Distributed Opportunistic Scheduling With Two-Level Channel Probing

Chandrashekhar Thejaswi P.S., Junshan Zhang, Vincent Poor

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***Index Terms*—Opportunistic scheduling, channel probing, optimal stopping theory, threshold policy.**

I. Introduction

Channel-aware scheduling has recently emerged as a promising technique to harness the rich diversities inherent in wireless networks. In channel-aware scheduling, a joint physical layer (PHY)/medium access control (MAC) optimization is utilized to improve network throughput by scheduling links with good channel conditions for data transmissions [1], [6], [8], [12]. While most existing studies in the literature focus on centralized scheduling (see, e.g., [3], [5], [6], [7], [8], [12]), some initial steps have been taken by the authors to develop distributed opportunistic scheduling (DOS) to reap multiuser diversity and time diversity in wireless ad-hoc networks [13], [14].

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The DOS framework considers an ad-hoc network in which many links contend for the same channel using random access, i.e. carrier-sense multiple-access (CSMA). However, random access protocols provide no guarantee on that a successful channel contention is necessarily attained by a link with good channel condition. From a holistic perspective, a successful link with a poor channel condition should forgo its data transmission, and let all links re-contend for the channel. This is because after further channel probing, it is more likely for a link with a better channel condition to take the channel, yielding possible higher throughput. In this way, multiuser diversity across links and time diversity across time can be exploited in a joint manner. However, each channel probing incurs a cost of the contention time. The desired tradeoff between the throughput gain from better channel conditions and the cost for further probing reduces to judiciously choosing the optimal stopping rule for channel probing and the transmission rate for throughput maximization. Using optimal stopping theory (OST), it is shown in [13] that the optimal scheduling scheme turns out to be a pure threshold policy: The successful link proceeds to transmit data only if its supportable rate is higher than the pre-designed threshold; otherwise, it skips the transmission opportunity and let all other links re-contend. In general, threshold-base scheduling uses local information only and hence it is amenable to easy distributed implementation in practical systems.

The initial study on DOS [13] hinges upon a key assumption that the channel state information (CSI) is perfectly available at the receiver. This assumption is later relaxed in [14] by considering channel conditions estimated with noisy observations. It is shown in [11] that the signal-to-noise ratio (SNR) estimated by the minimum mean squared error (MMSE) method is always larger than the "actual SNR". Thus, the transmission rate has to be backed off from the estimated rate in order to avoid transmission outages. We show in [14] that the optimal scheduling policy under noisy channel estimation still has a threshold structure. Since the optimal backoff schemes are analytically intractable, suboptimal linear backoff schemes are developed in [14] with the corresponding optimal backoff ratios and rate thresholds obtained via iterative algorithms.

Despite their robust performance under noisy channel estimation, the linear backoff schemes proposed in [14] back off the rate proportional to the channel estimation errors, which may lead to severe throughput degradation, especially in the low SNR regime, due to relatively excessive rate backoff. To circumvent this drawback, a plausible alternative is to mitigate the rate estimation errors by performing further channel probing. In the sequel, we call the the initial rate estimation performed *during* the channel contention as "*first-level probing*", whereas the subsequent probing performed *after* the successful contention is referred to as "*second-level probing*". Clearly, the improved rate estimation obtained with the second-level probing enables the desired link to make more accurate decisions. However, the advantages of second-level probing come at the price of additional delay. This gives rise to two important questions: 1) Is it worthwhile for the link with successful contention to perform further channel probing to refine the rate estimate, at the cost of additional probing? 2) While there is always a gain in the transmission rate due to the refinement, how much can one bargain with the additional probing overhead?

Specifically, we investigate the optimal design of DOS with two-level channel probing. Using a variant of OST, namely OST with two-level incomplete information [10], we provide a rigorous characterization of the optimal scheduling strategy that optimizes the tradeoff between the throughput gain achieved by second-level channel probing and the resulting additional delay. It is shown that the optimal scheduling strategy is threshold-based and is characterized by either one or two thresholds, depending on the settings. By establishing the corresponding necessary and sufficient conditions for these two cases, we show that the second-level channel probing can significantly improve the system throughput when the estimated rate via first-level probing falls in between the two thresholds. In such scenarios, the cost of addition delay can be well justified by the throughput enhancement using the second-level channel probing.

Our intuition is as follows: When the channel rate is small, it makes sense to give up the transmission, since the gain due to rate refinement would be marginal due to the poor link condition. On the other hand, when the rate is large enough, it may not be advantageous to perform additional probing as the refinement is meager. Then, it is natural to expect that there exists a "gray area" between these two extremes where significant gains are possible by refining the rate estimate with additional probing. We elaborate further on this in Section III. Finally, through numerical results, we illustrate the effectiveness of the proposed scheduling scheme.

Before proceeding further, major differences distinguishing this work from other exiting works should be emphasized. Despite the fact that both this work and [14] study distributed opportunistic scheduling with imperfect information, this work concentrates on proactively improving throughput by enhancing rate estimation, whereas [14] proposes to passively reduce data rate to avoid transmission outages. Furthermore, this work studies distributed opportunistic scheduling for ad hoc communications under noisy conditions where the rate estimate is available only after a successful channel contention; and clearly this is different from [11] which considers centralized scheduling assuming that the rate estimates of all links are available at the base station before scheduling. Finally, OST under two levels of incomplete information is addressed with the objective of maximizing the net return in [10]; in contrast, we study OST with two levels of probing as applied to DOS with the objective of maximizing the rate of return (i.e., the throughput).

The rest of the paper is organized as follows. In Section II-A, we present the system model, and provide in Section II-B the background on DOS with only first-level probing in noisy environments. We then present second-level channel probing and characterize the optimal DOS with two-level probing in Section III. Numerical results are presented in Section IV. Finally, some conclusions are drawn in Section V.

Notation: $|\cdot|$ denotes the amplitude of the enclosed complex-valued quantity. We use $E[\cdot]$ for expectation.

II. System Model and Background

A. System Model and Overview

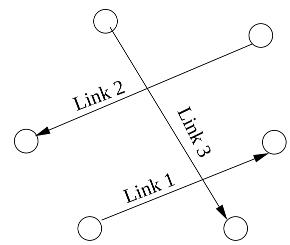

Fig. 1. Illustration of the ad hoc network under consideration.

