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Abstract

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Abstract: *We propose an energy-efficient polar decoding which employs optimized lookup tables (LUTs) to minimize the mutual information loss caused by finite-precision hardware implementations. We show that nearly lossless designs are feasible with few-bit LUTs.*

Keywords: *Coding and forward error correction for optical communications*

I. Introduction

A great advancement of soft-decision forward error correction (FEC) such as low-density parity-check (LDPC) codes has contributed to improving tolerance against linear and nonlinear distortion for reach expansion and throughput increase in modern optical communications. Recently, polar codes have drawn much attention in research community since they were selected in wireless 5G standard because of the competitive performance (approaching the Polyanskiy bound) against state-of-the-art LDPC codes. In this paper, we first introduce several recent efforts in developing practical polar codes, such as high-gain, low-power and high-throughput architectures, which may be suited for optical communications. Additionally, we discuss potential research challenges to be dealt with for further improvement. We then propose a hardware-friendly polar decoding scheme based on finite-precision lookup table (LUT) operations. By optimizing non-uniform quantization map through mutual information analysis, we can achieve excellent performance even for the case when small number of quantization levels is available under the constraint of hardware resource.

II. Polar Codes: Recent Progress

The polar decoding based on successive cancellation list (SCL) has a log-linear complexity order of $\mathbb{O}[LN\log_2(N)]$, where L and N are list size and codeword length, respectively. This nonlinear complexity can be a great drawback compared to linear-complexity belief-propagation (BP) decoding of LDPC codes. However, the nonlinear complexity turns to be advantageous when we shorten the block length. In [8, 15], it was shown that polar decoding can be lower complex than typical LDPC decoding in the short length regimes; specifically shorter than about $N = 65{,}000$ bits. For reference, some remaining challenges to be tackled in developing practical polar-coded systems are listed in Table I.

TABLE I
Challenges in hardware-friendly polar codes for high-speed optical communications

Specification	Potential solutions
High coding gain	Nonbinary [15], interleaver [6, 7], BICM-ID [12], concatenation, constellation shaping [13, 14], subcode
Low-complexity operation	Irregular polarization [8, 9, 10], offset min-sum, delta-min
Finite-precision operation	Mixed log/probability, quantization design [1, 2], lookup table design
Parallelism	Large kernel, multi-kernel, nonbinary [15], multibit decoding, belief propagation, neural network
Low latency	Spatial coupling, convolutional, short-length design
Low power consumption	Adaptive list, dynamic scheduling, flip decoding
High throughput	Turbo [11, 12] (unrolling, pipeline, massive parallel), analog decoding
Rate adaptation	Rate compatible, universal, puncture, shortening, adaptive modulation, nested, rateless, repeat request

A. *Why Polar Codes?*

The most random codes are known equally good when the block length is large enough. However, the decoding latency is usually increased with the block length. Hence, how to design short codes has been a challenging problem for decades. Unfortunately, LDPC codes generally do not perform well at short block lengths. From this reason, the fifth-generation wireless standard has chosen polar codes as an alternative FEC to LDPC codes for short-packet transmission. Here we introduce our research activities to improve the practicality of polar codes in high-throughput optical communications.

B. *Polar-Coded Modulation*

The conventional polar code construction assumes memoryless identical channel reliability, which is often not valid, e.g., when we use high-order QAM formats. In consequence, careful interleaver design which properly maps the coded bits to modulation bit-planes becomes important so as not to destruct the channel polarization. We have optimized [6, 7] an interleaver parameter for BICM systems to achieve more than 0.5 dB gain over random interleaving. We have further improved the performance by using an OFDM ordering and a geometric shaping with a super-Gaussian constellation [7, 13]. In [14], a joint probabilistic shaping and polar coding was realized without an external distribution matcher, through the use of dynamic frozen bits. A nonbinary-input coded modulation (NBICM) was investigated with nonbinary polar codes for high-order QAMs, showing about 1 dB gain over BICM. In addition, polar BICM-ID was studied in [12].

C. *Irregular Polar Codes*

We have proposed a new code construction method for irregular polar codes, whose polarization units are irregularly pruned to drastically reduce the computational complexity and decoding latency while achieving better performance [8, 9, 10]. Since pruned polarization units do not need any computation for both encoding and decoding, we can reduce the computational complexity. In addition, the decoding latency can be also reduced by carefully choosing the set of inactive polarization units to enable partially parallel computations. This is a great advantage for SCL decoding, which is usually difficult to parallelize. The proposed irregular polar codes showed slightly better performance than regular counterparts while reducing decoding complexity by at least 30% and decoding latency by 80%. It was also experimentally demonstrated that the irregular polar codes can outperform the state-of-the-art LDPC codes [8].

D. *Turbo Polar Codes*

One major drawback of polar SCL decoding lies in the difficulty of parallel implementation, leading to low throughput. To tackle this issue, we have proposed polar turbo product codes (TPC) [11, 12] which constitutes spatially-coupled parallel short-block polar codes. With parallel and pipeline SCL decoding, the proposed polar-TPC $(256,239)^2$ achieves roughly 256-times faster decoding throughput. In addition, the use of irregular polarization [8] reduces the decoding power by 72% and latency by 88%. The polar-TPC offers 0.5 dB gain over the conventional TPC based on BCH codes.

