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Abstract

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Compound conditional source coding, Slepian-Wolf list decoding, and applications to media coding

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I. Introduction

Distributed source coding ideas of the sort first developed by Slepian and Wolf [14] are finding application in a wide range of applications from sensor networks to security to multimedia coding. In this paper we describe how some of of these emerging applications are more exactly described as "compound conditional source coding." They share commonalities with both distributed source coding and conditional source coding [9], but are distinct from both. We provide motivating examples from the video coding literature. We develop both first-order achievability results and second-order reliability function results for this problem. In developing the latter we also derive an achievable list-decoding reliability function of Slepian-Wolf coding. Finally, we present a protocol that builds on Slepian-Wolf list decoding to get error-exponent results for compound conditional source coding.

Figure 1 gives a block diagram that depicts both Slepian-Wolf and conditional source coding. In both scenarios we want to transmit a length-n random source sequence $\mathbf{x}$ over a rate-constrained noiseless channel. In addition, the decoder observes the length-n side information $\mathbf{y}$ where the pair $(\mathbf{x}, \mathbf{y})$ is distributed according to $p_{\mathbf{x},\mathbf{y}}(\mathbf{x}, \mathbf{y})$. The difference between conditional and distributed source coding is in the encoder knowledge. In conditional source coding the encoder observes the side information, i.e., switch (a) in Fig. 1 is closed. In distributed source coding switch (a) is open. The encoder knows of only the existence of $\mathbf{y}$ and the statistics $p_{\mathbf{x},\mathbf{y}}(\mathbf{x}, \mathbf{y})$.

The problem we introduce in this paper–compound conditional source coding–is depicted in Fig. 2. The novel twist is that the encoder is told that $\mathbf{y}$ is one of a certain (small finite) set of possibilities $\{\mathbf{y}_1, \mathbf{y}_2, \ldots, \mathbf{y}_P\}$, but is not told which.

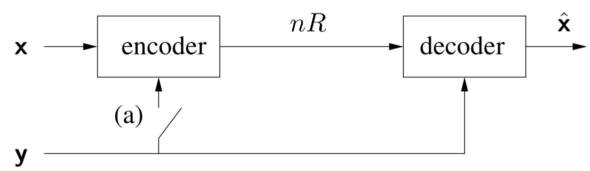

Fig. 1. In conditional source coding switch (a) is closed. In distributed source coding switch (a) is open. In both the objective is to reconstruct $\mathbf{x}$ with high probability and no distortion, i.e., $\hat{\mathbf{x}} = \mathbf{x}$.

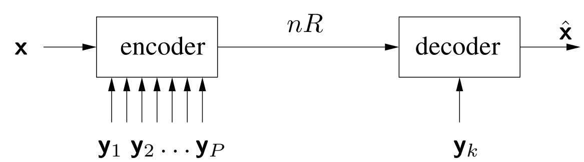

Fig. 2. In compound conditional source coding a set of P possible side information realizations $\mathbf{y}_1, \mathbf{y}_2, \ldots, \mathbf{y}_P$ are observed at the encoder. Only one, $\mathbf{y}_k$, is observed at the decoder. All P joint distribution $p_{\mathbf{x},\mathbf{y}_p}(\mathbf{x}, \mathbf{y}_p)$, $p \in \{1, 2, \ldots, P\}$ are known, but only the decoder knows the identity k of its observation. The objective is to reconstruct $\mathbf{x}$ with high probability and no distortion.

In contrast, in conditional source coding $P = 1$, and in distributed source coding the encoder knows only that $\mathbf{y}$ belongs to the typical set of possibilities and hence $P \simeq 2^{nH(y|x)}$. Because in our compound problem the encoder does not know which of the P possibilities is observed by the decoder, conditional (a.k.a. "predictive") coding fails. On the other hand, distributed source coding will work. However, because the set of possibilities has been narrowed so vastly (from an exponential to a sub-exponential number), the encoder should be able to operate more intelligently than a straightforward application of Slepian-Wolf techniques.

