# A dCDD-Based Transmit Diversity for NOMA Systems 

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#### Abstract

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#### Abstract

In this paper, a new transmit diversity scheme for cooperative non-orthogonal multiple access (NOMA) is proposed without perfect channel state information at the transmitter (CSIT). To support two users under a near-far user pairing constraint, a distributed cyclic delay diversity (dCDD) scheme is adjusted into NOMA by dividing a set of remote radio heads (RRHs) into two groups for multiple cyclic-prefixed single carrier transmissions. Using only a limited channel relevant information needed to make dCDD work, a new RRH assignment and power allocation mechanism is proposed. After then, closed-form expressions for the rates of two users achieved by the proposed RRH assignment and power allocation mechanism are derived. For various scenarios, link-level simulations verify that superior rates can be achieved by NOMA with dCDD over the traditional orthogonal multiple access with dCDD.


Index Terms-Distributed cyclic delay diversity, NOMA, cyclicprefixed single carrier transmission, near-far user pairing, rate.

## I. Introduction

To meet a high spectral efficiency, ultra-reliability, and lowlatency required by tactile Internet, mobile edge computing, and beyond fifth generation (B5G) networks, non-orthogonal multiple access (NOMA) has emerged as a key technique for the upcoming decade of wireless communication evolution [1], [2]. With the aid of superposition coding in the power-domain, multiple users are multiplexed on the same time-frequency resource block with different power levels. To mitigate intracluster interference inherited from NOMA principles, successive interference cancellation (SIC) is in general applied at the receiver to recover the signals [3].

To enhance system performance and provide new degrees of freedom (DoF), various diversity techniques have been applied to NOMA, including multiple-input multiple-output (MIMO) antennas techniques and cooperative relaying schemes. Compared to conventional MIMO orthogonal multiple access (OMA), MIMO-NOMA can achieve a larger diversity order while serving severing more users [4]. It has been shown in [4] and [5] that channel gain disparity affects the diversity order achieved by NOMA users. To achieve full diversity for NOMA systems, transmit power allocation was proposed in

[^1][6]. To avoid the sophisticated channel ordering required by SIC, MIMO-NOMA was decomposed into single-input singleoutput (SISO) NOMA with a greatly decreased complexity [3]. However, power allocation and user ordering in MIMONOMA systems make the system performance evaluation more challenging over that of SISO-NOMA systems [7]. By sorting the users according to quality of service ( QoS ) requirements, the impact of relay selection schemes on performance of cooperative NOMA systems was investigated in [8] and [9], where two-stage relay selection protocols were proposed for cooperative NOMA systems with fixed power allocation and adaptive power allocation, respectively. The results showed that diversity gain for the two-stage decode-and-forward (DF) relaying scheme is proportional to the number of relays. The same results were obtained in [10], where the users were ordered by channel conditions.
In contrast to existing work, our main contributions can be summarized as follows.

- To achieve the transmit diversity gain without perfect channel state information at the transmitter (CSIT), the distributed cyclic delay diversity (dCDD) scheme [11], [12] is employed among spatially distributed central unit (CU), remote radio heads (RRHs), and two users. Furthermore, to support the near-far user pairing for NOMA [13], the original dCDD scheme is adjusted as follows to transmit two information signals simultaneously.
- The CU divides a set of available RRHs into two groups based on the channel magnitudes, so that two users need to feed back a channel relevant information to the CU. This makes the CU control each group to transmit its own information signal to two users simultaneously. Thus, the proposed transmission scheme is different from the existing NOMA scheme that transmits the superposed signal.
- For two separate groups, the CU allocates a different power to support the near-far user pairing.
- Due to different geographical locations of two users, non-identical frequency selective fading channels from RRHs to two users are considered. Over realistic and challenging channels, we provide an analytical framework jointly taking into account a different degree of RRH cooperation via dCDD protocol and power allocation.


## A. Notation

$\mathbb{C}$ denotes the set of complex numbers; $\boldsymbol{I}_{m \times n}$ denotes the $m \times n$ matrix of ones; and $\mathbf{0}_{m \times n}$ denotes the $m \times n$ zero matrix. $\mathcal{C N}\left(\mu, \sigma^{2}\right)$ denotes the circularly symmetric complex

Gaussian distribution with mean $\mu$ and variance $\sigma^{2} ; F_{\varphi}(\cdot)$ and $f_{\varphi}(\cdot)$, respectively, denote the cumulative distribution function (CDF) and probability density function (PDF) of the random variable $(\mathrm{RV}) \varphi$; and $\binom{n}{k} \triangleq \frac{n!}{(n-k)!k!}$ denotes the binomial coefficient. Cardinalities of a vector $\boldsymbol{a}$ and a list $\mathbb{S}$ are respectively denoted by $|\boldsymbol{a}|$ and $|\mathbb{S}|$. In addition, subscripts are used to identify an element from a particular set. For a set of continuous random variables, $\left\{x_{1}, x_{2}, \ldots, x_{N}\right\}$, $x_{\langle i\rangle}$ denotes the $i$ th largest random variable. For the order statistics, $\left\{x_{\langle 1\rangle}, \ldots, x_{\langle N\rangle}\right\}$, we define the spacing statistics by the set $\left\{y_{1}, \ldots, y_{N}\right\}$, each of which is defined with $y_{1}=x_{\langle 1\rangle}$ and $y_{k}=x_{\langle k\rangle}-x_{\langle k-1\rangle}$ for $2 \leq k \leq K$, so that we have $x_{\langle k\rangle}=\sum_{i=1}^{k} y_{i}$.

## II. System and Channel Model



Fig. 1. Illustration of the proposed dCDD-based cooperative NOMA system with $L$ RRHs and two users, $\mathrm{UE}_{1}$ and $\mathrm{UE}_{2}$.

Fig. 1 illustrates a block diagram of the cooperative NOMA system, which comprises a single CU, $L$ RRHs, and two users, i.e., $\mathrm{UE}_{1}$ and $\mathrm{UE}_{2}$. The set of RRHs, $\left\{\mathrm{RRH}_{i}\right\}_{i=L-M+1}^{L}$, appears as the first group. Similarly another set of RRHs, $\left\{\mathrm{RRH}_{i}\right\}_{i=1}^{L-M}$, appears as the second group. Only one antenna is assumed to be deployed at each of the two users and RRHs due to hardware and power constraints. It is assumed that $\mathrm{UE}_{1}$ is farther away than $\mathrm{UE}_{2}$ from the RRHs, so that $\mathrm{UE}_{1}$ appears as a far user in the considered system. For this cooperative and distributed system, the CU specifies how to control $L$ RRHs [11] via dedicated highly reliable backhauls [14], $\left\{b_{l}\right\}_{l=1}^{L}$. The CU forms two independent data signals, $s_{1}$ and $s_{2}$ being transmitted simultaneously to $\mathrm{UE}_{1}$ and $\mathrm{UE}_{2}$. Half-duplex constraint is assumed for all the users and RRHs.

