Schalkwijk- Kailath Feedback Error Correction for Optical Data Center Interconnects

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TR2018-147 September 26, 2018

Abstract

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European Conference on Optical Communication (ECOC)

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Introduction

Capacity-approaching forward error correction (FEC) such as low-density parity-check codes has enabled high-throughput optical transmissions. However, strong FEC requires a high-power decoder in general. It is of great importance to trade-off the decoding power and coding gain for FEC design. In theory, it is known [1] since the 50's that FEC is not necessary to achieve the channel capacity if we can exploit channel feedback. Although the use of channel feedback does not increase the capacity in a memoryless channel, it is known to improve error exponent. This was discussed in various Schalkwijk–Kailath (S-K) schemes [1–5], where doubly-exponential error decaying performance is realized without relying on algebraic FEC codes. In this paper, we apply the S-K feedback error correction to intra-data center interconnects (DCI), as a viable alternative to recent FEC codes.

The use of channel feedback has not been considered in most fiber-optic systems, in particular for long-haul communications, because the round-trip time (RTT) due to lightwave propagation is non-negligible, e.g., 42ms RTT for a great-circle path between New York and San Francisco. Nevertheless, the propagation latency in short-reach fibers is relatively small, i.e., 5μs for 1km. The channel feedback potentially gives us additional opportunity to introduce several approaches such as adaptive modulation & coding [6–8], precoding [9–11], interference alignment [12], automatic repeat request [13], and rateless codes [14]. As an alternative approach, we adopt an improved S-K feedback scheme [5], whose computational complexity is linear with the number of retransmissions, to achieve high gain without using power-hungry FEC codes. We show that the proposed scheme achieves a net coding gain (NCG) greater than 10.9 dB with few feedback occurrences, requiring a small number of arithmetic multiplications and limited latency for short-reach intra-DCI networks.

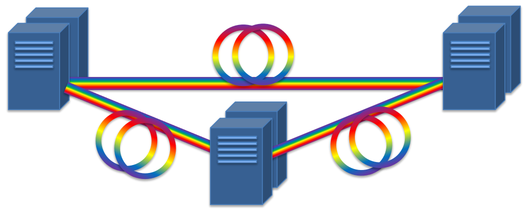

Fig. 1: Optical data center networks.

S-K feedback error correction

Fig. 1 depicts short-reach optical interconnects for intra-data center networks, which may not deploy long fibers so that latency of light propagation is constrained. For such intra-DCIs, we employ the S-K feedback error correction scheme [5], which is illustrated in Fig. 2. Let s be a modulated symbol (or sequence) to transmit, e.g., via M-ary quadrature-amplitude modulation (QAM). The S-K feedback scheme simply retransmits it multiple times utilizing feedback information from the receiver side. The transmission signals $\boldsymbol{x} = [x_1, x_2, \ldots, x_N]^{\mathrm{T}}$ over N-times retransmissions is expressed as follows:

$$\boldsymbol{x} = \boldsymbol{g}s + \boldsymbol{F}(\boldsymbol{y} - \boldsymbol{x} + \boldsymbol{w}), \tag{1}$$

where $\boldsymbol{y} = [y_1, y_2, \ldots, y_N]^{\mathrm{T}}$ denotes the received signal, modeled as

$$\boldsymbol{y} = \boldsymbol{x} + \boldsymbol{z}. \tag{2}$$

The received signal is directly sent back to the transmitter via feedback channel. Here, $\boldsymbol{w}$ and $\boldsymbol{z}$ are additive white Gaussian noises having variance of σ^2 and $1/\rho$ at feedback channel and forward channel, respectively. The vector $\boldsymbol{g} = [g_1, g_2, \ldots, g_N]^{\mathrm{T}}$ is used at encoding in (1) and also at decoding to obtain QAM estimates $\hat{s}$:

$$\hat{s} = \boldsymbol{g}^{\mathrm{T}}\boldsymbol{y}. \tag{3}$$

The improved S-K scheme [5] uses exponen-

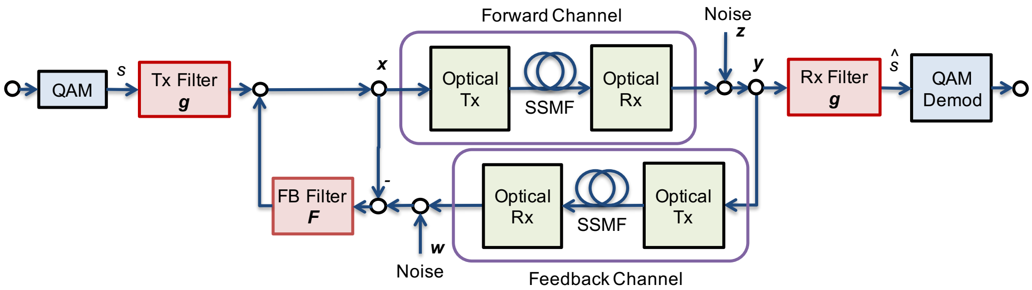

Fig. 2: Optical DCI employing S-K feedback error correction[5] with exponential decaying filters $\boldsymbol{g}$ and $\boldsymbol{F}$.