It immediately shown in Thm. 1 (cf. Sec. IV) that Slepian-Wolf coding is asymptotically optimal for compound conditional source coding in terms of rate. Consequently, to obtain sharp results demonstrating that Slepian-Wolf coding is sub-optimal, we show that the error exponent can be improved. Specifically, we show the error exponent of compound conditional source coding is at least as great as the list-decoding error exponent of Slepian-Wolf coding for the *realized* source statistics.

A larger error exponent implies shorter code block-length implying lower latency (which is valuable for real-time media applications) and a lower demand for communication resources. Still, a system designer may object that the complexity of list decoding exceeds the already infeasible complexity of decoding a random codebook and hence wonder how to interpret our result. Our main point is that using *encoder side information* (available in the form of the set of P possible side informations) can improve performance. System designers should search for ways to use this information.

The rest of the paper is outlined as follows. In Section II we formally describe the problem model, specify our coding protocol, and in Sec. III give examples of its relevance to multimedia. In Section IV we describe our main results. Derivations are given in Section V. We conclude in Section VI.

II. Problem description and Protocol

The compound conditional source coding problem is described as follows. A length-n random source $\mathbf{x}$ is drawn according to $p_{\mathbf{x}}(\mathbf{x})$ and observed by the encoder. A number P of candidate side informations $\mathbf{y}_1, \mathbf{y}_2, \ldots, \mathbf{y}_P$ are drawn according to $p_{\mathbf{y}_1|\mathbf{x}}(\mathbf{y}_1|\mathbf{x}), p_{\mathbf{y}_2|\mathbf{x}}(\mathbf{y}_2|\mathbf{x}), \ldots p_{\mathbf{y}_P|\mathbf{x}}(\mathbf{y}_P|\mathbf{x})$, respectively, and shown to the encoder. One of these P side information sequences $\mathbf{y}_k$ is shown to the decoder. We place no probability distribution on the choice of k. The encoder and decoder are both assumed to know all the joint distributions, $p_{\mathbf{x},\mathbf{y}_p}(\mathbf{x}, \mathbf{y}_p)$ for all $p \in \{1, 2, \ldots P\}$. Further, the decoder is assumed to know the index k of the side information selected. The validity of this last assumption will be clear when we describe the scenarios we are modeling.

The encoder sends a rate-R message over a noiseless channel to the decoder who attempts to recreate $\mathbf{x}$ losslessly with high probability.

A. List decoding

In this paper we present results on list decoding for Slepian-Wolf systems. In contrast to, e.g., maximum likelihood decoding when the output of the decoder is the single best guess of the source sequence, in list decoding the decoder outputs a length-L list of possibilities. A list decoder is in error only if the true source sequence $\mathbf{x}$ is not on the list $\mathcal{L}(\mathbf{y})$. List decoding results for channel coding first appeared in [6] and are also developed in [7, exercise 5.20].

B. Compound conditional source coding protocol

We now present the coding protocol for compound conditional source coding. The protocol builds on Slepian-Wolf list decoding. We encode using a randomized Slepian-Wolf code at rate $R \geq \max_p H(x|y_p)$. As is shown in Thm. 1 (cf. Sec. IV) this encoding rate guarantees successful decoding with high probability. To improve error exponent performance, in addition to the Slepian-Wolf bin index, the encoder also sends zero-rate resolution information. The encoder calculates the resolution information by list decoding with respect to each of the P side informations it observes. It encoders the location of the true sequence $\mathbf{x}$ on each list, which can be encoded with $\log L$ bits per list. Since there are P possible side informations the total number of resolution bits is $P \log L$. The rate of the resolution information is $P \log L/n$ which goes to zero as the block length n grows and thus, asymptotically, uses zero additional rate.