Consider a single-hop ad hoc network in which L links contend for the channel using random access, as illustrated in Fig. 1. A successful channel contention is achieved if only one link contends. Denote by p_ℓ the probability that link l contends for the channel, $\ell = 1, \ldots, L$, the overall successful contention probability, p_s, is then given by [2]

$$p_s = \sum_{\ell=1}^{L} \left(p_\ell \prod_{i \neq \ell} (1 - p_i) \right). \tag{1}$$

We define the random duration of achieving one successful channel contention as one round of channel probing. Clearly, the number of slots in each probing round, K, is a geometric random variable, i.e., $K \sim G(p_s)$. Denoted by τ the slot duration, the corresponding random duration of one probing round is thus $K\tau$ with expectation τ/p_s.

Let $s(n)$ denote the successful link in the n-th round of channel probing. Due to the nature of wireless channels, the rate in each probing round is random. Following the standard assumption on block fading channels in wireless communications [9], we assume that the channel remains constant for a duration of T (i.e., T is more or less the channel coherence time). When the transmission rate is available, the successful link may decide to transmit over a duration of T if the rate is high enough, or may skip it and allow all links to re-contend, in the hope that another link with a better channel will take the channel later.

To get a more concrete sense of joint channel probing and distributed scheduling, we depict in Fig. 2 an example with N

rounds of channel probing and one single data transmission. Specifically, suppose after the first round of channel probing with a duration of K_1 slots, the rate of link $s(1)$ is very small (indicating a poor channel condition); and as a result, $s(1)$ gives up this transmission opportunity and lets all the links re-contend. Then, after the second successful contention with a duration of K_2 slots, link $s(2)$ also gives up the transmission because rate of link is also small. This continues for N rounds until link $s(N)$ transmits because its transmission rate is good. Clearly, there exists a tradeoff between the throughput gain from better channel conditions and the cost for further probing.

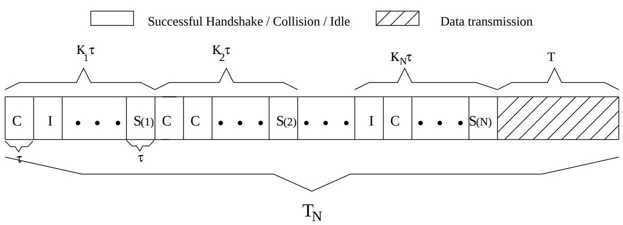

Fig. 2. A sample realization of channel probing and data transmission.

In [13], we show that the process of joint channel probing and distributed scheduling can be treated as a team game in which all links collaborate to *maximize the rate of return* (the average throughput). Specifically, as illustrated in Fig. 2, after one round of channel probing, a stopping rule N decides whether the successful link carries out data transmission, or simply skips this opportunity and let all the links re-contend. It turns out that the optimal DOS strategy achieving the maximum throughput hinges on the optimal stopping rule N^* that maximizes the rate of return:

$$N^* \triangleq \arg\max_{N \in Q} \frac{E[R_N T]}{E[T_N]}, \tag{2}$$

and

$$\theta^* \triangleq \sup_{N \in Q} \frac{E[R_N T]}{E[T_N]}, \tag{3}$$

where

$$Q \triangleq \{N : N \geq 1, E[T_N] < \infty\}. \tag{4}$$

It is clear that R_n plays a critical role in distributed opportunistic scheduling. In practice, rate estimates are seldom perfect. It is shown in [11] that the rate corresponding to the estimated SNR is always greater than the actual rate, and subsequently the transmission rate has to be backed-off from the estimated rate to avoid outages. Then, a natural question to ask is whether it is worthwhile for the link with successful contention to perform further channel probing to refine the channel estimate, at the cost of additional probing overhead. In other words, while there is always an improvement in the transmission rate due to the refinement, how much can one bargain with the additional probing overhead?

Intuitively speaking, when the transmission rate is small, it makes sense to give up the transmission, since the gain due to rate refinement would be marginal due to the poor link condition. On the other hand, when the rate is large enough, it may not be advantageous to perform additional probing as the refinement is meager. It is natural to expect that there exists a "gray area" between these extremes where significant gains are possible by refining the rate estimate with additional probing. In what follows, we seek a clear understanding of the above fundamental issues.

First, we present the PHY model. The received signal corresponding to $s(n)$ can be written as

$$Y_{s(n)}(n) = \sqrt{\rho} h_{s(n)}(n) X_{s(n)}(n) + \xi_{s(n)}(n), \tag{5}$$

where ρ is the *normalized* receiver signal-to-noise ratio (SNR), $h_{s(n)}(n)$ is the channel gain for link $s(n)$, $X_{s(n)}(n)$ is the transmitted signal with $E[|X_{s(n)}(n)|^2] = 1$, and $\xi_{s(n)}(n)$ is additive white Gaussian noise (AWGN) with unit variance. In this work, we consider a homogeneous network in which all links are subject to Rayleigh fading with identical channel statistics. More specifically, $h_{s(n)}(n)$ is modeled as $\mathcal{CN}(0,1)$ and remains constant over multiple slots. [1] Without loss if generality, we focus on the n-th probing round and omit temporal index n, whenever possible, for notational simplicity. For presentational simplicity, we use Y_n, X_n and ξ_n and h_n to denote $Y_{s(n)}(n), X_{s(n)}(n)$, $\xi_{s(n)}(n)$ and $h_{s(n)}(n)$, respectively, in the sequel.

When perfect CSI is available to the source node as assumed in [13], the instantaneous supportable data rate is given by the Shannon channel capacity :

$$R_n = W \log(1 + \rho |h_n|^2), \tag{6}$$

where W is the bandwidth and $\{R_n,\ n = 1, \ldots,\}$ are independent due to the independence assumption on h_n.

To facilitate our analysis, we concentrate our following investigation in the low SNR (wideband) regime, assuming $\rho \to 0$ and $W = \Theta(\frac{1}{\rho})$. It is well known that a decrease of SNR estimation error can only increase the rate of communication. For cases with wideband signaling (e.g. in the low SNR regime), where an increase in the SNR results in a linear increase in the throughput, obtaining more accurate estimates of the SNR can yield substantial benefits.

