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### **Abstract**

Exhaustive searches were applied to uniformly-excited sparse linear arrays to find the configurations with lowest peak sidelobe levels. By removing interior elements from the non-sparse array and moving the remaining elements to intermediate positions, we found configurations with substantially lower peak sidelobe levels than the non-sparse case with minimal broadening of the main lobe. An interactive, visual browser allows a designer to compare configurations and understand the trade-offs between optimal and sub-optimal designs.

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# Sidelobe Minimization of Uniformly-Excited Sparse Linear Arrays using Exhaustive Search and Visual Browsing

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Exhaustive searches were applied to uniformly-excited sparse linear arrays to find the configurations with lowest peak sidelobe levels. By removing interior elements from the non-sparse array and moving the remaining elements to intermediate positions, we found configurations with substantially lower peak sidelobe levels than the non-sparse case with minimal broadening of the main lobe. An interactive, visual browser allows a designer to compare configurations and understand the trade-offs between optimal and sub-optimal designs.

## Introduction

Our current work involves optimizing the peak sidelobe levels of sparse linear arrays, which could be applied to a distributed array radar problem [3]. Because we are interested in the true optimum and not an approximation to it, it was necessary to exhaustively search the parameter space of each problem and store the best results. We have done previous work in computer-human antenna optimization [1], including the use of exhaustive searches for smaller problems [2] ( $\sim 10^4$  configurations). The current problems are large enough ( $\sim 10^9$  configurations) to require the use of a supercomputer and special techniques to handle the generated data.

## The Problem

We consider a linear array of length  $120\lambda$  on which array elements can be placed. Each element is  $10\lambda$  wide and is itself composed of 16 isotropic radiators spaced  $0.625\lambda$  apart. One element is always placed at each end of the array, leaving room for up to 10 elements in the interior. Elements can be placed at  $\lambda/2$  boundaries only, and may not overlap each other or extend beyond the ends of the array. We further consider each case of  $n$  elements in the interior, where  $n$  ranges from 1 to 10. Because the positions are quantized, it is possible to enumerate all possible unique ways of placing the  $n$  elements according to the above rules. We use a recursive algorithm to do this. Figure 1 shows a typical configuration. For each enumerated configuration,

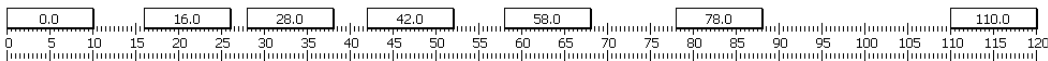


Figure 1: A typical array configuration with 5 interior elements. Each element is labeled with its position, i.e. the distance in units of  $\lambda$  from the beginning of the array.

we compute the gain pattern (assuming uniform excitation of the elements) and calculate the width of the main lobe (full-width half maximum) and the gain and angular position of the peak sidelobe. Because the number of configurations is very large for some  $n$  (hundreds of millions – see Table 1), the computations are done on a Beowulf cluster supercomputer. The results are then pruned down to the one million best configurations which can be explored with our visual browser.

## The Visual Browser

We developed a prototype browsing interface to explore the generated data. Such a visual interface allows designers to easily understand the trade-offs between different configurations. When trying to understand large data sets it is crucial to provide high-level overviews of the data along with the ability to focus-down and examine particular regions of interest in the parameter space, and eventually individual configurations. To support these complimentary tasks, Our interface has two components: an **Overview Pane** and an **Inspect Pane** (left and right sides of Figure 2, respectively).

