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Andreaas F. Molisch, Xinying Zhang

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Abstract

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Andreas F. Molisch, *Senior Member, IEEE,* and Xinying Zhang

***Abstract*—In this letter, we address the antenna subset selection problem in spatially correlated multiple-input multiple-output (MIMO) channels. To reduce the severe performance degradation of the traditional antenna selection scheme in correlated channels, we propose to embed fast Fourier transform operations in the RF chains. The resulting system shows a significant advantage both for diversity schemes and for the capacity of spatial multiplexing, while requiring only a minor hardware overhead.**

***Index Terms*—Antenna selection, diversity, MIMO.**

I. INTRODUCTION

MULTIPLE-INPUT multiple-output (MIMO) wireless systems, i.e., systems with multiple antennas at both link ends, can improve data rates or increase the diversity order, as demonstrated by analytical and simulation studies [4], [11]. One major drawback of conventional MIMO systems is the requirement for multiple RF chains (one for each antenna element), which leads to high implementation costs. For this reason, recent papers [6]–[9], have suggested antenna selection schemes that optimally choose a subset of the available transmit and/or receive antennas, and process the signals associated with those antennas. This allows to maximally benefit from the multiple antennas within the RF cost constraint.

These antenna selection schemes work well for the uncorrelated MIMO channels (e.g., i.i.d. Rayleigh fading at each antenna element). Hybrid selection–maximum ratio combining (HS-MRC) schemes perform almost as well as pure maximum ratio combining with the same number of antenna elements (and thus more RF chains) [9]. Similarly, spatial multiplexing MIMO systems with antenna selection (HS-MIMO) show high capacity in uncorrelated channels as long as the number of RF chains is at least as large as the number of available data streams [6], [8]. Also space-time codes combined with antenna selection perform well [5].

However, most practically occurring cellular channels exhibit fading correlation due to a nonuniform power azimuth spectrum (APS) at the base station (BS) [3]. In such channels, HS schemes performs considerably worse than full-complexity schemes [9]. In the current letter, we present a novel, simple but highly effective, hybrid antenna selection scheme that performs as well as full-complexity schemes in fully correlated channels, and as well as HS in uncorrelated channels. This is achieved by a fast Fourier fransformation (FFT) of the (spatial) received signal vector, which can be realized in the RF domain by sending the antenna signals through a Butler matrix. This system shows significant performance improvements for HS-MIMO as well. For simplicity only receive antenna selection is discussed in this paper, while the transmitter fully exploits all available antennas. However, the transmit selection can be handled in duality.

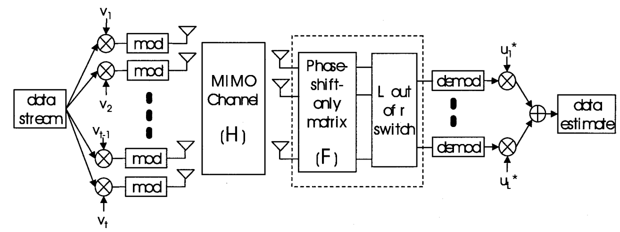

Fig. 1. MIMO channel model and system diagram.

II. SYSTEM AND CHANNEL MODEL

We consider a multiple antenna system with uniform linear arrays containing t transmit and r receive antenna elements, respectively, where $\mathbf{H}$ is the $r \times t$ transfer function of the MIMO channel. We adopt the widely used model [2], [7], [10]

$$\mathbf{H} = \mathbf{R}^{\frac{1}{2}} \mathbf{W} \left(\mathbf{T}^{\frac{1}{2}} \right)^T \tag{1}$$

where $\mathbf{W}$ is a matrix with i.i.d. complex Gaussian entries $\sim \mathcal{N}_C(0,1)$, and $\mathbf{R}, \mathbf{T}$ are $r \times r$, $t \times t$ matrices denoting receive and transmit correlations, respectively, and superscript ^{T} denotes transposition. Such a model is usually valid when assuming independent transmit and receive correlations. We furthermore assume the directions-of-arrival at the receiver are Gaussian-distributed around the mean values: $\theta = \theta_r + \epsilon; \epsilon \sim \mathcal{N}(0, \sigma_r^2)$. This allows a closed-form computation of the entries of $\mathbf{R}$, see [1]. The directions-of-arrival at the transmitter are uniformly distributed, so that $\mathbf{T}$ is a $t \times t$ identity matrix $\mathbf{I}_t$. This is a reasonable model for the uplink of a cellular system.