B. DOS with first-level probing

In this section, we briefly review the DOS with first-level channel probing [14]. Let M be the probing packet length. Thus $\tau = MT_s$, where T_s is the channel symbol duration. Assume that the rate estimation is performed based on the MMSE principle. Then, from the orthogonality principle, one can express the channel gain h_n in terms of its MMSE estimate $\hat{h}_n^{(1)}$ and the estimation error $\tilde{h}_n^{(1)}$ as

$$h_n = \hat{h}_n^{(1)} + \tilde{h}_n^{(1)}, \tag{7}$$

[1] It should be emphasized that the results reported in this work can be extended to frequency-selective fading channels by replacing scalar fading parameters with vectors.

where

$$\hat{h}_n^{(1)} \sim \mathcal{CN}\left(0, \frac{\rho M}{\rho M+1}\right), \tag{8}$$

$$\tilde{h}_n^{(1)} \sim \mathcal{CN}\left(0, \frac{1}{\rho M+1}\right), \tag{9}$$

with $\rho M = \Theta(1)$.

Without perfect CSI, the source node employs the *estimated* SNR $\{\rho|\hat{h}_n^{(1)}|^2, n = 1, \dots\}$ as the basis for distributed scheduling, despite that fact that the *actual* SNR is given by

$$\lambda_n^{(1)} = \frac{\rho|\hat{h}_n^{(1)}|^2}{1+\rho|\tilde{h}_n^{(1)}|^2}, \tag{10}$$

where we have modeled the channel estimation error as additive Gaussian noise [11].

Inspection of (10) reveals that $\lambda_n^{(1)}$ is always *smaller* than the estimated SNR $\{\rho|\hat{h}_n^{(1)}|^2\}$, in the presence of channel estimation errors. As a result, an outage occurs if the source node transmits at a data rate specified by $\{\rho|\hat{h}_n^{(1)}|^2\}$. To circumvent this problem, a linear backoff scheme has been proposed to reduce the data rate in [14]. More specifically, the estimated SNR is linearly backed off to $\sigma_M \rho|\hat{h}_n^{(1)}|^2$, where σ_M is the backoff factor with $0 < \sigma_M < 1$. Under imperfect channel information [14], the transmission rate in the low-SNR wideband region simplifies to

$$R_n^{(1)} \approx \rho W \sigma_M |\hat{h}_n^{(1)}|^2. \tag{11}$$

It can be shown that the optimal DOS policy with noisy channel estimation remains threshold-based with the optimal threshold $\hat{\theta}$ given as the solution to the following optimality equation [14]:

$$E\left[R_n^{(1)} - \theta\right]^+ = \frac{\theta\tau}{p_s T}. \tag{12}$$

III. DOS WITH TWO-LEVEL CHANNEL PROBING

In this section, we characterize the optimal DOS with two-level probing, i.e., the links have an opportunity of refining their rate estimates before making a decision on whether to transmit or not. In the following, we detail the procedure with second-level probing, and then cast DOS with two-level probing as a problem of maximal rate of return under the framework of optimal stopping theory. We then characterize the corresponding structure and provide a complete description of the optimal strategy.

A. Second-level channel probing

We illustrate, in Fig. 3 and Fig. 4, the underlying rationale behind DOS with two-level probing. To improve the channel estimation accuracy, the receiver of the successful link can request its transmitter to send another pilot packet. More specifically, the receiver refines the estimate of h_n by exploiting the newly transmitted pilot symbols during second-level probing, in addition to the pilot those sent during the first-level probing. We can show that the MMSE estimate of h_n obtained via two-level probing obeys $\hat{h}_n^{(2)} \sim \mathcal{CN}\left(0, \frac{\rho 2M}{\rho 2M+1}\right)$.

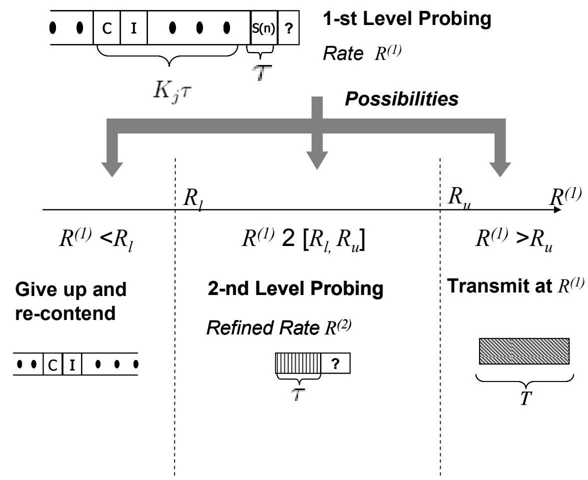

Fig. 3. A sketch of the first-level probing in DOS.

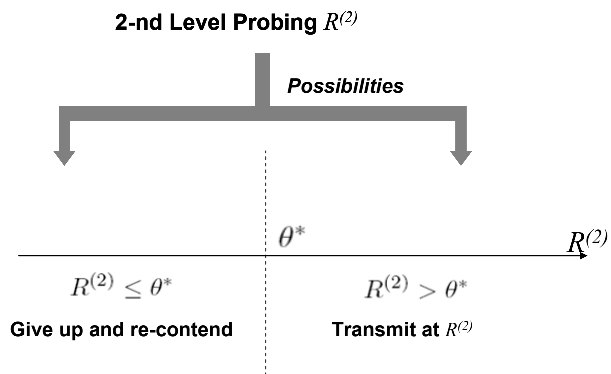

Fig. 4. A sketch of the second-level probing in DOS.

Finally, the resulting data rate is computed as

$$R_n^{(2)} = \rho W \sigma_{2M} |\hat{h}_n^{(2)}|^2, \tag{13}$$

where σ_{2M} is the corresponding linear rate back-off factor.

Next, we establish the relationship between the estimates due to first-level and second-level probings. To this end, we apply the principle of linear estimation to represent $\hat{h}_n^{(2)}$ as the linear combination of orthogonal components e and $\hat{h}_n^{(1)}$ as

$$\hat{h}_n^{(2)} = \hat{h}_n^{(1)} + e, \tag{14}$$

where $e \sim \mathcal{CN}(0, \sigma_e^2)$ with

$$\sigma_e^2 = \frac{M\rho}{(M\rho+1)(2M\rho+1)}. \tag{15}$$

By orthogonality, we have

$$E[|\hat{h}_n^{(2)}|^2] = E[|\hat{h}_n^{(1)}|^2] + \sigma_e^2. \tag{16}$$

Also, it follows that the rates corresponding to first-level and second-level probings, $R_n^{(1)}$ and $R_n^{(2)}$, obey the following relationship:

$$R_n^{(2)} = c_r R_n^{(1)} + z,$$

where $c_r = \frac{\sigma_{2M}}{\sigma_M}$ and $z \sim \mathcal{CN}(0, R_e)$ with $R_e = \sigma_{2M} W \rho \sigma_e^2$. We note that R_e can be interpreted as the expected rate gain due to the second level probing.