The decoder performs Slepian-Wolf list decoding with respect to the side information $\mathbf{y}_k$ it observes, and its joint distribution $p_{\mathbf{x},\mathbf{y}_k}$. It then uses the appropriate (kth) list index to select its final source estimate from the resulting list. As long as the true source sequence is on the list, decoding is successful. Therefore the error probability performance of compound conditional source coding is as good as that of Slepian-Wolf list decoding under the realized statistics $p_{\mathbf{x},\mathbf{y}_k}$.

Any finite $L > 1$ can improve the error exponent. This is the main technical point of our paper. In fact, since L does not effect the rate it can be chosen large, but finite. One can further optimize the proposed protocol using a more detailed analysis to choose the optimal L as a function of n and P. This can make the resolution information non-negligible, which would require more care in the error exponent calculation. We postpone these calculations for future work.

This protocol is *only* possible in the compound conditional source coding setting because the encoder can see the possible side informations. This allows the encoder to simulate the decoder, identify the lists the decoder would produce, and deduce the correct resolution informations. In the distributed source coding setting, the encoder has no idea what the side information realization is and cannot produce such lists.

III. Media Applications

We now give examples from the multimedia literature that can be abstracted into this setting. These examples all fall under the general moniker of "Wyner-Ziv" video coding. Wyner-Ziv [19] coding is the rate-distortion version of Slepian-Wolf coding. At a high level, a Wyner-Ziv system is just a traditional vector quantizer, followed by a Slepian-Wolf encoder and decoder, and finished off with post-processing consisting of a joint estimate of $\mathbf{x}$ based on the decoded vector quantization of $\mathbf{x}$ and the side information $\mathbf{y}$. The Slepian-Wolf core is thus the only distributed aspect of a Wyner-Ziv system. Therefore, without loss of too much generality we concentrate on the Slepian-Wolf core of these problems in this paper.

The insight that video encoding can be treated as a distributed source coding problem was first realized in [3], [12]. In terms of Fig. 1 the switch (a) is open, $\mathbf{x}$ corresponds to the next frame in the video sequence and the side information $\mathbf{y}$ corresponds to the already-decoded previous frame. If switch (a) were closed we could encode the innovation between the two frames which is how traditional predictive video encoders work. As the following examples illustrate there are a number of reasons to consider distributed coding frameworks.

Finally, we note that compound conditional source coding is somewhat similar to "source encoding with side-information under ambiguous state of nature" (SEASON) [10]. Both are attempts to develop a theoretical basis for the application of distributed source coding to media. Major differences are as follows. In SEASON the side informations are all marginally identically distributed, the side information realizations are not

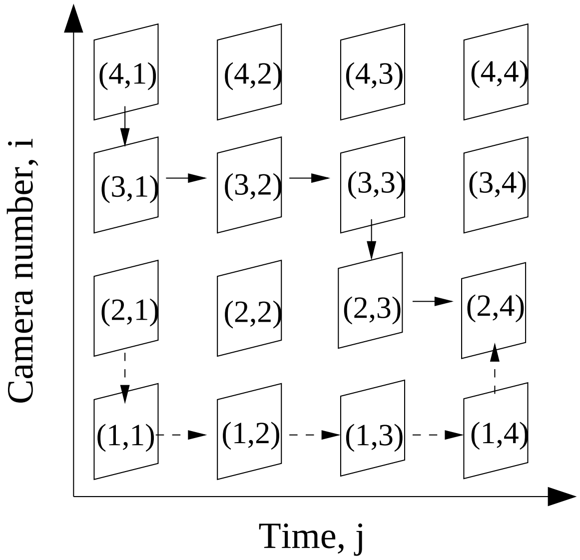

Fig. 3. Multiview coding.

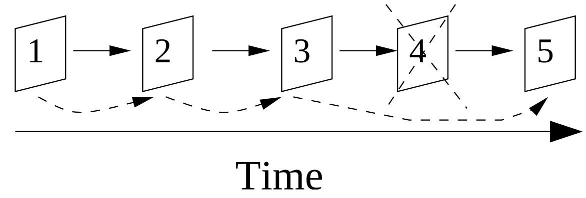

Fig. 4. Robust video coding.

