MITSUBISHI ELECTRIC RESEARCH LABORATORIES
http://www.merl.com

Nonbinary Polar Coding for Multilevel Modulation

Cayci, S.; Koike-Akino, T.; Wang, Y.

TR2018-191 March 06, 2019

Abstract

We investigate nonbinary polar-coded modulations, which achieve a significant performance gain of at least 1 dB compared to binary counterparts at a short block-length of 2048 bits.

Optical Fiber Communication Conference and Exposition and the National Fiber Optic Engineers Conference (OFC/NFOEC)

Nonbinary Polar Coding for Multilevel Modulation

Semih Cayci[1], Toshiaki Koike-Akino[2], Ye Wang[2]

[1]*Electrical & Computer Engineering, The Ohio State University, Columbus, OH 43212, USA.*
[2]*Mitsubishi Electric Research Laboratories (MERL), 201 Broadway, Cambridge, MA 02139, USA.*

Abstract: We investigate nonbinary polar-coded modulations, which achieve a significant performance gain of at least 1 dB compared to binary counterparts at a short block-length of 2048 bits.

OCIS codes: (060.4510) Optical communications, (060.1660) Coherent communications, (060.4080) Modulation.

1. Introduction

Widely used bit-interleaved coded modulation (BICM) based on binary codes has a fundamental limit compared to the theoretical bound, in particular for high-order modulation schemes. By employing BICM iterative demodulation (BICM-ID), the performance can be significantly improved [1]. However, BICM-ID requires soft-decision feedback from the decoder to the demodulator. Hence, BICM-ID can be less practical due to the high complexity and large latency. In contrast, the use of nonbinary (NB) codes [2–5] can achieve the theoretical bound without requiring turbo demodulation. This scheme called nonbinary-input coded modulation (NBICM) is in particular more important for high-order and high-dimensional modulation in order to exploit the full benefit of such recent modulation techniques.

In the optical research community, the NBICM technique has been studied mostly focusing on low-density parity-check (LDPC) codes [2–5]. To date, there are few investigation considering other cutting-edge NB coding methods. In this paper, we study NB polar codes to show the potential as an alternative. Recently, binary polar codes [6] have shown a promising performance [7–9] which is highly competitive to the state-of-the-art LDPC codes thanks to the introduction of successive cancellation list (SCL) decoder [7], which can approach the Polyanskiy bound for finite block lengths [8]. Nevertheless, there exist no studies yet on NB polar codes in the context of NBICM.

This paper verifies that a quaternary polar code designed for 16-ary quadrature-amplitude modulation (16QAM) can provide 1 dB performance gain compared to the binary counterpart at a relatively short codeword length of 2048 bits. The performance gain can be even higher for longer block sizes. Since quaternary polar SCL decoding can be parallelized every 2-bit pairs, roughly 2-fold improvement in decoding throughput and latency is realized. Although Galois field operation doubles the computational complexity, one less polarization stage is needed compared to binary codes. Considering the reduction in latency due to parallelism and no more than 2-times complexity increase, NB polar codes could be of important practical use. To the best of our knowledge, this is the first practical use of NB polar-coded QAM that significantly outperforms binary polar-coded QAM in optical communications systems.

2. Nonbinary Polar Coding

According to the polarization theory [10,11] for nonbinary alphabets, it is shown that there exist polar kernels that lead to polarization for any input alphabet size. The application of the original kernel to the channels with input alphabet sizes $Q = 2^q$ were investigated by Park & Barg [12], and it was shown that a different sense of polarization is observed, which could be used in unequal error protection. Nonbinary polar coding for arbitrary input alphabet sizes was also investigated in source coding applications [13]. A variety of explicit code construction algorithms for nonbinary polar codes are proposed as well [14, 15]. Nevertheless, there is few literature investigating its potential in the context of NBICM, which uses NB polar codes integrated with high-order modulation schemes.