As a transmit diversity, dCDD is employed [11], [12]. It has been known that the maximum number of RRHs for dCDD is limited by the transmission symbol block size ${ }^{1}$ and the maximum number of multipath components over channels connected from $L$ RRHs to $\mathrm{UE}_{1}$ and $\mathrm{UE}_{2}$. Thus, this paper investigates only a finite-sized cooperative NOMA system comprising a finite number of $L$ RRHs for full dCDD operation and two users. By employing appropriate channel sounding schemes or channel reciprocity [15], we further

[^2]assume that each user is able to know the maximum number of multipath components of the channels connected to itself. Since the considered system employs intersymbol interference (ISI)-free cyclic-prefixed single carrier (CP-SC) transmissions [16], [17], each user needs to feed back the maximum number of multipath components of the channels connected to itself.

The following channels are assumed in the proposed system.

- Channels from $L$ RRHs to $\mathrm{UE}_{1}$ and $\mathrm{UE}_{2}$ : The multipath channels from the $m$ th RRH to $\mathrm{UE}_{1}$ and $\mathrm{UE}_{2}$ are respectively given by

$$
\begin{equation*}
\boldsymbol{g}_{m}=\sqrt{\left(d_{1, m}\right)^{-\epsilon_{\mathrm{L}}}} \tilde{\boldsymbol{g}}_{m} \text { and } \boldsymbol{f}_{m}=\sqrt{\left(d_{2, m}\right)^{-\epsilon_{\mathrm{L}}}} \tilde{\boldsymbol{f}}_{m} \tag{1}
\end{equation*}
$$

where $\tilde{\boldsymbol{g}}_{m}$ and $\tilde{\boldsymbol{f}}_{m}$ identify the $m$ th frequency selective fading channel with $N_{g, m} \triangleq\left|\tilde{\boldsymbol{g}}_{m}\right|$ and $N_{f, m} \triangleq\left|\tilde{\boldsymbol{f}}_{m}\right|$ multipath components. The distances from the $m$ th RRH to $\mathrm{UE}_{1}$ and $\mathrm{UE}_{2}$ are respectively given by $d_{1, m}$ and $d_{2, m}$. The path loss exponent over channels $\boldsymbol{g}_{m}$ and $\boldsymbol{f}_{m}$, assumed to be identical, is denoted by $\epsilon_{\mathrm{L}}$. In addition, we assume that $d_{1, m} \neq d_{1, n}$ and $d_{2, m} \neq d_{2, n}, \forall m, n$.

- The multipath components of all frequency selective fading channels are assumed to be independent and identically distributed (i.i.d.) according to $\mathcal{C N}(0,1)$. However, due to different distances from $L$ RRHs to $\mathrm{UE}_{1}$ and $\mathrm{UE}_{2}$, a composite frequency selective fading channel comprising small and large-scale fading is distributed independently but non-identically distributed (i.n.i.d.) in the whole system.


## A. Summary of dCDD for CP-SC transmissions

1) Limited information fed back by $\mathrm{UE}_{1}$ and $\mathrm{UE}_{2}$ :

- Without perfect CSIT of the channels connected to $\mathrm{UE}_{1}$ and $\mathrm{UE}_{2}$, it is necessary to compute the maximum number of multipath components, that $\quad$ is, $\quad N_{g, \mathrm{CP}} \triangleq \max \left(\left\{N_{g, m}\right\}_{m=1}^{L}\right) \quad$ and $N_{f, \mathrm{CP}} \triangleq \max \left(\left\{N_{f, m}\right\}_{m=1}^{L}\right)$. Based on them, the CU first computes $N_{\mathrm{CP}}$ as $N_{\mathrm{CP}} \triangleq \max \left(N_{g, \mathrm{CP}}, N_{f, \mathrm{CP}}\right)$. After then, the CU computes the number of RRHs for dCDD as $K=\left\lfloor B / N_{\mathrm{CP}}\right\rfloor$, where $\lfloor\cdot\rfloor$ denotes the floor function. Due to an assumption of an underpopulated distributed system, the considered system has $L \leq K$.
- The channel magnitude of $\hat{\boldsymbol{g}}_{m}$ estimated by $\mathrm{UE}_{1}$ is given by $\lambda_{m} \triangleq\left\|\hat{\boldsymbol{g}}_{m}\right\|^{2}$. Due to very reliable channel estimation made by $\mathrm{UE}_{1}$, we assume $\hat{\boldsymbol{g}}_{m} \approx \boldsymbol{g}_{m}, \forall m$. For available $L$ channel estimates, $\mathrm{UE}_{1}$ arranges them as follows: $0<\lambda_{\langle 1\rangle} \leq \cdots \leq \lambda_{\langle L\rangle}<\infty$. After then it feeds back a list, $\mathbb{X}_{g} \triangleq[\langle 1\rangle,\langle 2\rangle, \ldots,\langle L\rangle]$, specifying channel magnitudes arranged in an ascending order. Based on $N_{\mathrm{CP}}$ and $\mathbb{X}_{g}$, the CU can divide the whole RRHs into two groups, after then apply CDD delays.


## B. dCDD for CP-SC-NOMA Transmissions

Considering that $\mathrm{UE}_{1}$ and $\mathrm{UE}_{2}$ are respectively far and near users [13], more transmit power should be allocated to the signal of $\mathrm{UE}_{1}$ according to the NOMA principal. To
meet these heterogeneous constraints and considering only $L$ available RRHs, we modify dCDD as follows:

- Based on $\mathbb{X}_{g}$, the CU divides $L$ RRHs into two groups. The first group composed of $M$ RRHs, and indexed by the set $\mathbb{S}_{1} \triangleq[\langle L-M+1\rangle, \ldots,\langle L\rangle] \subset \mathbb{X}_{g}$, transmits $\boldsymbol{s}_{1}$. Similarly, the second group, composed of the remaining $(L-M)$ RRHs, transmits $s_{2}$. The second group is indexed by the set $\mathbb{S}_{2} \triangleq[\langle 1\rangle, \ldots,\langle L-M\rangle] \triangleq \mathbb{X}_{g}-\mathbb{S}_{1}$.
- Since $\mathbb{S}_{2}$ is determined by the CU based on the channel connected to $\mathrm{UE}_{1}$, it should be shared with $\mathrm{UE}_{2}$.
Having applied dCDD, the received signal at $\mathrm{UE}_{1}$ is given by