B. Optimal scheduling strategy

In what follows, we devise DOS with two levels of probing using optimal stopping theory. Drawing on the ideas from [4], we show that the optimizing the network throughput via DOS can be cast as a *maximal rate of return* problem.

Consider the network model in Fig. 1. It takes a total duration of $\sum_{j=1}^{n} K_j \tau$ to reach the n-th round of probing. After the n-th round of probing, the successful link has the following three options after computing its rate $R_n^{(1)}$:

1) Transmit at rate $R_n^{(1)}$;
2) Defer transmission and let all nodes re-contend;
3) Perform second-level probing to obtain the new rate $R_n^{(2)}$, and then decide to transmit at $R_n^{(2)}$ or to defer and re-contend.

Clearly, the basis for distributed opportunistic scheduling with two-level probing is the observation sequence $\left\{R_n^{(1)}, R_n^{(2)}; n = 1, \ldots\right\}$ with the option of skipping $R_n^{(2)}$. We introduce θ as the *rate of return* which can be interpreted as the "shadow price" paid per unit time. Then, the first successful channel probing incurs an average cost of $\theta\tau/p_s$, whereas the data transmission and the second-level probing entail costs of θT and $\theta\tau$, respectively.

Let $\phi_n : \mathcal{R}^+ \to \{0, 1, 2\}$ and $\psi_n : \mathcal{R}^+ \to \{0, 1\}$ be the decision sequences after $R_n^{(1)} = x$ is observed. In particular, $\phi_n(x) = 1$ refers to transmitting at the current rate, $\phi_n(x) = 0$ means giving up the transmission and re-contend, while $\phi_n(x) = 2$ indicates engaging in the second-level probing. Furthermore, when $\phi_n(x) = 2$, the final decision hinges on $R_n^{(2)} = y$: if $\psi_n(y) = 1$, the link transmits at the refined rate, whereas if $\psi_n(y) = 0$, the link gives up the transmission and lets all nodes re-contend.

Next, let N denote the stopping rule

$$N = \inf_n \{n \geq 1 | \phi_n = 1 \text{ or } \phi_n = 2 \text{ and } \psi_n = 1\}.$$

Then, the expected net reward is given by

$$r \quad = \quad E\left[R_N T - \theta T_N\right], \tag{17}$$

where R_n is the transmission rate after the n-th probing round and reads

$$R_n = I(\phi = 1) \cdot R_n^{(1)} + I(\phi = 0) I(\psi = 1) \cdot R_n^{(2)}$$

and

$$T_n = \sum_{j=1}^{n} K_j \tau + I(\psi = 1) \cdot \tau + T$$

is the total time defined as the sum of total contention time and the data transmission duration (and second-level probing time when second-level probing is performed). It can be shown that the maximum expected return is given by

$$r_0 \quad = \quad \sup_{N \in Q} E\left[R_N T - \theta T_N\right]. \tag{18}$$

Define the maximal rate of return (i.e. the throughput) in DOS with two-level probing as

$$\theta^* = \sup_{N \in Q} \frac{E\left[R_N T\right]}{E\left[T_N\right]}.$$

One principal objective is to characterize the optimal scheduling scheme that maximizes the network throughput θ. The following lemma shows the existence of such an optimal stopping rule.

Lemma 1: *For DOS with two-level probing, the optimal stopping rule N^* exists. Furthermore, θ^* is attained at N^*, and θ^* satisfies*

$$r_0 = \sup_{N \in Q} E\left[R_N T - \theta^* T_N\right] = 0,$$

Proof: See Appendix A.

Next, we derive the optimality equation for the DOS with two-level probing. Without loss of generality, we assume the transmission duration to be unity, i.e. $T = 1$.

We begin with considering the option of second-level probing and introducing its associated reward function. Suppose after observing $R_n^{(1)} = x$, the link performs a second-level probing to obtain $R_n^{(2)}$, and then uses an optimal strategy thereafter. Then, depending on $R_n^{(2)} = y$, it may choose to transmit at rate y if the associated reward is greater than r (the expected net reward); otherwise it would defer and re-contend. The reward associated with the data transmission is $y - \theta$ in this case. In a nutshell, the expected net reward corresponding to the second-level probing is then given by

$$h_\theta(x, r) \triangleq rG(r + \theta | x) + \int_{r+\theta}^{\infty} (y - \theta) G(dy|x) - \theta\tau, \tag{19}$$

where $G(y|x)$ is the conditional cumulative distribution function (cdf) of $R_n^{(2)}$, given $R_n^{(1)} = x$. Note that $G(y|x)$ is non-central χ^2 with two degrees of freedom. Furthermore, both $R_n^{(1)}$ and $R_n^{(2)}$ are exponentially distributed. We use F and F_1 respectively, to denote the cdfs $R_n^{(1)}$ and $R_n^{(2)}$. Finally, it can be shown that $\lim_{x \to \infty} G(y|x) = 0$ and $E\left[y|x\right] = c_r x + R_e$.

In summary, upon observing $R_n^{(1)} = x$ after the n-th probing round, the link $s(n)$ can obtain one of the following three rewards:

1) $x - \theta$: the reward by transmitting at a rate x;
2) r_0: the reward obtained by forgoing the current opportunity and re-contending;
3) $h_\theta(x, r_0)$: the reward by resorting to refining the rate via second-level probing.

