_1 \mathbf{A}_1, \dots, \mathbf{H}_l \mathbf{A}_l)$.

To prove parts (i) and (ii) of Theorem 4, we continue as

$$\begin{aligned} & I(\mathbf{A}_0; \mathbf{H}_1 \mathbf{A}_1, \dots, \mathbf{H}_l \mathbf{A}_l) \\ & \leq I(\mathbf{A}_0; \mathbf{H}_1 \mathbf{A}_0, \dots, \mathbf{H}_l \mathbf{A}_0) \\ & = H(\mathbf{H}_1 \mathbf{A}_0, \dots, \mathbf{H}_l \mathbf{A}_0) - H(\mathbf{H}_1 \mathbf{A}_0, \dots, \mathbf{H}_l \mathbf{A}_0 | \mathbf{A}_0) \\ & = H(\mathbf{H}_1 \mathbf{A}_0, \dots, \mathbf{H}_l \mathbf{A}_0) \\ & = \text{rank}(\mathbf{H}_1, \dots, \mathbf{H}_l). \end{aligned}$$

The inequality is due to the information processing inequality and the fact that $\mathbf{A}_0 - \mathbf{H}_1 \mathbf{A}_0, \dots, \mathbf{H}_j \mathbf{A}_0 - \mathbf{H}_1 \mathbf{A}_1, \dots, \mathbf{H}_l \mathbf{A}_l$ forms a Markov chain. This inequality holds with equality for noiseless enrollments. The last equality follows from Lemma 1.

APPENDIX F
PROOF OF THEOREM 5

- (i) This is an immediate corollary of Theorem 1(v): Similar to the single system case, knowledge of \mathbf{S}_j or $(\mathbf{A}_j, \mathbf{K}_j)$ – which is sufficient to generate \mathbf{S}_j – allows the adversary to authenticate with probability one.
- (ii) The adversary can set the attack vectors \mathbf{C} to \mathbf{A}_i and \mathbf{J} to \mathbf{K}_j . The authentication attack succeeds with probability at least as large as $1 - P_{FR}(j)$ since, for $0 \leq p_i \leq p_j < 0.5$ and $0 \leq \alpha < 0.5$, the noise level between \mathbf{A}_j and \mathbf{A}_i

can only be lower than the noise level between \mathbf{A}_j and a legitimate probe biometric \mathbf{B} .

- (iii) The compromised data \mathcal{V} is independent of \mathbf{S}_j since it does not contain \mathbf{K}_j , which is independent and i.i.d Bernoulli(0.5). Hence, any attack vectors \mathbf{C} and \mathbf{J} would be independent of \mathbf{S}_j and would result in a uniformly distributed decoding syndrome $q_{SS}(\mathbf{S}_j, \mathbf{C}, \mathbf{J})$ according to (2). This results in a probability of successful attack equivalent to the probability of false accept.
- (iv) Similar to part (iii) above, the compromised data \mathcal{V} is independent of \mathbf{A}_j , which is itself i.i.d Bernoulli(0.5). Hence, any attack vectors \mathbf{C} and \mathbf{J} result in a uniformly distributed decoding syndrome $q_{SS}(\mathbf{S}_j, \mathbf{C}, \mathbf{J})$ and a probability of successful attack equal to the probability of false accept.