$$
\begin{gather*}
\boldsymbol{r}_{1}=\sqrt{\alpha_{1} P_{s}} \sum_{m=L-M+1}^{L}\left(d_{1,\langle m\rangle}\right)^{-\epsilon_{\mathrm{L}} / 2} \tilde{\boldsymbol{G}}_{\langle m\rangle} \tilde{\boldsymbol{s}}_{1, m}+ \\
\quad\left[\sqrt{\alpha_{2} P_{s}} \sum_{m=1}^{L-M}\left(d_{1,\langle m\rangle}\right)^{-\epsilon_{\mathrm{L}} / 2} \tilde{\boldsymbol{G}}_{\langle m\rangle} \tilde{\boldsymbol{s}}_{2, m}\right]_{J_{1}}+\boldsymbol{z}_{1} \tag{2}
\end{gather*}
$$

where $P_{s}$ is the peak transmission power at all the RRHs, and $\tilde{\boldsymbol{G}}_{\langle m\rangle}$ denotes the right circulant channel matrix determined by $\tilde{\boldsymbol{g}}_{\langle m\rangle}$. The additive noise over all the channels is denoted by $\boldsymbol{z}_{1} \sim \mathcal{C N}\left(\mathbf{0}, \sigma_{z}^{2} \boldsymbol{I}_{B}\right)$. In addition, $\alpha_{1}$ and $\alpha_{2} \triangleq 1-\alpha_{1}$ respectively denote power allocation coefficients for $\mathrm{UE}_{i}$ in transmitting $s_{i}$. The $m$ th transformed block symbols $\tilde{\boldsymbol{s}}_{1, m}$ and $\tilde{\boldsymbol{s}}_{2, m}$ are defined as $\tilde{\boldsymbol{s}}_{1, m} \triangleq \boldsymbol{P}_{\Delta_{m}} \boldsymbol{s}_{1}$ and $\tilde{\boldsymbol{s}}_{2, m} \triangleq \boldsymbol{P}_{\Delta_{\tilde{m}}} \boldsymbol{s}_{2}$, where $m \in \mathbb{S}_{1}$ and $\tilde{m} \in \mathbb{S}_{2}$. The permutation shifting matrices $\boldsymbol{P}_{\Delta_{m}} \in \mathbb{C}^{B \times B}$ and $\boldsymbol{P}_{\Delta_{\tilde{m}}} \in \mathbb{C}^{B \times B}$ can be obtained from the identity matrix $\boldsymbol{I}_{B}$ by respectively circularly shifting down by $\Delta_{m}$ and $\Delta_{\tilde{m}}$. Note that $[\cdot]_{J_{1}}$ is the interfering signal at $\mathrm{UE}_{1}$.

With the use of permutation matrices, we rewrite (2) as follows:

$$
\begin{align*}
\boldsymbol{r}_{1}= & \sqrt{\tilde{\alpha}_{1}} \sum_{m=L-M+1}^{L}\left(d_{1,\langle m\rangle}\right)^{-\epsilon_{\mathrm{L}} / 2} \tilde{\boldsymbol{G}}_{\langle m\rangle} \boldsymbol{P}_{\Delta_{m}} \boldsymbol{s}_{1}+ \\
& \sqrt{\tilde{\alpha}_{2}} \sum_{m=1}^{L-M}\left(d_{1,\langle m\rangle}\right)^{-\epsilon_{\mathrm{L}} / 2} \tilde{\boldsymbol{G}}_{\langle m\rangle} \boldsymbol{P}_{\Delta_{\tilde{m}}} \boldsymbol{s}_{2}+\boldsymbol{z}_{1} \tag{3}
\end{align*}
$$

where $\tilde{\alpha}_{1} \triangleq \alpha_{1} P_{s}$ and $\tilde{\alpha}_{2} \triangleq \alpha_{2} P_{s}$. In addition, $\tilde{\boldsymbol{G}}_{\langle m\rangle} \boldsymbol{P}_{\Delta_{m}}$ and $\tilde{\boldsymbol{G}}_{\langle m\rangle} \boldsymbol{P}_{\Delta_{\tilde{m}}}$ are right circulant matrices.

Similar to (3), the received signal at $\mathrm{UE}_{2}$ is given by

$$
\begin{align*}
& \boldsymbol{r}_{2}= \sqrt{\tilde{\alpha}_{1}} \sum_{m} \sum_{m \in \mathbb{S}_{1},\left|\mathbb{S}_{1}\right|=M}\left(d_{2, m}\right)^{-\epsilon_{\mathrm{L}} / 2} \tilde{\boldsymbol{F}}_{m} \boldsymbol{P}_{\Delta_{m}} \boldsymbol{s}_{1}+ \\
& \sqrt{\tilde{\alpha}_{2}} \sum_{\substack{m \in \mathbb{S}_{2} \\
\left|\mathbb{S}_{2}\right|=L-M}}\left(d_{2, m}\right)^{-\epsilon_{\mathrm{L}} / 2} \tilde{\boldsymbol{F}}_{m} \boldsymbol{P}_{\Delta_{\tilde{m}}} \boldsymbol{s}_{2}+\boldsymbol{z}_{2} \tag{4}
\end{align*}
$$

where we assume that $\boldsymbol{z}_{2} \sim \mathcal{C N}\left(\mathbf{0}, \sigma_{z}^{2} \boldsymbol{I}_{B}\right)$. Since $\tilde{\boldsymbol{F}}_{m}$, determined by $\tilde{\boldsymbol{f}}_{m}$, is right circulant, $\tilde{\boldsymbol{F}}_{m} \boldsymbol{P}_{\Delta_{m}}$ and $\tilde{\boldsymbol{F}}_{m} \boldsymbol{P}_{\Delta_{\tilde{m}}}$ are right circulant matrices as well. From (3) and (4), we can extract the following facts:

- The order statistics are related to large-scale and smallscale fading, and the CDD delay assignment for $\mathrm{UE}_{1}$, so that it is necessary to use them in analyzing the performance of $\mathrm{UE}_{1}$.
- The selection mechanism for $\mathbb{S}_{1}$ and $\mathbb{S}_{2}$ will be crucial in achieving the rates of $\mathrm{UE}_{1}$ and $\mathrm{UE}_{2}$ under the near-far
user pairing constraint. However, the CU has only $N_{\mathrm{CP}}$, $\mathbb{X}_{g}, \mathbb{S}_{1}$, and $\mathbb{S}_{2}$ for controlling the RRHs by the dCDD protocol.