Note that, in computing the rewards above, we have omitted the cost for obtaining the first successful channel probing, i.e. $\theta\tau/p_s$, since it is common to all the three returns. The optimal strategy for the link is to choose the option that yields the maximum of the above rewards. It follows that the optimal DOS strategy with two-level probing satisfies the following optimality equation [10]:

$$E\left[\max\left\{R^{(1)}-\theta, r_0, h_\theta\left(R^{(1)}, r_0\right)\right\}\right]-\frac{\theta\tau}{p_s}=r_0, \quad (20)$$

where $R^{(1)}$ has same distribution as $R_n^{(1)}$

From Lemma 1, when the throughput reaches maximum, we have that $r_0 = 0$ [4]. Thus, (20) can be rewritten as

$$E\left[\max\left\{R^{(1)}-\theta^*, h_{\theta^*}\left(R^{(1)}, 0\right)\right\}\right]^+=\frac{\theta^*\tau}{p_s}. \quad (21)$$

Inspection of (21) indicates that the second-level probing is optimal only when $h_{\theta^*}(x,0) > 0$ and $h_{\theta^*}(x,0) > x - \theta^*$ for some x.

C. Structure of optimal scheduling strategy

We now proceed to study the structure of the optimal scheduling strategy. Essentially, the optimal strategy takes a threshold form. Depending on the specific network setting, the optimal strategy may admit one of the two intuitively reasonable types, namely strategy A and strategy B (cf. Fig. 5 and 6). Generally speaking, under strategy A, it is always optimal to demand additional information when the estimated rate lies between two thresholds. Roughly speaking, this is the case when the gain due to second-level probing is comparable with the additional overhead. In contrast, under strategy B, there is never a need to appeal for a second-level probing. This case occurs for example, when the improvement due to the refinement is dominated by the probing overhead. An extreme example of this case is when perfect CSI is available to the transmitter.

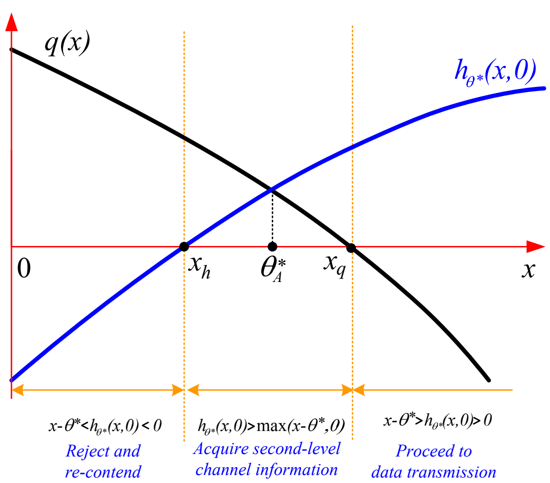

Fig. 5. A structural sketch for Strategy A.

Before we state the main result on the optimal strategy, we define $q(x) = h_{\theta^*}(x,0) - x + \theta^*$. Intuitively speaking, $q(x)$ represents the expected gain achieved by the second-level probing compared to directly transmitting at the current rate. Thus, if $q(x) > 0$, performing the second-level probing is a better option than directly proceeding to data transmission. We need the following lemmas before formulating the structure of the optimal scheduling strategy.

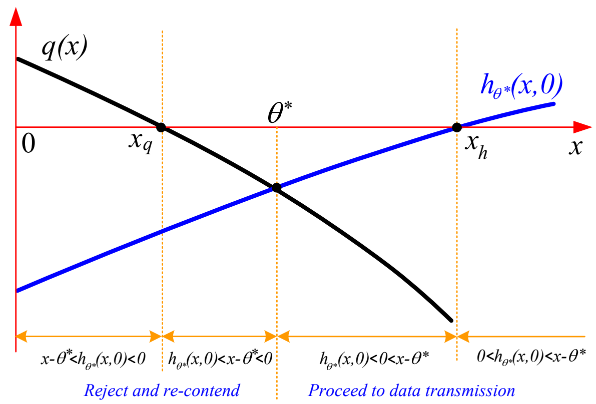

Fig. 6. A structural sketch for Strategy B.

Lemma 2: $h_{\theta^*}(x,0)$ and $q(x)$ are characterized by the following properties:

a) $h_{\theta^*}(x,0)$ is monotonically increasing in x with $\lim_{x\to\infty} h_{\theta^*}(x,0) = \infty$ and $\lim_{x\to 0} h_{\theta^*}(x,0) < 0$ for $R_e < \theta^*\tau$.

b) $q(x)$ is monotonically decreasing in x with $\lim_{x\to 0} q(x) > 0$ and $\lim_{x\to\infty} q(x) < 0$ for $R_e < \theta^*\tau$.

Proof: See Appendix B.

Lemma 3: There exists at most one solution, in terms of $\{x_h, x_q, \theta^*\}$, to the following system of equations:

$$\begin{cases} \int_{\theta^*}^{\infty}(1-G(u|x_h))du = \theta^*\tau, \\ R_e + \int_0^{\theta^*} G(u|x_q)du = \theta^*\tau, \\ \int_{x_h}^{x_q} h_{\theta^*}(u,0)\,dF(u) + \int_{x_q}^{\infty}(u-\theta^*)\,dF(u) = \frac{\theta^*\tau}{p_s}. \end{cases} \quad (22)$$

Recall that x_h and x_q are the solutions to $h_{\theta^*}(x,0) = 0$ and $q(x) = 0$, respectively. From Lemma 2, it is easy to see that there are at most one pair $\{x_h, x_q\}$ satisfying (22). Similarly, since $h_{\theta^*}(x,0)$ and $q(x)$ intercept at $x = \theta^*$, there exists at most one θ^* due to the monotonic nature of $h_{\theta^*}(x,0)$ and $q(x)$.

For convenience, let $\{x_h, x_q, \theta_A^*\}$ denote the solution to (22) with $x_h \le x_q$, and $\hat{\theta}$ be the solution to (12). Using the above lemmas, we obtain the following result on the structure of optimal scheduling strategy.