APPENDIX G
PROOF OF THEOREM 6

- (i) For $i \leq l$, since both \mathbf{S}_i and \mathbf{K}_i are compromised, the syndrome $\mathbf{H}_i \mathbf{A}_i$ is known to the adversary. In the case of noiseless enrollments, $\mathbf{H}_i \mathbf{A}_i = \mathbf{H}_i \mathbf{A}_0$. The linearly dependent rows of \mathbf{H}_j allow $\mathbf{H}_j \mathbf{A}_0$ to be determined as a function of the compromised data. The stored data for system j can be recovered as $\mathbf{S}_j = \mathbf{K}_j \oplus \mathbf{H}_j \mathbf{A}_j = \mathbf{K}_j \oplus \mathbf{H}_j \mathbf{A}_0$. By Theorem 5(i), the adversary can therefore falsely authenticate with system j with probability one.
- (ii) Since $\text{rank}(\mathbf{H}_1, \dots, \mathbf{H}_l, \mathbf{H}_j) = \text{rank}(\mathbf{H}_1, \dots, \mathbf{H}_l)$, each row of \mathbf{H}_j can be expressed as a linear combination of the rows of $\{\mathbf{H}_1, \dots, \mathbf{H}_l\}$. Let $\mathbf{H}_j = \mathbf{M}_j [\mathbf{H}_1^T \dots \mathbf{H}_l^T]^T$ where \mathbf{M}_j is an $m_j \times (m_1 + \dots + m_l)$ matrix of coefficients. Suppose that the attacker chooses the attack vector pair (\mathbf{C}, \mathbf{J}) , cf. Figure 2, such that $\mathbf{H}_j \mathbf{C} = \mathbf{M}_j [(\mathbf{H}_1 \mathbf{A}_1)^T \dots (\mathbf{H}_l \mathbf{A}_l)^T]^T$ (this can always be done) and $\mathbf{J} = \mathbf{K}_j$. Then, the syndrome formed in the authentication (decoding) step of the j -th two-factor secure sketch system would be

$$\begin{aligned} \mathbf{H}_j \mathbf{C} + \mathbf{H}_j \mathbf{A}_j & = \mathbf{H}_j \mathbf{C} + \mathbf{M}_j [(\mathbf{H}_1 \mathbf{A}_j)^T \dots (\mathbf{H}_l \mathbf{A}_j)^T]^T \\ & = \mathbf{M}_j [(\mathbf{H}_1 (\mathbf{A}_1 + \mathbf{A}_j))^T \dots (\mathbf{H}_l (\mathbf{A}_l + \mathbf{A}_j))^T]^T. \end{aligned}$$

If instead $\mathbf{D} = \mathbf{B}_j$, where \mathbf{B}_j is a legitimate probe vector for system- j , and $\mathbf{L} = \mathbf{K}_j$, then the syndrome formed in the authentication (decoding) step of the j -th two-factor secure sketch system would be

$$\mathbf{H}_j (\mathbf{B}_j + \mathbf{A}_j) = \mathbf{M}_j [(\mathbf{H}_1 (\mathbf{B}_j + \mathbf{A}_j))^T \dots (\mathbf{H}_l (\mathbf{B}_j + \mathbf{A}_j))^T]^T$$

- Since for all $0 \leq i \leq u$, the enrollment channel crossover probability $p_i \leq \alpha_j$, the probe channel crossover probability, each \mathbf{A}_i is a less “noisy” version of \mathbf{A}_0 than \mathbf{B}_j . Thus, the probability of system- j rejecting the specified attack vectors cannot be more than the probability that a legitimate probe vector \mathbf{B}_j is rejected (given by $P_{FR}(j)$). Thus the authentication attack will succeed with a probability that is at least $1 - P_{FR}(j)$.
- (iii) When the rows of \mathbf{H}_j are linearly independent, the syndrome $\mathbf{S}_j = \mathbf{H}_j \mathbf{A}_j$ is independent of the compromised data due to Lemma 1. Another way of seeing this is to

consider the authentication procedure in Section III-B. Using similar notation, the adversary seeks a $\hat{\mathbf{W}}$ such that

$$\hat{\mathbf{W}} = \arg \min_{\mathbf{W}: \mathbf{H}_j \mathbf{W} = \mathbf{H}_j \mathbf{D} \oplus \mathbf{S}_j} d(\mathbf{W}).$$

where the adversary synthesizes \mathbf{D} as a function of $\mathbf{H}_i \mathbf{A}_i$, $i = 1, 2, \dots, l$. But, since the rows of \mathbf{H}_j are linearly independent of the rows of $\mathbf{H}_1, \mathbf{H}_2, \dots, \mathbf{H}_l$, the decoding syndrome \mathbf{S}_j of the target system remains independent and uniformly distributed for any choice of \mathbf{D} made by the adversary based on the compromised data. Hence, the probability of successful attack is no larger than the false acceptance rate.