## III. Performance Analysis of CP-SC-NOMA Transmissions with dCDD

$\mathrm{UE}_{2}$ first decodes $\boldsymbol{s}_{1}$, after then decodes $\boldsymbol{s}_{2}$. According to (4), the receive SNR in decoding $s_{1}$ is given by

$$
\begin{equation*}
\gamma_{2, \boldsymbol{s}_{1}}=\frac{\alpha_{1} P_{s} \sum_{m \in \mathbb{S}_{1},\left|\mathbb{S}_{1}\right|=M}\left\|\boldsymbol{f}_{m}\right\|^{2}}{\alpha_{2} P_{s} \sum_{m \in \mathbb{S}_{2},\left|\mathbb{S}_{2}\right|=L-M}\left\|\boldsymbol{f}_{m}\right\|^{2}+\sigma_{z}^{2}} \tag{5}
\end{equation*}
$$

where we have assumed that $E\left\{s_{j}\right\}=\mathbf{0}$ and $E\left\{s_{j}\left(s_{j}\right)^{H}\right\}=$ $\boldsymbol{I}_{B}$, for $j=1,2$. In addition, $E\left\{\boldsymbol{s}_{i}\left(\boldsymbol{s}_{j}\right)^{H}\right\}=\boldsymbol{I}_{B} \delta_{i-j^{\prime}}$ with the Kronecker delta function, $\delta_{l}=\left\{\begin{array}{ll}0 & \text { if } l \neq 0 \\ 1 & \text { if } l=0 .\end{array}\right.$ After then, assuming $s_{1}$ is perfectly decoded, $\mathrm{UE}_{2}$ decodes $s_{2}$, so that the SNR in decoding $s_{2}$ is given by

$$
\begin{equation*}
\gamma_{2, \boldsymbol{s}_{2}}=\alpha_{2} \sum_{m \in \mathbb{S}_{2},\left|\mathbb{S}_{2}\right|=L-M}\left\|\tilde{\boldsymbol{f}}_{m}\right\|^{2} / \beta_{2, m} \tag{6}
\end{equation*}
$$

where $1 / \beta_{2, m} \triangleq P_{S}\left(d_{2, m}\right)^{-\epsilon_{\mathrm{L}}} / \sigma_{z}^{2}$. Based on (6), the PDF of $\gamma_{2, s_{2}}$ is given by
$f_{\gamma_{2, s_{2}}}(x)=\sum_{\substack{i_{3} \in \mathbb{S}_{2} \\\left|\mathbb{S}_{2}\right|=L-M}} \sum_{j_{3}=1}^{L_{g}, i_{3}} \frac{\theta_{L, L-M}\left(i_{3}, j_{3}\right)}{\Gamma\left(j_{3}\right)\left(\alpha_{2}\right)^{j_{3}}}(x)^{j_{3}-1} e^{-\frac{\beta_{2, i_{3} x}}{\alpha_{2}}}$
where $\theta_{L, L-M}\left(i_{3}, j_{3}\right)$ denotes the partial fraction coefficients. According to (3), the SNR for decoding $s_{1}$ by $\mathrm{UE}_{1}$ is given by

$$
\begin{equation*}
\gamma_{1, s_{1}}=\frac{\alpha_{1} C_{\max }^{L, M}}{\alpha_{2} D_{\min }^{L,-M}+1} \tag{7}
\end{equation*}
$$

where $C_{\max }^{L, M} \triangleq \sum_{m=L-M+1}^{L}\left\|\tilde{\boldsymbol{g}}_{\langle m\rangle}\right\|^{2} / \beta_{1,\langle m\rangle}, D_{\min }^{L, L-M} \triangleq$ $\sum_{m=1}^{L-M}\left\|\tilde{\boldsymbol{g}}_{\langle m\rangle}\right\|^{2} / \beta_{1,\langle m\rangle}$, and $1 / \beta_{1,\langle m\rangle} \triangleq \rho\left(d_{1,\langle m\rangle}\right)^{-\epsilon_{\mathrm{L}}}$. From (7), we can see that in contrast to the expression for $\gamma_{2, s_{1}}$, the order statistics are involved in the expression for $\gamma_{1, s_{1}}$. In addition, due to the use of the order statistics, $C_{\text {max }}^{L, M}$ and $D_{\min }^{L, L-M}$ are correlated with each other. From these challenging difficulties, it is necessary to derive their PDFs. We first derive the PDFs of $C_{\max }^{L, M}$ and $D_{\min }^{L, L-M}$ in the following Corollary 1.

Corollary 1: Denote $Y_{k} \triangleq\left\|\tilde{\boldsymbol{g}}_{\langle k\rangle}\right\|^{2} / \beta_{1,\langle k\rangle}$ the $k$ th smallest instantaneous SNR. Using the spacing statistics [18], [19], alternative forms for $C_{\max }^{L, M}$ and $D_{\min }^{L, L-M}$ can be derived, which are uncorrelated with each other. Then, we can compute the target PDFs as follows:

$$
\begin{align*}
f_{D_{\min }^{L, L-M}}(x) & =\sum_{\min } \sum_{i_{3}=1}^{L-M} \sum_{j_{3}=1}^{\nu_{i_{3}}+1} \frac{E_{\min ^{L, L-M}}\left(i_{3}, j_{3}\right)}{\Gamma\left(j_{3}\right)} e^{-\xi_{j_{3}} x} x^{j_{3}-1} \\
f_{C_{\max }^{L, M}}(x) & =\sum_{\max } \sum_{i_{4}=1}^{M} \sum_{j_{4}=1}^{\mu_{i_{4}}+1} \frac{E_{\max ^{L, M}}\left(i_{4}, j_{4}\right)}{\Gamma\left(j_{4}\right)} e^{-\zeta_{i_{4}} x} x^{j_{4}-1} \tag{8}
\end{align*}
$$

where unspecified terms, $\sum_{\max }, \sum_{\min },\left\{\xi_{j_{3}} s, \nu_{j_{3}} s\right\}$, and $\left\{\zeta_{j_{4}} s, \mu_{j_{4}} s\right\}$ are defined in Appendix A. In addition, $E_{\min ^{L, L-M}}(\cdot, \cdot)$ and $E_{\max ^{L, M}}(\cdot, \cdot)$ denote partial fraction coefficients.

Proof: See Appendix A.
Based on (8), the PDF of $\gamma_{1, s_{1}}$ can be derived as (9), provided at the top of the next page.