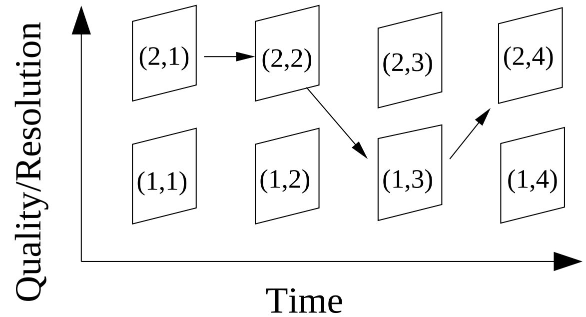

Fig. 5. Stream switching of multiresolution video coding.

revealed to the encoder, and all realizations are revealed to the decoder. Further, the decoder is not told which of the revealed side informations is the one that is jointly distributed with the source. SEASON is also posed in a rate-distortion setting but, as noted above, that difference is less important.

A. *Multiview coding*

In multiview video/image coding, a scene is captured using multiple cameras at each time instant. For example, in Fig. 3 (i, j) represents the camera number i and time instant j of a particular frame. Traditional predictive coding does not allow random access (i.e., decoding in any arbitrary order) while intra-coding has poor compression efficiency. In contrast, Wyner-Ziv coding [1], [15] allows random access (e.g., decoding in the order illustrated by either the solid or the dashed lines) while providing higher compression efficiency than independent intra-coding of each frame. When Wyner-Ziv techniques are used, this is an example of compound conditional source coding. The encoder knows the possible side information sequences in advance. For example, the prediction references for packet (2,4) could either be (1,4) or (2,3) depending on the desired decoding order.

B. *Robust video coding*

Various researchers [18], [11], [2] have proposed using Wyner-Ziv coding of video to mitigate error propagation when video packets are transmitted over a lossy channel. This application is illustrated in Fig. 4. For example, by using Wyner-Ziv coding at the appropriate bit rate, packet 5 can be decoded by using either packet 4 as a predictive reference (if packet 4 is received without error) or by using packet 3 as a predictive reference (if packet 4 is lost). This is another example of a compound conditional source coding problem since the encoder knows in advance the possible side informations (packet 4 or packet 3 or packet 2 or packet 1) that the decoder might use in decoding packet 5. While we do not place any probability distribution on the choice of realized side information, packet loss statistics could be used to refine the error model, although applications with extreme latency requirements will be limited by the worst case, as herein.

C. *Stream switching for multiresolution video coding*

A key issue in video streaming is that network bandwidth may vary over time. Some researchers have proposed using Wyner-Ziv video coding to allow the transmitter to vary the bitrate/resolution/quality of the video stream dynamically. Enabling the decoder to "switch" from one resolution to another is complicated by the fact that the decoder may not have the prediction references for the other stream. For example, if, as is depicted in Fig. 5, a decoder switches from high resolution to low resolution at time 3, it may not have the appropriate prediction references for low resolution. Methods of addressing this issue include forcing motion vectors for each resolution to be the same, only allowing resolution switches at intra "I" frames, and using SP/SI frames [16], [13], [17]. An alternative is to encode error residuals or texture information using Wyner-Ziv coding [15], giving more graceful resolution switching. Since the streams at all resolutions are known by the encoder, this is compound conditional source coding.

IV. MAIN TECHNICAL RESULTS

In this section we state our main technical results. For simplicity we state some results for the case of i.i.d. sources.