III. TRANSMIT/RECEIVE DIVERSITY

We first consider a system with a single data stream, see Fig.1. To maximize the diversity gain, the information stream is multiplied by a t-dimensional complex weighting vector before it is modulated to the passband and applied to each of the t transmitting antennas. The vector of complex-valued signals at the receive antenna elements $\vec{x}(k) \in \mathcal{C}^r$ is

$$\vec{x}(k) = \mathbf{H}\vec{v}s(k) + \vec{n}(k) \tag{2}$$

where $s(k) \in \mathcal{C}$ is the transmitting stream. The total transmission power is constrained to P. The thermal noises $\vec{n}(k) \in \mathcal{C}^r$

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A. F. Molisch is with Mitsubishi Electric Research Laboratory, Cambridge, MA 02139 USA, and also with the Department of Electroscience, Lund University, Lund 22100, Sweden (email: Andreas.Molisch@ieee.org).

X. Zhang is with the Department of Electrical Engineering, Princeton University, Princeton, NJ 08544 USA (email: xinying@ee.princeton.edu).

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are white i.i.d. Gaussian random processes with independent real and imaginary parts and variance $N\mathbf{I}_r$, and $\vec{v}$ is the t-dimensional transmitter weighting vector satisfying $||\vec{v}|| = 1$. In a conventional HS-MRC receiver, L out of the r observation streams are selected, downconverted, and linearly combined with the coefficient vector $\vec{u}^*$, where * denotes complex conjugation. In our new scheme, the observation streams are first passed through a $r \times r$ Fourier transformation before the selection.

For the determination of the optimum weights we introduce the singular value decomposition (SVD) of $\mathbf{H}$: $\mathbf{H} = \mathbf{U}\mathbf{\Sigma}\mathbf{V}^\dagger$, where $\mathbf{U}$ and $\mathbf{V}^\dagger$ are unitary matrices representing the left and right singular vector spaces of $\mathbf{H}$, respectively; and $\mathbf{\Sigma}$ is the diagonal matrices consisting of all the singular values of $\mathbf{H}$. $\lambda_i(A)$ denotes the ith largest singular value of a matrix A; † denotes Hermitian transpose.

1) *Using all Antennas:* In this case, it is obvious that MRT and MRC should be adopted to maximize the estimate signal-to-noise ratio (SNR), i.e. $\vec{u}(\vec{v})$ should be the singular vector in $\mathbf{U}$ ($\mathbf{V}$) corresponding to the largest singular value $\lambda_1(\mathbf{H})$. The resulting SNR is then $\rho\lambda_1^2(\mathbf{H})$ where $\rho = (P/N)$ is the nominal SNR.
2) *Antenna selection:* Now we assume that L out of the r antenna elements are selected at the receiver. Mathematically, each selection option corresponds to a reduced-size transfer function matrix, which is formed by extracting the L rows of $\mathbf{H}$ that are associated with the selected antennas. We denote the set of all such submatrices as $\mathcal{S}_L(\mathbf{H})$. Therefore for the pure L/r antenna selection, the optimal SNR achieved is

$$\max_{\tilde{\mathbf{H}} \in \mathcal{S}_L(\mathbf{H})} \rho\lambda_1^2(\tilde{\mathbf{H}}).$$