## A. Analysis for Achievable Rate

a) Achievable rate of $\mathrm{UE}_{2}$ : The achievable rate of $\mathrm{UE}_{2}$ is given by

$$
\left.\begin{array}{c}
R_{2,(L-M)}=\int_{0}^{\infty} \log _{2}(1+x) f_{\gamma_{2, s_{2}}}(x) d x \\
R_{2,(L-M)}=\frac{1}{\log (2)} \sum_{i_{3} \in \mathbb{S}_{2},\left|\mathbb{S}_{2}\right|=L-M} \sum_{j_{3}=1}^{L_{g, i_{3}}} \frac{\theta_{L, L-M}\left(i_{3}, j_{3}\right)}{\Gamma\left(j_{3}\right)\left(\alpha_{2}\right)^{j_{3}}} \\
\left(\frac{\beta_{2, i_{3}}}{\alpha_{2}}\right)^{-j_{3}} G_{3,2}^{1,3}\left(\left.\frac{\beta_{2, i_{3}}}{\alpha_{2}}\right|_{c} ^{1-j_{3}, 1,1}\right.  \tag{11}\\
1,0
\end{array}\right), ~(11)
$$

where the Laplace transform of a particular Meijer G-function [20, eq. (07.34.22.0003.01)], [21, eq. (2.24.3.1)] is used for the derivation.
b) Achievable rate of $\mathrm{UE}_{1}$ : The achievable rate of $\mathrm{UE}_{1}$ is provided in the following Theorem 1.

Theorem 1: The dCDD-based CP-SC-NOMA transmission provides $\mathrm{UE}_{1}$ with the rate provided by (12), which is provided at the top of the next page.

Proof: See Appendix B.
In (12), $\quad G_{p_{1}, q_{1}: p_{2}, q_{2}: p_{3}, q_{3}}^{m_{1}, 0: m_{2}, n_{2}: m_{3}: n_{3}}(x, y|\cdot| \cdot \mid$.$) denotes$ the generalized bivariate Meijer G-function [20, eq. (07.34.21.0081.01)] and [22].

## IV. Simulations

We assume the following simulation setting.

- For CP-SC transmissions, we assume that $B=32$ and $N_{\mathrm{CP}}=6$. Thus, the CU can support up to five RRHs for dCDD.
- Five RRHs are placed at $\{(0,12),(-12,12),(-3,12)$, $(-9,12),(-16.9145,6.1564)\}$ in a 2-D plane. When $L<5$ RRHs are selected for dCDD, we choose the first $L$ RRHs for the simulations.
- The two users, $\mathrm{UE}_{1}$ and $\mathrm{UE}_{2}$, are respectively placed at $(3,-3)$ and $(-3,3)$.
- According to [23], we assume that $\epsilon_{\mathrm{L}}=2.09$.
- We consider a non-identical number of multipath components and a non-identical distance between two nodes in the system. Thus, non-identical frequency selective fading is assumed for all the channels in the system.
- We use a fixed $P_{s}=1$ for the whole link-level simulations.
The curves obtained via link-level simulations are denoted by Ex, whereas analytically obtained performance curves are denoted by An.

Since the derivation of a closed-form expression for $R_{1,(L, M)}$, expressed by (12), is challenging in the proposed
system, we first verify its accuracy comparing with the corresponding exact rate provided in Fig. 2. The corresponding rate of $\mathrm{UE}_{2}$ is provided in Fig. 3.

For various combinations of $(L, M), \alpha_{1}, \alpha_{2}$, and $\left(N_{f, m}, N_{g, m}\right)$, Figs. 2 and 3 show that (12) and (11) provide accurate analysis for the rates of $\mathrm{UE}_{1}$ and $\mathrm{UE}_{2}$.

## A. Rate analysis

a) The impact of the cardinality of $\mathbb{S}_{1}$ : We use a fixed $\alpha_{1}=0.9$ and $\left\{N_{g, m}\right\}_{m=1}^{L}=2$ to investigate the impact of the cardinality of set $\mathbb{S}_{1}$ on the rate. In Fig. 4, we use fixed $\left|\mathbb{S}_{1}\right|=2$ and $\left|\mathbb{S}_{1}\right|=3$ for two values of $L$. This figure shows that at a fixed $\left|\mathbb{S}_{1}\right|$, more RRHs in $\mathbb{S}_{1}$ results in greater rate up to a certain SNR. However, in the high SNR region, an asymptotic rate advantage is determined by the ratio of $\left|\mathbb{S}_{1}\right|$ to $L$. Thus, we can see that $R_{1,(4,3)}>R_{1,(5,3)}$. Similar results can be seen with $\left|\mathbb{S}_{1}\right|=2$, i.e., $R_{1,(4,2)}>R_{1,(5,2)}$. Form these observations, a greater $\left|\mathbb{S}_{1}\right|$ does not guarantee a greater rate in the asymptotic SNR region.
b) The impact of power allocation to RRHs specified by $\mathbb{S}_{1}$ and $\mathbb{S}_{2}$ : For four RRHs and $\left|\mathbb{S}_{1}\right|=3$, we investigate the impact of $\alpha_{1}$ on the rate of $\mathrm{UE}_{1}$ and $\mathrm{UE}_{2}$ in Fig. 5. In this scenario, only one RRH out of four RRHs is used to transmit $s_{2}$. This figure shows that as $\alpha_{1}$ increases, $\mathrm{UE}_{1}$ achieves a greater rate, whereas $\mathrm{UE}_{2}$ achieve a lower rate since less power is allocated to RRHs specified by $\mathbb{S}_{2}$. This figure also shows that $R_{2,(L-M)}$ increases as the SNR increases, whereas $R_{1,(L, M)}$ is upper bounded by its limit since $\gamma_{1, s_{1}}$ is approximated by $\gamma_{1, s_{1}} \approx \frac{\alpha_{1} C_{\max }^{L, M}}{\alpha_{2} D_{\min }^{L, L-M}}$ as the SNR increases. That is, $\mathrm{UE}_{1}$ enters the interference-limited region. From these observations, the sum rate is like a double-edged sword. If the pair of $(L, M)$ and the power allocation are not properly handled, the sum of rate will be dominated by $\mathrm{UE}_{2}$, which violates the near-far user pairing constraint. Since the CU knows only $\mathbb{X}_{g}$ and controls $\alpha_{1}$, every pair of $(L, M)$ is not possible to meet this constraint. For $\left|\mathbb{S}_{1}\right|=2$, i.e., two RRHs are assigned to transmit $s_{2}$, we can see distinctive results in Fig. 6 comparing with those of Fig. 5. For the considered scenario, $\left(L=4, M=2, \alpha_{1}=0.6\right)$ and $(L=4, M=$ $\left.2, \alpha_{1}=0.7\right)$ are not feasible user pairs. Although other pairs $\left(L=4, M=2, \alpha_{1}=0.8\right)$ and $\left(L=4, M=2, \alpha_{1}=0.9\right)$ are feasible, they have limited operating SNR ranges to support the near-far user pairing. These observations suggest the following RRH assignment and power allocation mechanism:

- A larger $\left|\mathbb{S}_{1}\right|$ results in $R_{1,(L, M)}>R_{2,(L-M)}$, so that it is necessary to assign more RRHs to transmit $s_{1}$.
- Due to the existence of the interfering signal at $\mathrm{UE}_{1}$, the CU needs to assign more transmit power to the RRHs specified by $\mathbb{S}_{1}$.
c) Comparison with OMA with dCDD: For various system parameters, Figs. 3 and 4 show that NOMA with dCDD achieves superior rate comparing with OMA with dCDD for both users.