Theorem 1: *The optimal strategy for DOS with two-level probing takes one of the two forms:*

[Strategy A] *It is optimal for the successful link*

i) *to transmit immediately after the first-level probing if $R_n^{(1)} > x_q$; or*

ii) *to give up the transmission and let all the nodes re-contend if $R_n^{(1)} < x_h$; or*

iii) *to engage in the second-level probing if $R_n^{(1)} \in [x_h, x_q]$; upon computing the new rate $R_n^{(2)}$, transmit at the rate $R_n^{(2)}$ if $R_n^{(2)} > \theta_A^*$ or give up the transmission otherwise.*

TABLE I
THROUGHPUT GAIN OF DOS WITH TWO-LEVEL PROBING, $W = 300$ AND $M = 50$

ρ	0.03	0.04	0.05	0.06	0.07	0.08	0.09	0.10
θ^*	0.0215	0.0342	0.0449	0.0558	0.0671	0.0784	0.0899	0.1015
θ^L	0.0137	0.0201	0.0266	0.0332	0.0397	0.0463	0.0527	0.0592
$\gamma(\rho)$	56.3 %	70.0%	68.9%	68.5%	68.7%	69.6%	70.5%	71.6%

Furthermore, the throughput under strategy A is θ^*_A.
[Strategy B] *There is never a need to perform second-level probing. That is, it is optimal for the successful link to transmit at the current rate $R_n^{(1)}$ if $R_n^{(1)} > \hat{\theta}$; otherwise, it defers its transmission and re-contends. Furthermore, the throughput under strategy B is $\hat{\theta}$.*

Proof: See Appendix C.

D. Optimality Conditions

In previous sections, we have studied DOS with two-level probing within the OST framework, and characterized the structure of optimal scheduling strategy. Our findings reveal that optimal scheduling may take either of the two forms: strategy A or strategy B. Thus, the next key step is to determine the conditions on when it is optimal to use strategy A or strategy B. In what follows, we consider this problem, and show that it can be easily determined by performing a threshold test on the function $h_{\theta^*}(\cdot,\cdot)$. We have the following theorem.

Theorem 2: *Strategy A is optimal if $h_{\theta^*_A}(\theta^*_A, 0) \geq 0$; otherwise, Strategy B is optimal.*

Proof: See Appendix D.

IV. NUMERICAL RESULTS

In this section, we provide a numerical example to demonstrate the effectiveness of the proposed DOS with two-level probing under noisy channel estimation. Unless otherwise specified, we set $p_s = 0.8$. In this example, we compare the throughput performance obtained by the proposed DOS with two-level probing and that obtained by DOS with perfect CSI [13], DOS with first-level probing [14]. The baseline for comparison is the scheduling not exploiting the channel information, and we call it *PHY-oblivious scheduling*. In particular, we set $M = 50$ and $W = 300$, i.e. $\tau = \frac{50}{300} \approx 0.17$. Clearly, the throughput achieved by DOS with two-level probing substantially outperforms that obtained by PHY-oblivious scheduling. Furthermore, Fig. 7 suggests that the second-level probing provides noticeable improvement over the first-level probing for ρ larger than 0.04. Finally, the degradation of DOS with two-level probing with respect to DOS with perfect CSI is about 0.02 nats/s/Hz.

Table I tabulates the throughput gain defined as

$$\gamma(\rho) = \frac{\theta^* - \theta^L}{\theta^L} \tag{23}$$

where θ^* and θ^L are the throughput obtained by using DOS with two-level probing and PHY-oblivious scheduling, respectively. It can be seen from Table I that the throughput gain is quite significant in the low SNR region under consideration. It is easy to understand that the optimal throughput θ^*, as well as θ^L, increases with the increase of ρ. However, recall that the normalized channel estimation noise variance, $\frac{1}{\rho M}$, also decreases with the increase of ρ. As a result, the throughput gain by exploiting the additional M pilots increases as ρ increases.

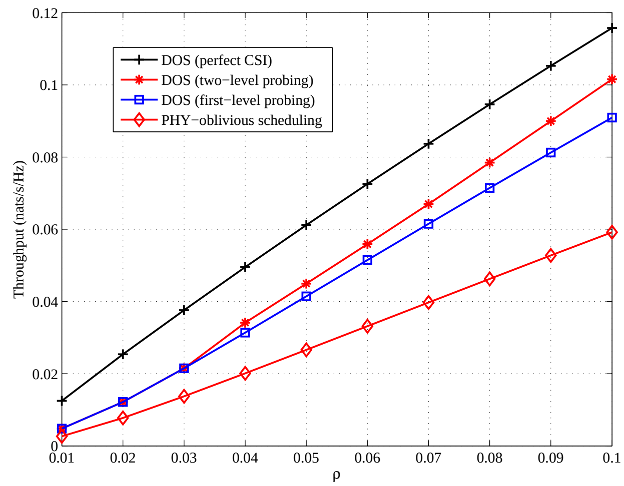

Fig. 7. Throughput as a function of ρ.

V. CONCLUSION

We have considered channel-aware distributed scheduling for single-hop ad-hoc networks in which many links contend for the same channel using random access. From a holistic perspective, if a link with successful contention observes a poor channel condition, it should forgo its data transmission and let all links re-contend. We note that each channel probing incurs a cost of the contention time. It is therefore critical to characterize the desired tradeoff between the throughput gain from better channel conditions and the cost for further probing. Furthermore, since in practice, link conditions are estimated with noisy observations, the transmission rate has to be backed off from the estimated rate to avoid transmission outages. Thus, another natural question is whether it is worthwhile for the link with successful contention to perform further channel probing to mitigate estimation errors, at the cost of additional probing. The main goal of this study is to seek a clear understanding of these issues.

Thus motivated, we have investigated DOS with two-level channel probing by optimizing the tradeoff between the throughput gain from more accurate rate estimation and the corresponding probing overhead. Capitalizing on optimal stopping theory with two-level incomplete information, we have showed that the optimal scheduling policy is threshold-based and characterized by either one or two thresholds, depending on system settings. We have also identified the optimality conditions. In particular, our analysis reveals that DOS with second-level channel probing is optimal when the first-level estimated rate falls in between the two thresholds. Finally, by a numerical example, we have illustrated the effectiveness of the proposed DOS with two-level channel probing.

We note that the proposed distributed scheduling with two-level probing provides a new framework to study joint PHY/MAC optimization in practical networks where noisy probing is often the case and imperfect information is inevitable. We believe that this study provides some initial steps towards opening a new avenue on exploring channel-aware distributed scheduling for ad hoc networks to enhance spectrum utilization; and this is potentially useful for enhancing MAC protocols for wireless local area networks (LANs) and wireless mesh networks.