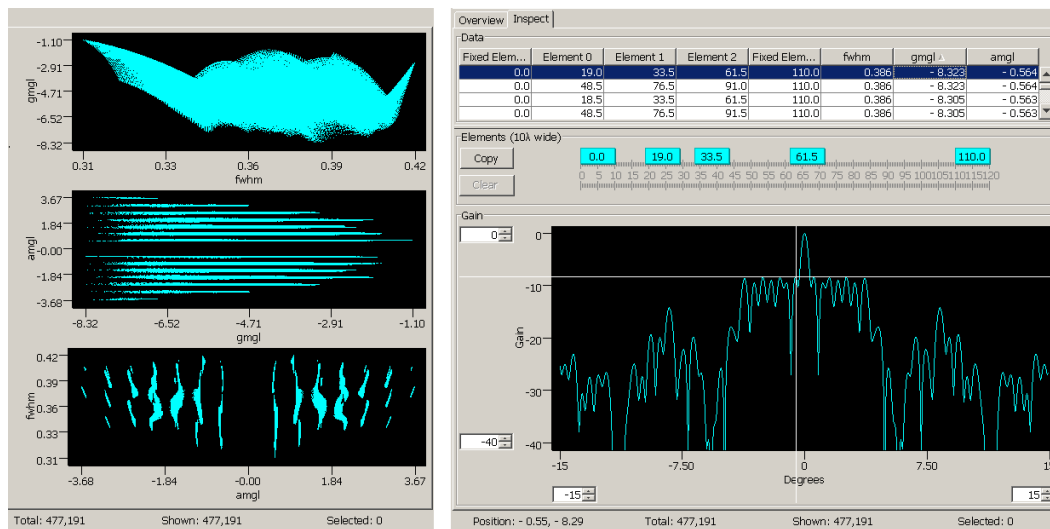


Figure 2: Visual browser with **Overview Pane** (left) and **Inspect Pane** (right).

The **Overview Pane** allows the designer to quickly identify the bounds of a data set by providing multiple graphs which compare various design parameters. For the problem explored in this paper these design parameters are the main lobe width and the gain and angle of the peak sidelobe; three graphs provide pair-wise comparisons of these variables. A designer can query the system by drawing a rectangle in the area of interest directly on any of the graphs. Separate selections can be made in each graph window. The graphs are linked so that query results (i.e., datapoints falling within the depicted range) are displayed simultaneous in all three graphs. The **Overview Pane** also provides zooming capability to allow the designer to focus in on successive query results to see more detail.

The **Inspect Pane** allows the designer to see details for individual antenna designs. It provides a table of the design parameters in a spreadsheet style, along with gain pattern display area. Selecting an antenna in the table causes its gain pattern to be displayed, along with a visual layout of that particular antenna's elements. The interface also allows a second antenna's gain pattern and elements to be displayed *and* the elements to be manually positioned to explore modified configurations.

## Results

Figure 3 shows graphically the results of the calculations. For larger  $n$  there is sufficient data so that the histograms have a near-Gaussian distribution. Because of the long, low histogram tails, the best results are not visible in the graph. This means the number of configurations near the optimum point is quite small compared to the total number of configurations, and thus would be difficult to find using a stochastic method. Table 1 provides a summary of the results, including the extrema. The actual configurations which provide the optimum results are presented in Table 2. Note that the case of  $n = 10$  is equivalent to the non-sparse case, and has a peak sidelobe at  $-13.3\text{dB}$ . There are configurations with  $n = 6-9$  which have better peak sidelobe suppression than the non-sparse case, but which widen the main lobe by only a few percent. It is therefore possible to realize sidelobe characteristics compatible with a non-sparse array in a distributed array with non-uniform spacing.