Theorem 1: (Compound conditional source coding) Let $p_{\mathsf{x},\mathbf{y}_p}(\mathbf{x}, \mathbf{y}_p) = \prod_{i=1}^{n} p_{x,y_p}(x_i, y_{p,i})$ where $p \in \{1, 2, \ldots P\}$ be the joint distributions of a length-n source sequence x with P side informations. The encoder observes $\mathbf{x}$ and $(\mathbf{y}_p, p)$ for all $p \in \{1, 2, \ldots P\}$. The decoder observes only $(\mathbf{y}_k, k)$ where

$k \in \{1,2,\ldots,P\}$. Then for any $\epsilon > 0$ there exists an $n_0 > 0$ such that for all $n > n_0$ there exists an encoder/decoder pair with $\Pr[\hat{\mathbf{x}} \neq \mathbf{x}] < \epsilon$ if

$$R > \max_{p \in \{1,2,\ldots,P\}} H(x|y_p) \tag{1}$$

Theorem 2: (List-decoding for Slepian-Wolf systems) Let $p_{\mathbf{x},\mathbf{y}}(\mathbf{x},\mathbf{y})$ be the joint distribution of a pair of length-n random sequences $(\mathbf{x},\mathbf{y})$ where $\mathbf{x}$ is the source observed at the encoder and $\mathbf{y}$ is the decoder side information. There exists a rate-R encoder/list-decoder pair, where the list $\mathcal{L}(\mathbf{y})$ is of size $|\mathcal{L}(\mathbf{y})| = L$, such that the average probability of list decoding error is bounded for any choice of ρ, $0 \leq \rho \leq L$ as

$$\Pr[\mathbf{x} \notin \mathcal{L}(\mathbf{y})] \leq 2^{-n\rho R} \sum_{\mathbf{y}} \Big(\sum_{\mathbf{x}} p_{\mathbf{x},\mathbf{y}}(\mathbf{x},\mathbf{y})^{\frac{1}{1+\rho}} \Big)^{1+\rho}. \tag{2}$$

In the special case of an i.i.d. source $p_{\mathbf{x},\mathbf{y}}(\mathbf{x},\mathbf{y}) = \prod_{i=1}^{n} p_{x,y}(x_i,y_i)$. Maximizing over the free parameter ρ, $0 \leq \rho \leq L$, we get the following error exponent.

Corollary 1: For i.i.d. sources $\Pr[\mathbf{x} \notin \mathcal{L}(\mathbf{y})] \leq 2^{-nE}$ for all $E \leq E_{\text{SW,list}}(p_{x,y},R,L)$ where $E_{\text{SW,list}}(p_{x,y},R,L) =$

$$\max_{0 \leq \rho \leq L} \rho R - \log \sum_{y} \Big(\sum_{x} p_{x,y}(x,y)^{\frac{1}{1+\rho}} \Big)^{1+\rho}. \tag{3}$$

Corollary 2: (Error exponent of compound conditional source coding) Consider the compound conditional source coding problem of Thm. 1. Let k denote the realized decoder side information where $k \in \{1,2,\ldots,P\}$. Then

$$-\frac{\log \Pr[\hat{\mathbf{x}} \neq \mathbf{x}]}{n} \geq E_{\text{SW,list}}(p_{x,y_k},R,L) \tag{4}$$

To understand the impact of list decoding on the achieved error exponent, one should compare (2) and (3) to the exponential error bounds for maximum likelihood Slepian-Wolf decoding [8]. In particular the argument of the maximization in (3) is the same. List-decoding only effects the domain of the free variable ρ. While in maximum likelihood decoding [8] $0 \leq \rho \leq 1$, in length-L list decoding $0 \leq \rho \leq L$. This additional freedom can translate into a large increase in the exponent at higher rates. This is the same effect as when list decoding is used in channel coding (where the large increase occurs at lower rates). We again emphasize the dependence on the realized k. The error exponent is at least as large as the Slepian-Wolf list decoding error exponent for list size L and for the *realized* joint statistics p_{x,y_k}. Recall k is known by the decoder but not the encoder. Encoding is universal. It does not depend on the joint statistics; decoding does.