This scheme shows good performance when the fading at the antenna elements is independent, but not for strongly correlated fading. In the limit of a single incident fading wave, selection diversity does not improve performance, and maximum ratio combining reduces to pure beamforming; the beamforming gain is proportional to the number of combined signals r.
3) *FFT-Based Selection:* In this new scheme, we send all received observation streams through a (spatial) Fourier transform before selection and downconversion. This can be implemented easily by means of a Butler matrix, which performs—in the RF domain—a r-point FFT described by the $r \times r$ matrix $\mathbf{F}$. Thus the selection is performed on the virtual channel $\mathbf{FH}$. In the meanwhile, the thermal noises are also multiplied by $\mathbf{F}$, resulting in a (different) vector of i.i.d. Gaussian noise variables; note that the *statistics* of the noise are preserved by this operation.

Let us next give an intuitive argument for the use of the FFT. The output of the FFT can be regarded as "beams" oriented into different directions in space. Each beam implicitly has a beamforming gain proportional to the dimension of the FFT, which is r. In a strongly correlated channel, the scheme just picks the strongest beam, and is thus as good as MRC. When the PAS is uniform, the FFT has no effect on the performance: selecting the best L beams and combining them with maximum ratio combining gives the same performance as selecting the best L antenna signals. This interpretation is supported by Fig. 2 which

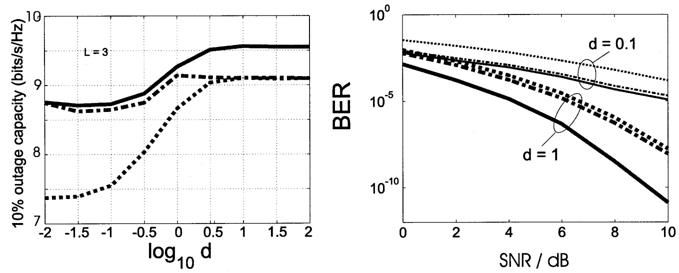

Fig. 2. Left graph: 10% outage capacity with respect to the relative antenna spacing d for $\rho = 20$ db. Right graph: BER of BPSK as a function of SNR for $d = 0.1$ and $d = 1$. Full-complexity receiver (solid surve), pure antenna selection (dotted curve) and FFT-antenna selection (dash-dotted curve) with $t = 2$, $r = 8$, $\theta_r = (\pi/4)$ and $\sigma_r = (\pi/12)$. with same channel and system parameters.

shows the 10% outage capacity (as defined by [4]) as a function of the fading correlation for a system with 2 transmit antennas, $r = 8$ receive antenna elements, and $L = 3$ RF chains. We see that for large correlation (meaning a small ratio of antenna spacing to correlation length of the channel), antenna selection performs considerably worse than the FFT-based selection or the full-complexity scheme. At low correlation, FFT-based selection shows the same performance as antenna selection. Fig. 3 shows the capacity distribution functions for all three schemes with different numbers of receiver chains L. We see that our FFT-based selection outperforms antenna selection especially for small L

We stress that $\mathbf{F}$ is not the optimum transformation matrix; rather, a matrix that adapts to the instantaneous channel realization is required to give the best performance for i.i.d. fading. However, the use of such a matrix requires an adaptive implementation of a general transformation matrix in RF. We suggest the FFT matrix because it gives good performance for a wide range of channel configurations, does not need adaptive elements, and is very simple to implement. A compromise solution, which uses adaptive phase shifters only, was recently suggested by us [12].

IV. Spatial Multiplexing

Contrary to the transmission of a single information stream over multiple antennas in Section III, different streams can be applied on different antenna elements to provide a maximal data rate, which is illustrated in Fig. 4. In this case, the system model is

$$\vec{x}(k) = \mathbf{H}\vec{s}(k) + \vec{n}(k), \tag{3}$$

where $\vec{s}(k)$ is now a $t \times 1$ vector denoting the transmit sequences. As the channel realization is unavailable at the transmitter, we assume even power distribution, i.e., $\mathcal{E}[\vec{s}(k)\vec{s}^*(k)] = (P/t)\mathbf{I}_t$. With spatial multiplexing, capacity is the vital parameter to evaluate the system performance.