tially decaying weights $\boldsymbol{g} = \alpha[1, \beta, \beta^2, \ldots, \beta^{N-1}]^{\mathrm{T}}$, where $\alpha = \sqrt{(1-\beta^2)/(1-\beta^{2N})}$ is a power normalization constant. The factor $0 \leq \beta \leq 1$ is a control parameter to adjust the output signal-to-noise ratio (SNR), and the optimal value is the positive minimum root of the following equation:

$$\beta^{2N} - (1 + (1+\sigma^2)\rho\gamma)N\beta^2 + N - 1 = 0, \quad (4)$$

where $0 \leq \gamma \leq 1$ denotes noise canceling parameter to be optimized. The linear feedback coding matrix $\boldsymbol{F}$ is a strictly lower-triangular matrix having exponentially decaying column as follows:

$$\boldsymbol{F} = \alpha' \cdot \mathrm{Toeplitz}\big([0, 1, \beta, \beta^2, \ldots, \beta^{N-2}]^{\mathrm{T}}\big), \quad (5)$$

where $\alpha' = -(1-\beta^2)/(1+\sigma^2)\beta$ is a power normalization constant, and $\mathrm{Toeplitz}(\boldsymbol{b})$ denotes a lower-triangular Toeplitz matrix having a vector $\boldsymbol{b}$ in the first column. Because of the Toeplitz property, the S-K encoding operation in (1) can be implemented in a complexity order of $\mathcal{O}[N]$ not $\mathcal{O}[N^2]$. Specifically, the nth transmission can be efficiently generated in a recursive manner:

$$x_n = g_n s + c_n, \quad (6)$$

$$c_{n+1} = \beta c_n + \alpha'(y_n - x_n + w_n), \quad (7)$$

for $c_1 = 0$. The linear feedback scheme can increase the effective SNR as follows:

$$\rho' = \frac{(1+\sigma^2)N(1-\gamma)\rho}{\sigma^2 + \beta^{2(N-1)}}. \quad (8)$$

Over N-times retransmission, the S-K encoder requires at most $3N-2$ arithmetic multiplications for (6) and (7) in total. The decoder needs just N-times multiplications in (3). This linear computational complexity is one of major benefits of the S-K feedback scheme. The increase of decoding latency is also linear as $(N-1)\tau$, where τ is the RTT. For short-reach DCIs, the latency can be maintained at a small value, e.g., $\tau \simeq 10\mu$s for 1km reach. Although additional delay is induced in practice for channel feedback, no complicated signal processing is required to send back the received signals.

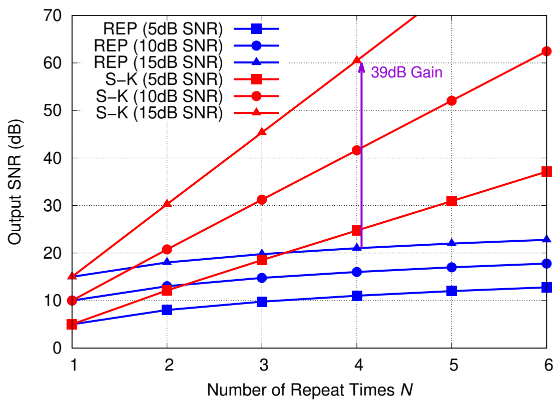

Fig. 3: Output SNR after linear feedback error correction.

More importantly, the S-K scheme can achieve an exponentially increasing SNR output because of the $\beta^{2(N-1)}$ term in (8). The achievable SNR for an ideal feedback with $\sigma^2 = 0$ is shown in Fig. 3, where we can see that the effective SNR can be significantly increased with the number of retransmissions N. It should be noted that the output SNR for a simple repetition scheme (denoted by 'REP') with $\beta = 1$ and $\boldsymbol{F} = \boldsymbol{0}$ will be linear $N\rho$, which is significantly lower than the S-K scheme. For example, the S-K scheme achieves an output SNR greater than 60 dB for channel input SNR of 15 dB by $N = 4$ transmissions, whereas the conventional repetition scheme achieves 21 dB. Although the repetition can increase the output SNR, the spectral efficiency decreases with N if the constellation size M is constant. In order to keep the spectral efficiency constant over repetitions, the modulation order should be scaled up to M^N for pre-determined number of repeats N.