Let $\boldsymbol{U}_0^{N-1}$ be a vector of independent and identically distributed random variables over $\mathbb{GF}(Q)$ where $N = 2^n$ for a positive integer n. The channel input $\boldsymbol{X}_0^{N-1}$ is obtained by the transformation $\boldsymbol{X}_0^{N-1} = \boldsymbol{B} \cdot \boldsymbol{F}^{\otimes n} \cdot \boldsymbol{U}_0^{N-1}$ where $\boldsymbol{B}$ denotes the bit-reversal permutation, $[\cdot]^{\otimes n}$ represents the nth Kronecker power and $\boldsymbol{F}$ is the following kernel [11]: $\boldsymbol{F} = \begin{bmatrix} 1 & 0 \\ \alpha & 1 \end{bmatrix}$, for some $\alpha \in \mathbb{GF}(Q)$. If Q is a prime integer, then $\alpha = 1$ leads to polarization [10]. On the other hand, if Q is not a prime, then it is shown [10, 12] that polarization phenomenon does not always happen. Specifically, if α is chosen uniformly at random among the non-zero elements of $\mathbb{GF}(Q)$ [10] or it is fixed to a primitive element of $\mathbb{GF}(Q)$ [11], polarization occurs. In this paper, we investigate the case where $Q = 2^q$ for $q > 1$. Then, $\alpha = 2$ is always a primitive element of $\mathbb{GF}(Q)$, thus this choice for $\boldsymbol{F}$ leads to polarization.

The transformation $\boldsymbol{U}_0^{N-1} \mapsto \boldsymbol{X}_0^{N-1}$ is a straightforward extension of the polar transform [6], and can be performed in a log-linear complexity order of $\mathscr{O}[N \log N]$. For $Q = 2^q$, q bitwise exclusive-or (XOR) operations are performed at each node along with a multiplication in $\mathbb{GF}(Q)$, which implies that the complexity grows linearly in Q. The elements of $\boldsymbol{U}_0^{N-1}$ corresponding to frozen bit locations $\mathscr{F}$ are set to 0.

Likewise binary SCL decoder, the recursive structure for NB polar decoding is preserved without major modification. After one-step polarization transform, the channel transition probabilities are given as follows:

$$W^-(y_0^1|u_0) = \frac{1}{Q}\sum_{u\in\mathbb{F}_Q} W(y_0|u_0 \oplus 2u)W(y_1|u), \qquad W^+(y_0^1, u_0|u_1) = \frac{1}{Q}W(y_0|u_0 \oplus 2u_1)W(y_1|u_1),$$

where $W(y|u)$ denotes the likelihood of y conditioned on u. Using this, the channel transition probabilities can be computed recursively in a similar way to the binary case. In addition, the probability convolution is more efficiently computed by the fast Fourier transform (FFT) over the Galois field, having a complexity of $\mathscr{O}[qQ]$ per q bits. This recursive structure enables the use of the SCL decoder [7]. As in the binary case, SCL can be implemented with $\mathscr{O}[qQLN\log_2(N)/2]$ complexity for a list size of L. For the same block length of qN bits, the computational complexity of binary polar decoding is $\mathscr{O}[qLN\log_2(qN)]$. Hence, quaternary codes have no more than double complexity of binary codes as shown in Fig. 1a. By using irregular pruning to reduce complexity [9], quaternary polar codes can be lower complex than most typical LDPC codes for codewords shorter than 10^4 bits. More importantly, NB polar SCL decoding is parallelizable for q-bit tuples, and hence, q-fold improvement in decoding throughput is possible.

There is a variety of efficient code construction algorithms for binary and nonbinary polar codes [15–17]. We use the no-loss greedy code construction algorithm [14] for NB polar codes with an adaptation to the modified kernel $\boldsymbol{F}$, which enables fast and accurate construction.

3. Nonbinary vs binary polar codes

Here, we present bit-error-rate (BER) performance results for the nonbinary polar coding. For comparison, we consider the binary polar codes as a benchmark. In order for fairness, we assume that same amount of information bits is conveyed in both cases; specifically, we consider $(N/q, K/q)_Q$ polar codes over $\mathbb{GF}(Q = 2^q)$ to have identical N-bit codeword lengths for both binary ($Q = 2$) and quaternary ($Q = 4$) codes. The code construction is performed by assuming that the channel outputs for the AWGN channels are quantized according to the channel degradation scheme, both with maximum output alphabet size of $M = 16$. The systematic encoding is employed.