1) *Capacity Using all Antennas:* The channel capacity of the original MIMO system in (3) at equal power distribution is well known to be

$$C = \sum_{i=1}^{t} \log_2 \left[1 + \frac{\rho}{t}\left|\lambda_i(\mathbf{H})\right|^2\right]. \tag{4}$$

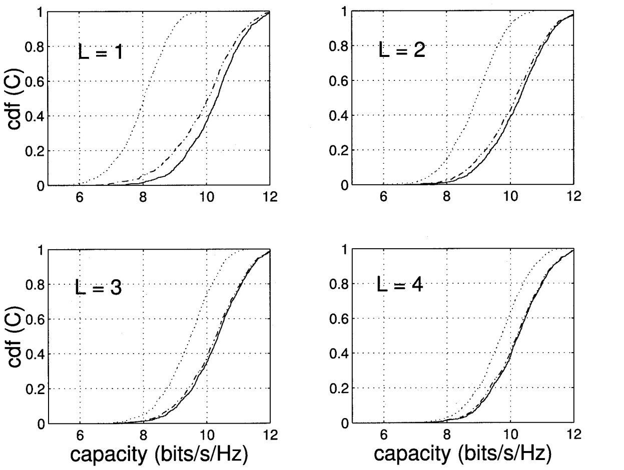

Fig. 3. Capacity cdf: full-complexity system (solid surve), pure antenna selection (dotted curve) and FFT-antenna selection (dash-dotted curve) with $t = 2, r = 8, \rho = 20$ db, $d = 0.5, \theta_r = (\pi/12)$ and $\sigma_r = (\pi/12)$.

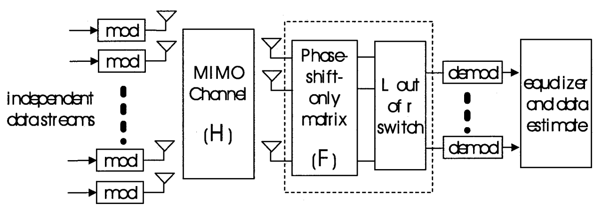

Fig. 4. System and channel model for spatial multiplexing.

2) *Antenna Selection:* With antenna selection in the receiver end, the optimal choice that maximizes the resulting capacity is

$$C = \max_{\tilde{\mathbf{H}} \in S_L(\mathbf{H})} \sum_{i=1}^{L} \log_2 \left[1 + \frac{\rho}{t} \left| \lambda_i(\tilde{\mathbf{H}}) \right|^2 \right]. \tag{5}$$

3) *FFT-Based Selection:* Similarly, with FFT involved, the optimal achievable capacity after antenna selection is given by (5) with $\mathbf{H}$ replaced by $\mathbf{FH}$.

Again, we see that the FFT-based selection shows considerably better performance than antenna selection (Fig. 5). For $L = 1$, the performance of both antenna selection and FFT-based selection is much worse than for a full-complexity scheme, since the number of receive RF chains is smaller than the number of transmit antennas, so that the separation of data streams becomes difficult. For $L \geq t$, both selection schemes can support the t data streams, but the FFT-based scheme outperforms antenna selection scheme by about 2 b/s/Hz, because it has better SNR.

V. Summary and Conclusions

We presented a new antenna selection scheme that shows excellent performance for arbitrary fading correlation of the received signals. The received signals are first spatially Fourier-transformed, and then the best L out of the total r received signals are downconverted and processed. We show that this scheme performs as well as L/r HS-MRC in uncorrelated-fading channels, and much better, namely as well as r-signal MRC in strongly correlated channels. It has (apart from a Butler matrix) the same hardware effort as L/r HS-MRC, which means the saving of $r - L$ RF chains compared to r-signal MRC. Computer experiments confirm our conclusions. We note that while the formulation here was given for selection at the receiver, the scheme can be implemented in a completely analogous fashion at the transmitter, or at both link ends.