APPENDICES

A. Proof of Lemma 1

For a given θ, let $N(\theta)$ be a stopping rule such that

$$\begin{aligned} r_0(\theta) &= E\left[R_{N(\theta)} - \theta T_{N(\theta)}\right], && (24)\\ &= \sup_{N\in Q} E\left[R_N T - \theta T_N\right]. && (25)\end{aligned}$$

Thus, it follows from Theorem 1 in [4, Chapter 3] that $N(\theta)$ exists if the following conditions are satisfied:

$$\begin{aligned} E[\sup_n Z_n] &< \infty && (26)\\ \limsup_{n\to\infty} Z_n &= -\infty, \text{ a.s.}, && (27)\end{aligned}$$

where $Z_n \triangleq R_n T - \theta T_n$.

It is easy to verify that $\limsup_{n\to\infty} Z_n = -\infty$ a.s.. Furthermore, we have

$$\begin{aligned} E\left[\sup_n Z_n\right] &\leq E\left[\sup_n\left(\max\{R_n^{(1)}, R_n^{(2)}\} - n\cdot\theta\frac{\tau}{p_s}\right)\right]\\ &+ \theta\tau E\left[\sup_n \sum_{j=1}^{n}\left(\frac{1}{p_s} - K_j\right)\right] - \theta T.\end{aligned}$$

Appealing to [4, Chapter 4], we conclude that the right hand side (RHS) of the above equation is finite and therefore $E\left[\sup_n Z_n\right] < \infty$. The second part of the lemma follows directly from Theorem 1 in [4, Ch.6].

B. Proof of Lemma 2

a) Using Fubini's theorem, we rewrite $h_\theta(x,r)$ as

$$h_{\theta^*}(x,0) = \int_{\theta^*}^{\infty}(1 - G(u|x))du - \theta^*\tau. \qquad (28)$$

Since $G(y|x)$ monotonically decreases with x, $h_{\theta^*}(x,0)$ is also monotonically increasing in x. Note that $\lim\limits_{x\to\infty}(1 - G(u|x)) = 1$. Then, by Lebesgue's convergence theorem, we have $\lim_{x\to\infty} h_{\theta^*}(x,0) = \infty$. Next, recall that $|z|^2 \sim \exp(R_e)$. It follows that

$$\lim_{x\to 0} G(y|x) = G_{|z|^2}(y) = 1 - e^{-\frac{y}{\sigma_z^2}}, \qquad (29)$$

and consequently

$$\begin{aligned} \lim_{x\to 0} h_{\theta^*}(x,0) &= \int_{\theta^*}^{\infty}\left(1 - G_{|z|^2}(u)\right)du - \theta^*\tau\\ &= R_e e^{-\frac{\theta^*}{R_e}} - \theta^*\tau. && (30)\end{aligned}$$

It follows that under the condition $\theta^*\tau > R_e$,

$$\lim_{x\to 0} h_{\theta^*}(x,0) < 0. \qquad (31)$$

b) Using Fubini's theorem, we can rewrite $h_{\theta^*}(x,r)$ as

$$h_{\theta^*}(x,0) = x + R_e + \int_0^{\theta^*} G(u|x)du - \theta^*(1+\tau) \qquad (32)$$

Based on (32), we get

$$q(x) = R_e + \int_0^{\theta^*} G(u|x)du - \theta^*\tau. \qquad (33)$$

Recall that $G(y|x)$ monotonically decreases with x, we can conclude that $q(x)$ is also monotonically decreasing in x. Thus, we have

$$\lim_{x\to 0} q(x) = R_e e^{-\frac{\theta^*}{R_e}} + (1-\tau)\,\theta^* > 0, \qquad (34)$$

and

$$\lim_{x\to\infty} q(x) = R_e - \theta^*\tau < 0, \qquad (35)$$

where the last inequality is derived under the same condition $\theta^*\tau > R_e$ employed in (31).

C. Proof of Theorem 1

Let x_h and x_q be solutions to $h_{\theta^*}(x,0) = 0$ and $q(x) = 0$ respectively. From Lemma 2, we have

$$h_{\theta^*}(x,0)\begin{cases} < 0 & \text{if } x < x_h\\ = 0 & \text{if } x = x_h\\ > 0 & \text{if } x > x_h\end{cases} \qquad (36)$$

and

$$q(x)\begin{cases} < 0 & \text{if } x > x_q\\ = 0 & \text{if } x = x_q\\ > 0 & \text{if } x < x_q.\end{cases} \qquad (37)$$

Thus, one of the following two possibilities holds.

1) The case with $x_q \geq x_h$:

From the above discussions and the monotonicity properties of $h_{\theta^*}(\cdot,0)$ and $q(\cdot)$, it follows that

$$\max\left[x-\theta^*, h_{\theta^*}(x,0)\right]^+ = \begin{cases} x-\theta^* & \text{if } x > x_q\\ h_{\theta^*}(x,0) & \text{if } x\in[x_h, x_q]\\ 0 & \text{if } x < x_h\end{cases} \qquad (38)$$

Furthermore, from (38) and the optimality equation (21), we have that

$$\int_{x_h}^{x_q} h_{\theta^*}(u,0)\,dF(u) + \int_{x_q}^{\infty} (u-\theta^*)\,dF(u) = \frac{\theta^*\tau}{p_s}. \quad (39)$$

Subsequently, it is clear that the optimal strategy is

$$\phi_n(R_n^{(1)}) = \begin{cases} 1 \text{ (transmit)} & \text{if } R_n^{(1)} > x_q \\ 2 \text{ (refinement)} & \text{if } R_n^{(1)} \in [x_h, x_q] \\ 0 \text{ (re-contend)} & \text{if } R_n^{(1)} < x_h \end{cases} \quad (40)$$

and when $\phi_n(R_n^{(1)}) = 2$, the strategy is

$$\psi_n(R_n^{(2)}) = \begin{cases} 1 \text{ (transmit)} & \text{if } R_n^{(2)} \geq \theta_A^* \\ 0 \text{ (re-contend)} & \text{if } R_n^{(2)} < \theta_A^* \end{cases} \quad (41)$$

where θ_A^* is the solution to (39). It can be seen that thresholds x_h and x_q are found as the solutions to $h_{\theta^*}(x,0) = 0$ and $q(x) = 0$ respectively. Thus, $\{x_h, x_q, \theta_A^*\}$ is the solution to the system (3). An illustration of Strategy A is depicted in Fig. 5.