Performance results

The S-K feedback scheme does not require any FEC codes to approach the channel capacity when a large number of retransmissions is available. However, multiple retransmissions increase the decoding latency in a linear manner. Hence, we should limit the number of retransmissions N, and integrate with low-power FEC scheme to

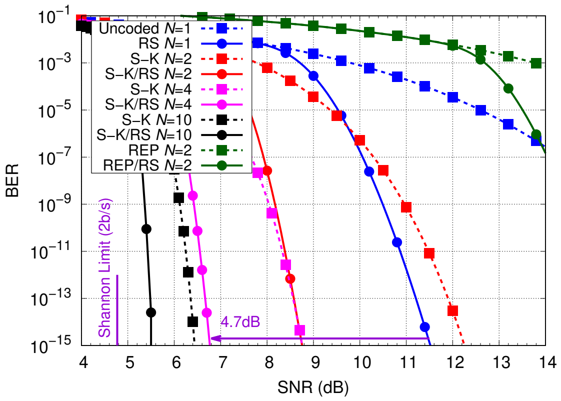

Fig. 4: BER performance of S-K feedback error correction with/without RS$(255, 239)$ code for $M = 4$ QAM.

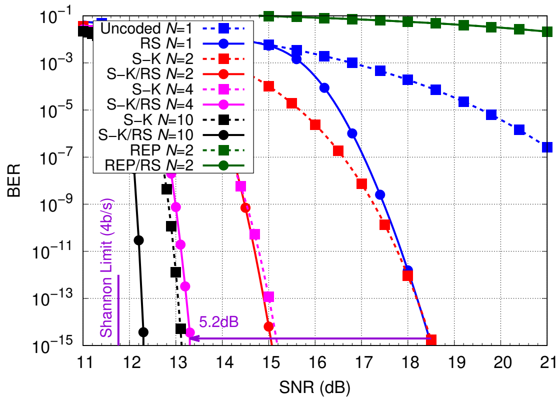

Fig. 5: BER performance of S-K feedback error correction with/without RS$(255, 239)$ code for $M = 16$ QAM.

compensate in practice. We consider the first-generation FEC of Reed–Solomon code RS$(255, 239)$, widely used for various standards. The use of RS codes enables us to evaluate closed-form bit-error rate (BER) even below 10^{-15}. At such a BER target, the NCG of the RS code is 6.2 dB.

Fig. 4 shows BER performance analysis as a function of SNR for $M = 4$ QAM transmission. We can see that the S-K scheme without the RS code for a single feedback ($N = 2$) offers almost comparable performance to the RS code without feedback. Given three-times feedback ($N = 4$), the S-K scheme with the RS code achieves 4.7 dB gain over the RS code without feedback. Since the S-K scheme itself has no code rate loss, the S-K scheme in conjunction with the RS code achieves an NCG of 10.9 dB at a BER of 10^{-15}. It is also observed that the S-K scheme approaches closely the Shannon limit with a relatively large number of retransmissions for $N \geq 10$. Note that the simple repetition scheme ('REP') performs very poorly. This is because SNR gain is only 3 dB for $N = 2$ while the number of constellation points need to be increased as $M^2 = 16$ QAM not 4-QAM in order to provide a fair comparison.

We show the BER performance at a higher spectral efficiency with $M = 16$ QAM transmission in Fig. 5. Comparing to the case of $M = 4$ in Fig. 4, the gain of the S-K scheme is slightly improved to be closer to the Shannon limit. For $N = 4$, the S-K feedback scheme with the RS code achieves an NCG of 11.4 dB in total.

Conclusions

We introduced S-K feedback scheme for intra-DCI systems, to achieve high gain without relying on powerful FEC codes. The proposed S-K scheme integrated with the legacy RS code shows an NCG of 11.4 dB with $N = 4$ feedback for 16-QAM transmissions. The S-K scheme can be realized by a few number of multiplications, and the latency can be maintained at a small value for short-reach DCI applications. To deploy the feedback error correction paradigm in practical systems, there remain further investigations such as buffering, dispersion and polarization distortions.

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