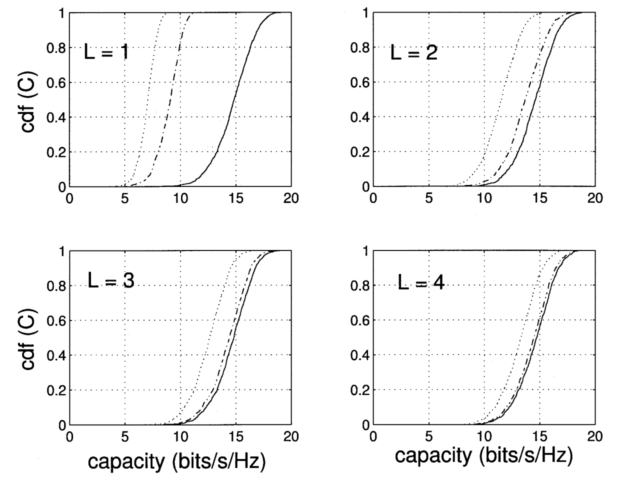

Fig. 5. Capacity cdf with spatial multiplexing: full-complexity (solid surve), antenna selection (dotted curve) and beam selection (dash-dotted curve) with $t = 2, r = 8, \rho = 20$ dB, $d = 0.5, \theta_r = (\pi/12)$ and $\sigma_r = (\pi/12)$.

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References

[1] D. Asztely, "Antenna arrays in mobile communication systems: fast fading and GSM base station receiver algorithms," Royal Inst. of Technology, Stockholm, Sweden, Tech. Rep. IR-S3-SB-9611, Mar. 1996.

[2] H. Bölcskei and A. J. Paulraj, "Performance analysis of space-time codes in correlated Rayleigh fading environments," in *Proc. Asilomar Conf. Signals, Systems, and Computers*, Pacific Grove, CA, Nov. 2000, pp. 687–693.

[3] R. B. Ertel, P. Cardieri, K. W. Sowerby, T. S. Rappaport, and J. H. Reed, "Overview of spatial channel models for antenna array communication systems," *IEEE Trans. Pers. Commun.*, vol. 41, pp. 10–22, Feb. 1998.

[4] G. J. Foschini and M. J. Gans, "On limits of wireless communications in a fading environment when using multiple antennas," *Wireless Pers, Commun.*, vol. 6, pp. 311–335, 1998.

[5] A. Ghrayeb and T. M. Duman, "Performance analysis of MIMO systems with antenna selection over quasifading channels," in *2002 Proc. IEEE Symp. Information Theory*, 2002, p. 333.

[6] D. Gore, R. Nabar, and A. Paulraj, "Selection of an optimal set of transmit antennas for a low rank matrix channel," in *Proc. IEEE ICASSP 2000*, pp. 2785–2788.

[7] D. A. Gore and A. J. Paulraj, "MIMO antenna subset selection with space-time coding," *IEEE Trans. Signal Processing*, vol. 50, pp. 2580–2588, Oct. 2002.

[8] A. F. Molisch, M. Z. Win, and J. H. Winters, "Capacity of MIMO systems with antenna selection," in *Proc. ICC*, Helsinki, Finland, 2001, pp. 570–574.

[9] ——, "Reduced-complexity transmit/receive-diversity systems," IEEE Trans. Signal Processing, vol. 51, pp. 2729–2738, Nov. 2003, to be published.

[10] D. Shiu, G. J. Foschini, M. J. Gans, and J. M. Kahn, "Fading correlation and its effect on the capacity of multi-element antenna systems," *IEEE Trans. Commun.*, vol. 48, pp. 502–513, Mar. 2000.

[11] J. H. Winters, "On the capacity of radio communication systems with diversity in Rayleigh fading environments," *IEEE J. Select. Areas Commun.*, vol. 5, pp. 871–878, June 1987.

[12] X. Zhang, A. F. Molisch, and S. Y. Kung, "Antenna selection under spatially correlated MIMO channels," in *Proc. Globecom 2003*.