## V. Conclusions

In this paper, we have proposed a new transmit diversity scheme for the two-user CP-SC NOMA system. To support the

$$
\begin{align*}
f_{\gamma_{1, s_{1}}}(x)= & \sum_{\max } \sum_{i_{4}=1}^{M} \sum_{j_{4}=1}^{\mu_{i_{4}}+1} \sum_{\min } \sum_{i_{3}=1}^{L-M} \sum_{j_{3}=1}^{\nu_{i_{3}}+1} \frac{E_{\max ^{L, M}}\left(i_{4}, j_{4}\right)}{\Gamma\left(j_{4}\right)} \frac{E_{\min ^{L, L-M}}\left(i_{3}, j_{3}\right)}{\Gamma\left(j_{3}\right)} \\
& \left(\frac{1}{\alpha_{1}}\right)^{j_{4}} \sum_{l=0}^{j_{4}}\binom{j_{4}}{l}\left(\alpha_{2}\right)^{l} \Gamma\left(l+j_{3}\right) e^{-\frac{\zeta_{i_{4}}}{\alpha_{1}} x} x^{j_{4}-1}\left(\frac{\alpha_{2} \zeta_{i_{3}}}{\alpha_{1}} x+\xi_{j_{3}}\right)^{-j_{3}-l} \tag{9}
\end{align*}
$$

$$
\left.\left.\begin{array}{rl}
R_{1,(L, M)}= & \frac{1}{\log (2)} \sum_{\max } \sum_{i_{4}=1}^{M} \sum_{j_{4}=1}^{\mu_{i_{4}}+1} \sum_{\min } \sum_{i_{3}=1}^{L-M} \sum_{j_{3}=1}^{\nu_{i_{3}}+1} \frac{E_{\max ^{L, M}\left(i_{4}, j_{4}\right)}}{\Gamma\left(j_{4}\right)} \frac{E_{\min L, L-M}\left(i_{3}, j_{3}\right)}{\Gamma\left(j_{3}\right)} \\
& \left(\frac{1}{\alpha_{1}}\right)^{j_{4}} \sum_{l=0}^{j_{4}}\binom{j_{4}}{l}\left(\alpha_{2}\right)^{l} \Gamma\left(l+j_{3}\right)\left(\alpha_{2}\right)^{-l-j_{3}}\left(\frac{\zeta_{i_{4}}}{a_{1}}\right) G_{1,0: 2,2: 1,1}^{1,0: 1,2: 1,1}\left(\frac{\zeta_{i_{3}}}{\alpha_{1}}, \left.\frac{\xi_{j_{3}}}{\alpha_{2}}|\cdot| \cdot \right\rvert\, j_{4}, j_{4}-1\right. \tag{12}
\end{array} j_{4}, j_{4} \mid 1-l-j_{3}\right) .0 .1\right) .
$$



Fig. 2. Achievable rate of $\mathrm{UE}_{1}$ for various system parameters.


Fig. 3. Achievable rate of $\mathrm{UE}_{2}$ for various system parameters.
near-far user pairing, a new joint RRH assignment and power allocation mechanism for dCDD based CP-SC transmissions without requiring perfect CSIT of the whole channels at the CU has been proposed. For i.n.i.d. frequency selective fading channels, new closed-form expressions for the rates of two users have been derived. Its accuracy has also been verified. By link-level simulations, it has been shown that every pair of $(L, M)$ is not supported for the whole range of power


Fig. 4. Achievable rate of $\mathrm{UE}_{1}$ for various pairs of $(L, M)$.


Fig. 5. Achievable rate of $\mathrm{UE}_{1}$ and $\mathrm{UE}_{2}$ for various values of $\alpha_{1}$ with $N_{g, m}=N_{f, m}=\{2,4,3,3\}, L=4$, and $\left|\mathbb{S}_{1}\right|=3$.
allocation for the CP-SC NOMA system due to the near-far user pairing constraint.

## Appendix A: Derivation of Corollary 3

Let $\tilde{\lambda}_{\langle m\rangle}$ be defined by $\tilde{\lambda}_{\langle m\rangle} \triangleq\left\|\tilde{\boldsymbol{g}}_{\langle m\rangle}\right\|^{2} / \beta_{1,\langle m\rangle}$.


Fig. 6. Achievable rate of $\mathrm{UE}_{1}$ and $\mathrm{UE}_{2}$ for various values of $\alpha_{1}$ with $N_{g, m}=N_{f, m}=\{2,4,3,3\}, L=4$, and $\left|\mathbb{S}_{1}\right|=2$.
a) PDF of $D_{\min }^{L, L-M}$ : The joint PDF of $Y_{1} \triangleq \tilde{\lambda}_{\langle 1\rangle}, \ldots, Y_{L-M} \triangleq \tilde{\lambda}_{\langle L-M\rangle}$ can be written as $f_{Y_{1}, Y_{2}, \ldots, Y_{L-M}}\left(y_{1}, y_{2}, \ldots, y_{L-M}\right)=\frac{1}{M!} \operatorname{Per}\left(\boldsymbol{A}_{\min }^{L, L-M}\right)$ where the permanent matrix, $\operatorname{Per}\left(\boldsymbol{A}_{\text {min }}^{L, L-M}\right)$, is defined as follows:

$$
\boldsymbol{A}_{\min }^{L, L-M}=[\begin{array}{cccc}
f_{Y_{1}}\left(y_{1}\right) & \ldots & f_{Y_{1}}\left(y_{L-M}\right) & \tilde{F}_{Y_{1}}\left(y_{L-M}\right) \\
\vdots & \vdots & \vdots & \\
\underbrace{f_{Y_{L}}\left(y_{1}\right)} & \cdots & f_{Y_{L}\left(y_{L-M}\right)}^{1} & \underbrace{1}
\end{array} \underbrace{M}_{Y_{Y_{L}}\left(y_{L-M}\right)} \quad]
$$

where $\tilde{F}_{Y_{k}}\left(y_{k}\right)$ and $f_{Y_{k}}\left(y_{k}\right)$ are the complementary CDF (CCDF) and PDF of $Y_{k}$, i.e., the $k$ th smallest SNR. Their expressions are given by

$$
\begin{align*}
\tilde{F}_{Y_{k}}\left(y_{k}\right) & =\frac{\gamma_{l}\left(N_{g, k}, \beta_{1, k} y_{k}\right)}{\Gamma\left(N_{g, k}\right)} \text { and } \\
f_{Y_{k}}\left(y_{k}\right) & =\frac{\left(\beta_{1, k}\right)^{N_{g, k}}}{\Gamma\left(N_{g, k}\right)} e^{-\beta_{1, k} y_{k}}\left(y_{k}\right)^{N_{g, k}-1} \tag{A.2}
\end{align*}
$$