V. Derivations

A. Compound conditional source coding

The logic behind Thm. 1 is the same as that for compound channels [4]. By the random-binning achievability of the regular Slepian-Wolf coding theorem [14] we can decode $\mathbf{x}$ correctly with high probability for source $p_{x,y_p}(\mathbf{x},\mathbf{y}_p)$ if $R \geq H(x|y_p)$. If we pick $R \geq \max_p H(x|y_p)$ we can decode reliably since the random binning used in a Slepian-Wolf encoder does not depend on the source statistics (only the binning rate does).[1] Since the number of distributions is sub-exponential, by the union bound we can find a binning that is good for all P joint distributions. Finally, by the standard converse we cannot decode reliably for source p if $R < H(x|y_p)$. Therefore, $R \geq \max_p H(x|y_p)$ is required.

B. List-decoding for Slepian-Wolf systems

The Slepian-Wolf encoding technique we use in deriving our list decoding results is the standard one. The encoder assigns all $|\mathcal{X}|^n$ source sequences independently and uniformly to 2^{nR} sets or "bins". The (random) bin containing the source sequence $\mathbf{x}$ is denoted $\mathcal{B}(\mathbf{x})$. The decoder decodes to a list $\mathcal{L}(\mathbf{y})$ of the $L \geq 1$ most likely source sequences given the side information $\mathbf{y}$. The average probability of list decoding error, occurring when $\mathbf{x}$ is not on the decoded list, averaged over the choice of bin assignments can be bounded as follows.

$$\Pr[\mathbf{x} \notin \mathcal{L}(\mathbf{y})] = \sum_{\mathbf{x},\mathbf{y}} \Pr[\mathbf{x} \notin \mathcal{L}(\mathbf{y}) | \mathbf{x},\mathbf{y}] p_{\mathbf{x},\mathbf{y}}(\mathbf{x},\mathbf{y}) \tag{5}$$

$$\leq \sum_{\mathbf{x},\mathbf{y}} \Pr[\exists\ L\ \tilde{\mathbf{x}} \in \mathcal{B}(\mathbf{x}) \text{ s.t. } p_{\mathbf{x}|\mathbf{y}}(\tilde{\mathbf{x}}|\mathbf{y}) \geq p_{\mathbf{x}|\mathbf{y}}(\mathbf{x}|\mathbf{y}) | \mathbf{x},\mathbf{y}] \; p_{\mathbf{x},\mathbf{y}}(\mathbf{x},\mathbf{y}). \tag{6}$$

Equation (6) says that an error can occur only if there are L elements in the bin $\mathcal{B}(\mathbf{x})$ containing the realized source sequence, such that all L are conditionally more likely than $\mathbf{x}$. To get a handle on this event we define $M(\mathbf{x},\mathbf{y})$ to be the cardinality of the set of sequences that are more likely than $\mathbf{x}$ when conditioned on $\mathbf{y}$,

$$M(\mathbf{x},\mathbf{y}) = |\{\tilde{\mathbf{x}} \text{ s.t. } p_{\mathbf{x}|\mathbf{y}}(\tilde{\mathbf{x}}|\mathbf{y}) \geq p_{\mathbf{x}|\mathbf{y}}(\mathbf{x}|\mathbf{y})\}|. \tag{7}$$

For any source/side-information pair $(\mathbf{x},\mathbf{y})$, we enumerate the length-L lists of source sequences that can lead to list-decoding errors as $\mathcal{L}_i(\mathbf{x},\mathbf{y})$. The index $i \in \Big\{1,2,\ldots \binom{M(\mathbf{x},\mathbf{y})-1}{L}\Big\}$. For compactness we write $\mathcal{L}_i$, implicitly understanding the dependency on $(\mathbf{x},\mathbf{y})$. If any of these lists is fully contained in the bin of source sequences that contains $\mathbf{x}$, i.e., $\mathcal{L}_i \subset \mathcal{B}(\mathbf{x})$ for some i, then a list decoding error occurs. We define the event $A_{\mathcal{L}_i,\mathbf{x}}$ to be the event that list $\mathcal{L}_i \subset \mathcal{B}(\mathbf{x})$. The error probability (6) can be expressed as