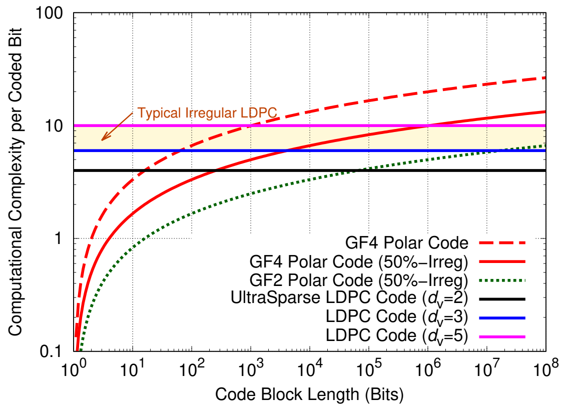
Fig. 1b compares the performance of $(256, 128)_4$ quaternary polar code over $\mathbb{GF}(2^2)$ designed for 16QAM and $(512, 256)_2$ binary polar code under SCL decoding with list sizes of $L = 1, 16$. Here, we assume an outer Bose–Chaudhuri–Hocquenghem (BCH) code of 7% overhead, having a threshold at 10^{-3} to realize the final BER of 10^{-15}. We observe that a significant performance gain up to 0.7 dB is attained by using the quaternary polar code for the same length of $N = 512$. This performance gain is particularly remarkable since the nonbinary polar code usually has a disadvantage in terms of polarization speed as the quaternary polar transform is applied one stage less than the binary polar codes for fairness. The performance gains become even more significant at large block lengths as below.

In Figs. 1c and 1d, we consider longer polar codes for $N = 2048$ and $N = 4096$, respectively. We observe that the performance gain by using a nonbinary polar code increases greater than 1 dB, suggesting that nonbinary codes can be more advantageous for larger block-lengths. This result is intuitive as the loss in the polarization speed for using one less stage of polarization becomes negligible as the number of stages increases.

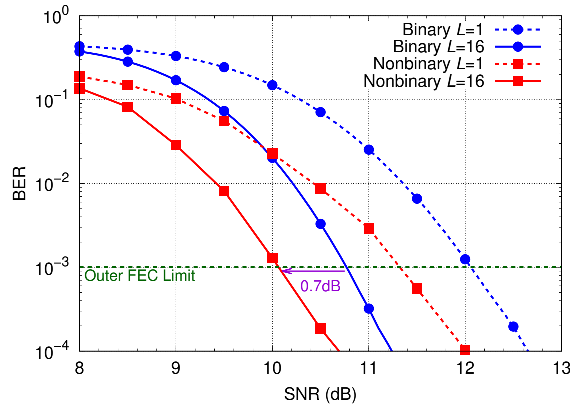
As a final remark, let us note that the computational complexity of SCL can be further reduced by employing a look-up table at each node of the decoding tree at the expense of slight performance degradation. If m symbols are used for indexing the likelihoods at each node, a look-up table scheme can be constructed by using memory space in the order of $\mathscr{O}[N\log_2(N)m^2\log_2(m)]$ bits. This scheme is particularly useful for NBICM since it reduces the complexity of SCL to an index search only. Rigorous analysis of quantization loss will be presented in our further work.

4. Conclusions

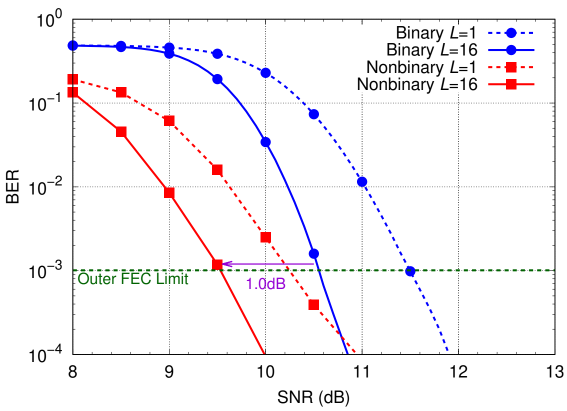
In this paper, we proposed a nonbinary polar coding scheme for high-order modulation, and compared its performance with binary polar codes in a fair manner. We showed that the nonbinary polar codes designed for 16QAM can significantly outperform binary polar codes even at moderate block-lengths. The performance gains for NBICM compared to BICM become even more significant at high block-length. This performance gain for Q^2-QAM for $Q = 2^q$ along with