2) The case with $x_q < x_h$:

From (36) and (37), we have

$$\max\left[x-\theta^*, h_{\theta^*}(x,0)\right]^+ = \begin{cases} x-\theta^* & \text{if } x \geq \theta^* \\ 0 & \text{if } x < \theta^* \end{cases} \quad (42)$$

and $h_{\theta^*}(x,0) < \max\left[x-\theta^*, 0\right]$. Therefore, it is never optimal to perform second-level probing. From (42) and the optimality equation (21) we obtain

$$\int_{\theta^*}^{\infty} (x-\theta^*)\,dF(x) = \frac{\theta^*\tau}{p_s},$$

which is equivalent to (12). Thus from (42), the optimal strategy is

$$\phi(R_n^{(1)}) = \begin{cases} 1 \text{ (transmit)} & \text{if } R_n^{(1)} \geq \hat{\theta} \\ 0 \text{ (re-contend)} & \text{if } R_n^{(1)} < \hat{\theta}, \end{cases} \quad (43)$$

where the threshold $\hat{\theta}$ is the solution to (12). An illustration of Strategy B is depicted in Fig. 6.

D. Proof of Theorem 2

Suppose $h_{\theta_A^*}(\theta_A^*, 0) \geq 0$. Then, this implies that $h_{\theta_A^*}(\theta_A^*, 0) \geq \max[x-\theta_A^*, 0]$ when $x = \theta_A^*$. Specifically, when $R_1^{(1)} = \theta_A^*$, performing a second-level probing and using an optimal strategy thereafter yield an expected reward of $h_{\theta_A^*}(\theta_A^*, 0)$, which is at least as good as using strategy B. Equivalently, we show that there exists at least one value of x (θ_A^* in this case) for which performing second-level probing is optimal. We conclude that strategy A is optimal.

Next, we assume Strategy A is optimal and show that $h_{\theta_A^*}(\theta_A^*, 0) \geq 0$. Under such an assumption, there must exist some x_1 for which it is beneficial to demand additional information, i.e.

$$h_{\theta_A^*}(x_1, 0) \geq \max[x_1 - \theta_A^*, 0]. \quad (44)$$

We now investigate $h_{\theta_A^*}(\theta_A^*, 0)$ in two different cases, namely $\theta_A^* \geq x_1$ and $\theta_A^* < x_1$.

1) The case with $\theta_A^* \geq x_1$:
In this case,

$$h_{\theta_A^*}(\theta_A^*, 0) \geq h_{\theta_A^*}(x_1, 0) \geq \max[x_1 - \theta_A^*, 0] = 0, \quad (45)$$

where the first and second inequalities are due to the monotonicity of $h(\cdot, 0)$ and the assumed optimality of Strategy A, respectively.

2) The case with $\theta_A^* < x_1$:
In this case,

$$h_{\theta_A^*}(\theta_A^*, 0) \geq h_{\theta_A^*}(x_1, 0) - x_1 + \theta_A^* \geq 0, \quad (46)$$

where the first inequality follows from the fact that $h_{\theta_A}(x,0) - x + \theta$ is decreasing in x and the second inequality is due to (44).

Summarizing the above two cases, we conclude that $h_{\theta_A^*}(\theta_A^*, 0) \geq 0$ is a necessary and sufficient condition for the optimality of strategy A. Thus from contraposition, it follows that strategy B is optimal if $h_{\theta_A^*}(\theta_A^*, 0) < 0$.

References

[1] M. Andrews, K. Kumaran, K. Ramannan, A. Stolyar, R. Vijaykumar, and P. Whiting, "CDMA data QoS scheduling on the forward link with variable channel conditions," *Bell Labs Tech. Memo.*, 2000.

[2] D. P. Bertsekas and R. Gallager, *Data Networks*. Upper Saddle River, New Jersey: Prentice-Hall, 1992.

[3] S. Borst, "User-level performance of channel-aware scheduling algotirthms in wireless data networks," in *Proc. IEEE INFOCOM'03*, 2003.

[4] T. Ferguson, *Optimal Stopping and Applications*. available at http://www.math.ucla.edu/˜tom/Stopping/Contents.html, 2006.

[5] S. Guha, K. Munagala, and S. Sarkar, "Jointly optimal transmission and probing strategies for multichannel wireless systems," in *Proceedings of CISS'06*, Princeton, NJ, 2006.

[6] Z. Ji, Y. Yang, J. Zhou, M. Takai, and R. Bagrodia, "Exploiting medium access diversity in rate adaptive wireless LANs," in *Proc. MOBICOM'04*, Philadelphia, PA, September 2004.

[7] X. Liu, E. K. Chong, and N. B. Shroff, "A framework for opportunistic scheduling in wireless networks," *Computer Networks*, vol. 41, no. 4, pp. 451–474, Mar. 2003.

[8] X. Qin and R. Berry, "Exploiting multiuser dieversity for medium access control in wireless networks," in *Proc. IEEE INFOCOM'03*, San Francisco, CA, April 2003.

[9] B. Sadeghi, V. Kanodia, A. Sabharwal, and E. Knightly, "Opportunistic media access for multirate ad hoc networks," in *Proceedings of ACM/IEEE MOBICOM'02*, Atlanta, GA, 2002.

[10] W. Stadje, "An optimal stopping problem with two levels of incomplete information," *Mathematical Methods of Operations Research*, vol. 45, no. 1, pp. 119–131, 1997.

[11] A. Vakili, M. Sharif, and B. Hassibi, "The effect of channel estimation error on the throughput of broadcast channels," in *Proc. IEEE International Conference on Acoustics, Speech, and Signal Processing (ICASSP)*, Toulouse, France, May 2006.

[12] P. Viswanath, D. N. C. Tse, and R. Laroia, "Opportunistic beamforming using dumb antennas," *IEEE Trans. Info. Theory*, vol. 48, no. 6, pp. 1277–1294, Jun. 2002.

[13] D. Zheng, W. Ge, and J. Zhang, "Distributed opportunistic scheduling for ad-hoc communications: An optimal stopping approach," in *Proc. Mobihoc 2007*, Montreal, Canada, 2007.

[14] D. Zheng, M. Pun, W. Ge, J. Zhang, and H. Poor, "Distributed opportunistic scheduling for ad-hoc communications under noisy channel estimation," in *Proc. IEEE International Conference on Communications*, Beijing, China, May 2008.