$$\begin{aligned}
&\sum_{\mathbf{x},\mathbf{y}} \Pr[\cup_i A_{\mathcal{L}_i,\mathbf{x}} | \mathbf{x},\mathbf{y}] p_{\mathbf{x},\mathbf{y}}(\mathbf{x},\mathbf{y}) \\
&\leq \sum_{\mathbf{x},\mathbf{y}} \min\Big\{1, \sum_{i} \Pr[A_{\mathcal{L}_i,\mathbf{x}} | \mathbf{x},\mathbf{y}]\Big\} p_{\mathbf{x},\mathbf{y}}(\mathbf{x},\mathbf{y}) \\
&= \sum_{\mathbf{x},\mathbf{y}} \min\Big\{1, \sum_{i} 2^{-nLR}\Big\} p_{\mathbf{x},\mathbf{y}}(\mathbf{x},\mathbf{y})
\end{aligned} \tag{8}$$

[1]In an implementation of these idea using, e.g., low-density parity-check codes (LDPCs) to encode (i.e., syndrome generation) and belief propagation (BP) to decode, certain design issues arise that are not revealed by this analysis. For example, while for an idealized random binning encoder the detailed source statistics are not important (only the entropy is), this is not the case when using LDPCs. Rather, the degree distribution of the code should be tuned to the side-information channel using density-evolution, EXIT-charts, or some other design technique. Further, a degree distribution that is universally good for all P source distributions will in general not exist. Therefore, in a compound setting the code would need to be designed according to the worst-case joint distribution, akin to how the rate is set in Thm 1.

$$= \sum_{\mathbf{x},\mathbf{y}} \min\left\{1, \binom{M(\mathbf{x},\mathbf{y})-1}{L} 2^{-nLR}\right\} p_{\mathsf{x},\mathsf{y}}(\mathbf{x},\mathbf{y})$$

$$\leq \sum_{\mathbf{x},\mathbf{y}} \min\left\{1, (M(\mathbf{x},\mathbf{y})-1)^L 2^{-nLR}\right\} p_{\mathsf{x},\mathsf{y}}(\mathbf{x},\mathbf{y})$$

$$\leq \sum_{\mathbf{x},\mathbf{y}} \min\left\{1, \Big(\sum_{\tilde{\mathbf{x}}} 1\left[p_{\mathsf{x}|\mathsf{y}}(\tilde{\mathbf{x}}|\mathbf{y}) \geq p_{\mathsf{x}|\mathsf{y}}(\mathbf{x}|\mathbf{y})\right]\Big)^L 2^{-nLR}\right\} p_{\mathsf{x},\mathsf{y}}(\mathbf{x},\mathbf{y})$$

$$\leq \sum_{\mathbf{x},\mathbf{y}} \min\left\{1, \left(\sum_{\tilde{\mathbf{x}}} \min\left\{1, \frac{p_{\mathsf{x}|\mathsf{y}}(\tilde{\mathbf{x}}|\mathbf{y})}{p_{\mathsf{x}|\mathsf{y}}(\mathbf{x}|\mathbf{y})}\right\}\right)^L 2^{-nLR}\right\} p_{\mathsf{x},\mathsf{y}}(\mathbf{x},\mathbf{y})$$

$$\leq \sum_{\mathbf{x},\mathbf{y}} \left(\sum_{\tilde{\mathbf{x}}} \frac{p_{\mathsf{x}|\mathsf{y}}(\tilde{\mathbf{x}}|\mathbf{y})^s}{p_{\mathsf{x}|\mathsf{y}}(\mathbf{x}|\mathbf{y})^s}\right)^{L\rho_0} 2^{-nL\rho_0 R} \; p_{\mathsf{x},\mathsf{y}}(\mathbf{x},\mathbf{y}) \tag{9}$$