where $\gamma_{l}(\cdot, \cdot)$ denotes the lower incomplete gamma function. Furthermore, $\left[\begin{array}{ll}\underbrace{\boldsymbol{a}_{1}} & \underbrace{\boldsymbol{a}_{2}}\end{array}\right]$ denotes $i$ copies of $\boldsymbol{a}_{1}$ and $j$ copies of $\boldsymbol{a}_{2}$. The moment generating function (MGF) of $D_{\text {min }}^{L, L-M}$ can be defined as
$M_{D_{\min }^{L, L-M}}(s)=$
$\int_{0}^{\infty} \int_{y_{1}}^{\infty} \ldots \int_{y_{L-M-2}}^{\infty} \int_{y_{L-M-1}}^{\infty} I_{\min }^{L, L-M}(s) d y_{L-M} \ldots d y_{2} d y_{1}$ where $I_{\min }^{L, L-M}(s)$ is given by

$$
\begin{equation*}
I_{\min }^{L, L-M}(s)=\frac{1}{(M)!} e^{-s\left(\sum_{j=1}^{L-M} y_{j}\right)} \operatorname{Per}\left(\boldsymbol{A}_{\min }^{L, L-M}\right) \tag{A.3}
\end{equation*}
$$

Having applied spacing, that is, $Y_{k}=\sum_{l=1}^{k} X_{k}, M_{D_{\min }^{L, L-M}}(s)$ can be evaluated as (A.4), provided at the top of the next page. In (A.4), we have defined $\beta_{j}=\beta_{1, j}, C_{1}=\prod_{j=1}^{L-M} \frac{\left(\beta_{n_{j}}\right)^{m_{n_{j}}}}{\Gamma\left(m_{n_{j}}\right)}$, $C_{2}=\prod_{j=L-M+1}^{L}\left(\beta_{n_{j}}\right)^{l_{j}} / \Gamma\left(l_{j}+1\right), \tilde{m}_{\min }=$
$\sum_{j=L-M+1}^{L} l_{j}+m_{n_{L-M}}-1$, and $\tilde{\beta}_{\text {min }}=\sum_{j=L-M}^{L} \beta_{n_{j}}$. In addition, $\quad C_{3} \triangleq(L-M)^{-\nu_{1}-1}(L-M-$ 1) ${ }^{-\nu_{2}-1} \ldots 2^{-\nu_{L-M-1}-1} \Gamma\left(\nu_{1}+1\right) \Gamma\left(\nu_{2}+1\right) \ldots \Gamma\left(\nu_{L-M-1}+\right.$ 1) $\Gamma\left(\nu_{L-M}+1\right), \quad \xi_{i}=\frac{1}{(L-M-i+1)} \sum_{j=i}^{L} \beta_{n_{j}}$, $\nu_{i}= \begin{cases}m_{n_{1}-M} 1+\sum_{j=2}^{L-M} p_{j, i} & \text { for } i=1 \\ \sum_{j=i}^{L-M} p_{j, i} & \text { for } 2 \leq i \leq L-M,\end{cases}$
$\sum_{\substack{p_{J, 1}, \ldots, p_{J, a} \\ \sum_{i, ~}^{a}=1 \\ p_{J, i}=b}}$ denotes the sum of positive integer indices $\sum_{i_{J}=1}^{a} p_{J, i}=b$
$\left\{p_{J, 1}, \ldots, p_{J, a}\right\}$ satisfying $\sum_{i=1}^{a} p_{J, i}=b$. After applying the partial fraction to $A_{1}$ in (A.4), and the inverse MGF, the PDF can be derived as follows:
$f_{D_{\min }^{L, L-M}}(t)=\sum_{\min } \sum_{i=1}^{L-M} \sum_{j=1}^{\nu_{i}+1} \frac{E_{\min ^{L, L-M}(i, j)}}{\Gamma(j)} e^{-\xi_{j} t} t^{j-1}$.
b) PDF of $C_{\max }^{L, M}$ : The joint PDF of $Y_{L-M+1} \triangleq \tilde{\lambda}_{\langle L-M+1\rangle}, \ldots, Y_{L} \triangleq \tilde{\lambda}_{\langle L\rangle}$ can be written as $f_{Y_{L-M+1}, \ldots, Y_{L}}\left(y_{1}, y_{2}, \ldots, y_{M}\right)=\frac{1}{(L-M)!} \operatorname{Per}\left(\boldsymbol{A}_{\max }^{L, M}\right)$
where the permanent matrix, $\operatorname{Per}\left(\boldsymbol{A}_{\text {max }}^{L, M}\right)$, is determined as follows:

$$
\boldsymbol{A}_{\max }^{L, M} \triangleq\left[\begin{array}{cccc}
F_{1}\left(y_{1}\right) & f_{1}\left(y_{1}\right) & \ldots & f_{1}\left(y_{M}\right)  \tag{A.7}\\
\vdots & \vdots & \vdots & \vdots \\
F_{L}\left(y_{1}\right) & f_{L}\left(y_{1}\right) & \cdots & f^{f_{L}\left(y_{M}\right)} \\
\underbrace{L^{-M}} & \underbrace{1} & \underbrace{1}
\end{array}\right]
$$

where $F_{k}(\cdot)=1-\tilde{F}_{k}(\cdot)$ denotes the CDF of $Y_{k}$. The MGF of $C_{\text {max }}^{L, M}$ can be defined as
$M_{C_{\text {max }}^{L, M}}(s)=\int_{0}^{\infty} \int_{y_{1}}^{\infty} \ldots \int_{y_{M-2}}^{\infty} \int_{y_{M-1}}^{\infty} I_{\max }^{L, M}(s) d y_{M} \ldots d y_{1}$
where $I_{\max }^{L, M}(s)$ is given by

$$
\begin{equation*}
I_{\max }^{L, M}=\frac{1}{(L-M)!} e^{-s\left(\sum_{j=1}^{M} y_{j}\right)} \operatorname{Per}\left(\boldsymbol{A}_{\max }^{L, M}\right) \tag{A.8}
\end{equation*}
$$