III. QUANTIZED POLAR DECODING

In this paper, we account for another practical aspect required for hardware implementation. In practical hardware, we generally need to consider finite-precision operations, which can be fixed-point arithmetic for typical energy-efficient interfaces. Most existing researches on polar codes have not considered such finite-precision operations. Hassani and Urbanke first analyzed polar decoding performance with uniform quantization in [1], and proved that ternary quantization can result in polarization. In [2], uniform quantization was optimized through quantized Gaussian approximation (GA), and it was shown that 6-bit quantization is sufficient to approach floating-point precision performance. In fact, analogous quantized density evolution (DE) was proposed in [3], while they did not focus on quantized decoding but frozen bit design to prevent exponentially increasing cardinality of DE. The technique was extended to nonbinary polar codes in [4]. In this paper, we propose an optimized non-uniform quantization to improve the resilience against precision errors, exploiting the quantized DE [3] to design lookup table (LUT). Our method is analogous to the LUT optimization for LDPC codes proposed in [5], where minimum rate loss is realized by analyzing mutual information transfer. Note that the factor graph of polar codes constitutes of nodes having no higher degree than two for incoming edges, and thus designing LUTs to minimize mutual information loss is rather straightforward than LDPC codes.

A. *Quantized DE for LUT Optimization*

We use L-value notation, i.e., log-likelihood ratio (LLR) defined as $L = \log \Pr(y|x=0)/\Pr(y|x=1)$, where x and y are transmitted data and received data, respectively. For BPSK in AWGN channels, it becomes $L = 2y/\sigma^2$ with σ^2 being the noise variance. For SCL decoding, the polarization unit performs sum-product message passing, taking two incoming log-likelihood ratio (LLR) messages L_i and L_j, to generate outputs for upper and lower branches as follows:

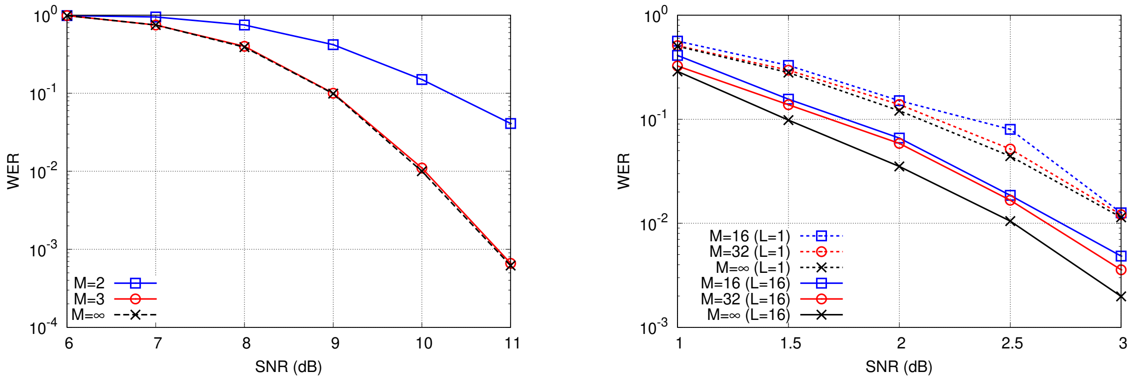
$$L'_i = L_i \boxplus L_j = 2\tanh^{-1}\left(\tanh\frac{L_i}{2} \times \tanh\frac{L_j}{2}\right), \qquad L'_j = (-1)^{u_i} L_i + L_j, \tag{1}$$

where $u_i \in \{0,1\}$ denotes the hard decision value in SCL decoding. There are various approximation methods to reduce the complexity of "box-plus" operations, such as min-sum and delta-min. In fact, when the L-values are quantized, we do not need any arithmetic operation or approximations but LUTs. The problem of LUT constructions, however, lies in the cardinality expansion. Specifically, letting Q be the alphabet size of incoming L-values, the outgoing L-values has at most Q^2 and $2Q$ possibilities, respectively for upper and lower branches. After $\log_2(N)$-stage polarizations, it will be at most Q^N cardinality. In [3], two merging operations, i.e., upgrading-merge and degrading-merge functions, were proposed to keep the cardinality constant for efficient design of frozen bit locations. We use the degrading-merge function to optimize LUTs, where the best pair of adjacent L-value outputs were sequentially merged such that the mutual information loss is minimized. Assuming symmetry, we quantize amplitude of L-value into M alphabets, and thus the LUT cardinality is constrained up to $Q = 2M$, including sign bit.

B. *Performance Analysis*

We first show the benefit of the LUT polar decoding in binary-input quantization in AWGN channels, corresponding to $Q = 2$ L-values from demodulator. With binary-symmetric channel modeling with proper crossover probability, soft information is propagated through SCL decoding. Fig. 1(a) shows word error rate (WER) of polar codes (256, 240) with varying maximum LUT size. It was shown that $M = 3$ LUT quantization already achieves floating-point performance.

For lower code rates and more list sizes, it was found that more quantization is required at lower SNR regimes. Fig. 1b shows AWGN channel performance given at least 32-ary quantization in soft-input L-values from demodulator, for a list size of 1 and 16. For 16-list decoding, there are some visible gap from floating-point performance. Nevertheless, the degradation is maintained marginal as low as 0.2 dB with $M = 32$ LUTs.



(a) Polar (256, 240) (b) Polar (256, 128)

Fig. 1. Performance evaluation of quantized polar decoding with optimized LUTs.

IV. Conclusions

We introduced few-bit quantized polar decoding using LUTs which were optimized to minimize the mutual information loss due to finite-precision operation. It was shown that our proposed method achieves excellent performance approaching idealistic performance with infinite precision even with very few quantization level. Our FEC may be well-suited for low-power optical interface hardware considering its simplicity based on few-bit LUTs. We note that the LUT design can be further improved by introducing non-deterministic LUT based on upgrading-merge function in [3].

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