$$\leq \sum_{\mathbf{y}} p_{\mathsf{y}}(\mathbf{y}) \sum_{\mathbf{x}} p_{\mathsf{x}|\mathsf{y}}(\mathbf{x}|\mathbf{y})^{1-L\rho_0 s} \Big(\sum_{\tilde{\mathbf{x}}} p_{\mathsf{x}|\mathsf{y}}(\tilde{\mathbf{x}}|\mathbf{y})^s\Big)^{L\rho_0}$$

$$= \sum_{\mathbf{y}} p_{\mathsf{y}}(\mathbf{y}) 2^{-n\rho R} \Big(\sum_{\mathbf{x}} p_{\mathsf{x}|\mathsf{y}}(\mathbf{x}|\mathbf{y})^{\frac{1}{1+\rho}}\Big)^{1+\rho} \tag{10}$$

$$= 2^{-n\rho R} \sum_{\mathbf{y}} \Big(\sum_{\mathbf{x}} p_{\mathsf{x},\mathsf{y}}(\mathbf{x},\mathbf{y})^{\frac{1}{1+\rho}}\Big)^{1+\rho} \tag{11}$$

Sequences are independently and uniformly assigned to the 2^{nR} bins, so in (8) the probability that all the sequences on any particular list are all assigned to $\mathcal{B}(\mathbf{x})$ is 2^{-nRL}. In (9) we twice use Gallager's refined union bound [7] where $0 \leq s \leq 1$ and $0 \leq \rho_0 \leq 1$. In (10) we define $\rho = L\rho_0$ and let $s = 1/(1+\rho)$, therefore in (11) $0 \leq \rho \leq L$, giving (2).

For i.i.d. $p_{\mathsf{x},\mathsf{y}}(\mathbf{x},\mathbf{y}) = \prod_{i=1}^n p_{x,y}(x_i, y_i)$, and (11) becomes

$$2^{-n\rho R}\Big[\sum_y \Big(\sum_x p_{x,y}(x,y)^{\frac{1}{1+\rho}}\Big)^{(1+\rho)}\Big]^n.$$

Since the bound holds for all ρ we get the tightest bound by maximizing over ρ, giving the Slepian-Wolf length-L list-decoding random binning error exponent for i.i.d. sources:

$$\frac{-\log \Pr[\mathbf{x} \notin \mathcal{L}(\mathbf{y})]}{n} \geq \max_{0\leq\rho\leq L} \rho R - \log \sum_y \Big(\sum_x p_{x,y}(x,y)^{\frac{1}{1+\rho}}\Big)^{1+\rho}.$$

C. Error exponent of compound conditional source coding

Corollary 2 is proved in Sec. II-B. We now understand the choice of encoding rate. We encode using a randomized Slepian-Wolf code, per the achievability of Thm. 2, at rate $R \geq \max_p H(x|y_p)$ per Thm. 1. Resolution information is calculated, and decoding is performed, per Sec. II-B.

VI. Conclusions

Much recent work on "Wyner-Ziv" video coding poses video compression as an application of distributed source coding. We make the point that some of these scenarios are more exactly described as "compound conditional" problems. Distributed source coding techniques, while centrally important to robustly address the compound nature of these problems, do not by themselves characterize the full range of operational possibilities. As an example, in this paper we demonstrate that the achievable error exponents of compound conditional problems exceed those of distributed source coding.

We expect that the distinction between distributed and compound conditional source coding can be leveraged in other ways to implement further improvements. For example, in the rate-distortion coding of non-jointly-Gaussian sources there is a rate-loss in the Wyner-Ziv problem. This begs the question of whether, in a compound setting, the best approach is based on a Wyner-Ziv distributed coding solution, a pair of conditionally-coded solutions, or some hybrid scheme.

Another direction is to explore the use of universal list decoders. This scenario arises when the joint source/side-information distributions are not known though, as herein, the index k of the realized side information is known by the decoder. In this case a universal Slepian-Wolf list decoder (see, e.g., [4], [5] for applications of universal binning decoders to (non-list) Slepian-Wolf and Wyner-Ziv problems) would be used in the place of the list decoder used herein.

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