After some manipulations, $M_{C_{\max }^{L, M}}(s)$ is given by (A.9) provided on the next page. In (A.9), we have defined $D_{1} \triangleq \prod_{j=1}^{M} \frac{\left(\beta_{n_{j}}\right)^{m_{n_{j}}}}{\Gamma\left(m_{n_{j}}\right)}, \tilde{k}_{l} \triangleq \sum_{t=0}^{m_{n_{l}}-1} t k_{l, t+1}, \tilde{m}_{\max } \triangleq m_{n_{1}}-$ $1+\sum_{l=M+1}^{L} \tilde{k}_{l}$, and $D_{2} \triangleq\left(\prod_{j=M+1}^{L} C_{q_{j}}\right)$ with $C_{q_{j}}=$ $\sum_{q_{j}=0}^{1}\binom{1}{q_{j}}(-1)^{q_{j}} \sum_{k_{j, 1}, \ldots, k_{j, m_{n}}} \frac{\prod_{t=0}^{m_{n_{j}}-1} \frac{\left(\beta_{n_{j}}\right)^{k_{j, t+1}}}{(t!)}}{k_{j, 1}!\cdots q_{k, m_{n_{j}}}!} . \quad$ In $k_{j, 1}+\ldots+k_{j, m_{n}}=1$
addition, $\zeta_{i}=\left\{\begin{array}{ll}\frac{\left(\sum_{j=1}^{M} \beta_{n_{j}}+\sum_{j=M+1}^{L} \beta_{n_{j}} q_{j}\right)}{M} & \text { for } i=1 \\ \frac{\sum_{j=i}^{M} \beta_{n_{j}}}{(M-i+1)} & \text { for } 2 \leq i \leq M\end{array}\right.$, $\mu_{i}=\left\{\begin{array}{ll}\tilde{m}_{\max }+\sum_{j=2}^{M} q_{j, 1} & \text { for } i=1 \\ \sum_{j=i}^{M} p_{j, i} & \text { for } 2 \leq i \leq M\end{array}, D_{3}=(M)^{-\mu_{1}-1}\right.$ $(M-1)^{-\mu_{2}-1} \ldots 2^{-\mu_{M-1}-1} \Gamma\left(\mu_{1}+1\right) \ldots \Gamma\left(\mu_{M}+1\right)$. Thus,

$$
\begin{align*}
& M_{D_{\min }^{L, L-M}}^{L,}(s)= \sum_{\substack{n_{1}, \ldots, n_{L-M} \\
n_{1} \neq n_{2} \neq \ldots \neq n_{L-M}}} C_{1} \sum_{l_{L-M+1}=0}^{m_{n_{L}-M+1}-1} \ldots \sum_{l_{L}=0}^{m_{n_{L}}-1} C_{2} \sum_{\substack{p_{p_{2,1}, p_{2,2}}^{2}}} \frac{\Gamma\left(m_{n_{2}}\right)}{\prod_{i=1}^{2} p_{2, i}!} \ldots \\
& \sum_{\substack{p_{2, i_{2}=m_{n}-1} \\
p_{L-M, 1}^{L-\ldots, p_{L-M, L-M}} \\
\sum_{i_{L-M}=1}^{L-M=1} p_{L-M, i}=\tilde{m}_{\min }}} \frac{\Gamma\left(\tilde{m}_{\min }+1\right)}{\prod_{i=1}^{L-M} p_{L-M, i}!} C_{3} \prod_{i=1}^{L-M}\left(\left(s+\xi_{i}\right)^{-\nu_{i}-1}\right)=\sum_{\min } \underbrace{\prod_{i=1}^{L-M}\left(\left(s+\xi_{i}\right)^{-\nu_{i}-1}\right)}_{A_{1}} . \tag{A.4}
\end{align*}
$$

$$
\begin{align*}
& M_{C_{\max }^{L, M}}(s)= \sum_{\substack{n_{1}, \ldots, n_{M} \\
n_{1} \neq n_{2} \neq \ldots \neq n_{M}}} D_{1} D_{2} \sum_{\substack{p_{2}, 1, p_{2,2} \\
p_{2,1}+p_{2,2}=m_{n_{2}-1}}} \frac{\Gamma\left(m_{n_{2}}\right)}{\prod_{i=1}^{2} p_{2, i}!} \sum_{\substack{p_{3,1}, p_{3,2}, p_{3,3} \\
p_{3,1}+p_{3,2}+p_{3,3}=m_{n}-1}} \frac{\Gamma\left(m_{n_{3}}\right)}{\prod_{i=1}^{3} p_{3, i}!} \ldots \\
& \sum_{\substack{p_{M, 1}, \ldots, p_{M, M} \\
p_{M, 1}+\ldots+p_{M, M}=m_{n_{M}-1}}} \frac{\Gamma\left(m_{\left.n_{M}\right)}\right.}{\prod_{i=1}^{M} p_{M, i}!} D_{3}\left(\prod_{i=1}^{M}\left(s+\zeta_{i}\right)^{-\mu_{i}-1}\right)=\sum_{\max } \underbrace{\left.\prod_{i=1}^{M}\left(s+\zeta_{i}\right)^{-\mu_{i}-1}\right)}_{A_{2}} \tag{A.9}
\end{align*}
$$

after applying the partial fraction to $A_{2}$ and then the inverse MGF, the PDF can be derived as follows:

$$
\begin{equation*}
f_{C_{\max }^{L, M}}(t)=\sum_{\max } \sum_{i=1}^{M} \sum_{j=1}^{\mu_{i}+1} \frac{E_{\max ^{L, M}}(i, j) e^{-\zeta_{i} t} t^{j-1}}{\Gamma(j)} \tag{A.10}
\end{equation*}
$$

## Appendix B: Derivation of Theorem 1

The rate, $R_{1,(M, L-M)}$, is given by

$$
\begin{equation*}
R_{1,(M, L-M)}=\int_{0}^{\infty} \log _{2}(1+x) f_{\gamma_{1, s_{1}}}(x) d x \tag{B.1}
\end{equation*}
$$

To compute (B.1), we first express the following functions of $x$ in terms of meijer-G functions:

$$
\left.\begin{array}{rl}
\log _{2}(1+x) x^{j_{4}-1} & =\frac{1}{\log (2)} G_{2,2}^{1,2}(x \mid \\
j_{4}, j_{4}-1
\end{array}\right),
$$

Having applied [24], we can derive (12).

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[^1]:    K. J. Kim and P. V. Orlik are with Mitsubishi Electric Research Laboratories (MERL), Cambridge, MA, USA.
    H. Liu is with Shandong Jiaotong University, Jinan, China.
    H. Lei is with Chongqing University of Posts and Telecommunications, Chongqing, China.
    Z. Ding is with The University of Manchester, Manchester, UK.
    H. V. Poor is with the Department of Electrical Engineering, Princeton University, Princeton, NJ, USA.
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[^2]:    ${ }^{1}$ Since CP-SC transmission is used in the considered system, transmission symbols $\boldsymbol{s}_{1}$ and $\boldsymbol{s}_{2}$ are composed of $B$ modulated symbols.