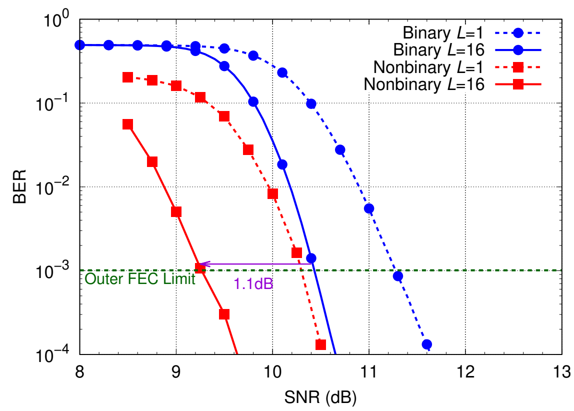
(a) Computational complexity comparisons.



(b) Binary $(512,256)_2$ vs. nonbinary $(256,128)_4$.



(c) Binary $(2048,1024)_2$ vs. nonbinary $(1024,512)_4$.



(d) Binary $(4096,2048)_2$ vs. nonbinary $(2048,1024)_4$.

Fig. 1: Binary vs. nonbinary systematic polar codes for 16QAM (L-list SCL decoding).

reduced latency due to parallelism is achieved at the expense of just a Galois field size (Q-fold) increase in complexity. It should be noted that the complexity increase can be even higher when we use BICM-ID compared to NBICM because of turbo iterations and soft-decision calculations.

References

1. T. Koike-Akino et al., "Coded modulation design for finite-iteration decoding and high-dimensional modulation," *OFC* (2015): W4K.1.
2. M. Arabaci, I. B. Djordjevic, R. Saunders, R. M. Marcoccia, "High-rate nonbinary regular quasi-cyclic LDPC codes for optical communications," *JLT* **27** 23 (2009): 5261–7.
3. I. Djordjevic, B. Vasic, "Nonbinary LDPC codes for optical communication systems," *IEEE PTL* **17** (2005): 2224–6.
4. T. Xia et al., "Nonbinary LDPC convolutional codes for high-dimensional modulations," *SPPCom* (2015): SpS3D-5.
5. T. Koike-Akino et al., "Experimental demonstration of nonbinary LDPC convolutional codes for DP-64QAM/256QAM," *ECOC* (2016): W.2.C.1.
6. E. Arıkan, "Channel polarization: A method for constructing capacity-achieving codes for symmetric binary-input memoryless channels," *IEEE TIT* **55** 7 (2009): 3051–73.
7. I. Tal, A. Vardy, "List decoding of polar codes," *IEEE TIT* **61** 5 (2015): 2213–26.
8. B. Li, H. Shen, D. Tse, "An adaptive successive cancellation list decoder for polar codes with cyclic redundancy check," *IEEE COMML* **16** 12 (2012): 2044–7.
9. T. Koike-Akino et al., "Irregular polar coding for complexity-constrained lightwave systems," *JLT* **36** 11 (2018): 2248–58.
10. E. Şaşoğlu, E. Telatar, E. Arıkan, "Polarization for arbitrary discrete memoryless channels," *ITW* (2009).
11. R. Mori, T. Tanaka, "Non-binary polar codes using Reed-Solomon codes and algebraic geometry codes," *ITW* (2010).
12. W. Park, A. Barg, "Polar codes for Q-ary channels, $q = 2^r$," *IEEE TIT* **59** 2 (2013): 955–69.
13. S. Cayci, O. Arikan, "Lossless polar compression of q-ary sources," *ISIT* (2013).
14. S. Cayci, O. Arikan, E. Arıkan, "Polar code construction for non-binary source alphabets," *SPCAC* (2012).
15. I. Tal, A. Sharov, A. Vardy, "Constructing polar codes for non-binary alphabets and MACs," *ISIT* (2012).
16. I. Tal, A. Vardy, "How to construct polar codes," *IEEE TIT* (2013).
17. P. Trifonov, "Efficient design and decoding of polar codes," *IEEE TOC* (2012).