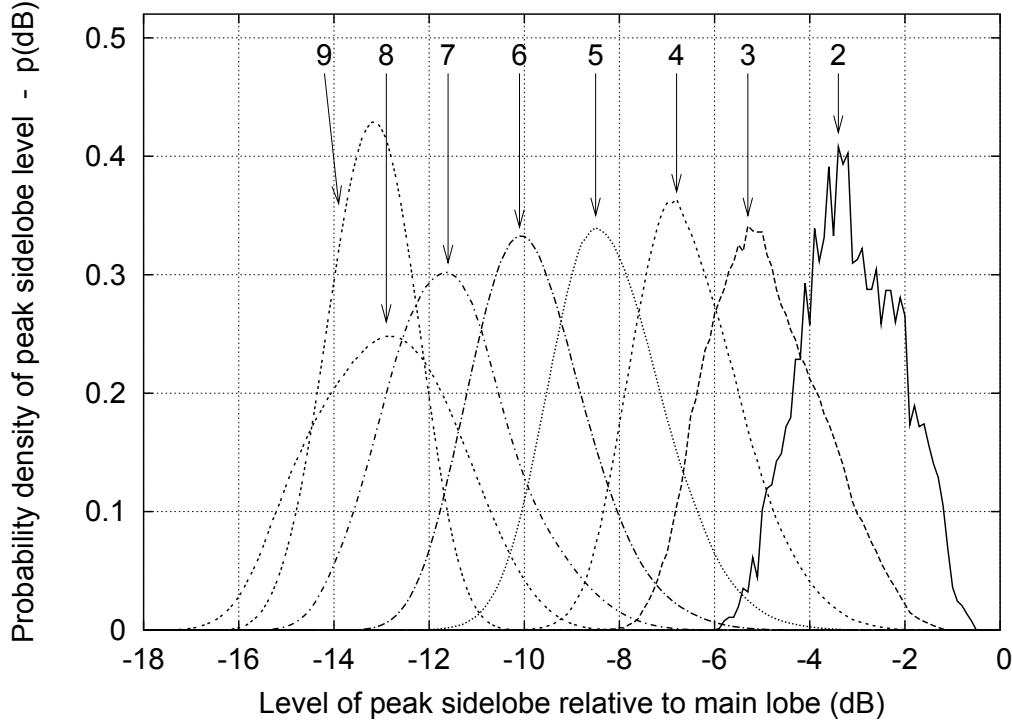


Figure 3: Histograms of the peak sidelobe level for configurations with 2 to 9 interior elements. The histograms have been normalized so that they integrate to 1, making them probability density functions.

Interior elements $n$	1	2	3	4	5
# configurations	181	13,041	477,191	9,381,251	96,560,646
Sidelobe max.	-0.4	-0.6	-1.1	-1.6	-2.5
Sidelobe mode	-1.1	-3.4	-5.3	-6.8	-8.5
Sidelobe min.	-2.3	-5.9	-8.3	-10.5	-13.1
Main lobe width	0.313	0.366	0.386	0.403	0.459
Interior elements $n$	6	7	8	9	10
# configurations	470,155,077	869,648,208	377,348,994	10,015,005	1
Sidelobe max.	-3.5	-5.2	-7.1	-9.9	-13.3
Sidelobe mode	-10.1	-11.6	-12.9	-13.1	-13.3
Sidelobe min.	-14.1	-16.2	-18.0	-16.3	-13.3
Main lobe width	0.434	0.432	0.439	0.443	0.423

Table 1: Summary of the results for each problem size  $n$ , including the total number of possible configurations, the maximum, mode and minimum peak sidelobes encountered (in dB relative to the main lobe) and the main lobe width (in degrees) for the configuration with the minimum peak side lobe.

Element number	1	2	3	4	5	6	7	8	9
1 element	30.0								
2 elements	29.0	46.0							
3 elements	19.0	33.5	61.5						
4 elements	16.0	29.0	47.5	72.0					
5 elements	14.5	24.5	34.5	44.5	58.5				
6 elements	13.0	24.5	38.0	49.5	63.0	80.0			
7 elements	12.5	27.0	40.5	54.0	65.0	75.0	88.0		
8 elements	11.5	24.5	36.0	46.5	56.5	67.0	78.0	91.0	
9 elements	10.0	20.0	30.0	40.0	50.0	60.0	70.0	80.0	90.0

Table 2: The positions of the elements (in units of  $\lambda$ ) for the configuration with lowest peak sidelobe for each  $n$ . The peak sidelobe levels and main lobe widths are in Table 1. The table does not include the two exterior elements whose positions do not change across configurations: they are at positions 0.0 and  $110.0\lambda$ .

## References

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