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Information-Theoretic Metrics in Coherent Optical Communications and their Applications

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Introduction

Achievable information rates (AIRs) have emerged as practical tools to design fiber optical communication systems[1,2]. AIRs can be used to design modulations formats and to predict the performance of forward error correction (FEC)[3–5]. FEC typically comes in two flavors: hard-decision (HD) and soft-decision FEC. In the former, the FEC decoder is fed with bits, while in the latter, the FEC decoder is fed with logarithmic likelihood ratios (LLRs). To increase spectrally efficiency, high order constellations such as QAM are required. The combination of these modulation formats and FEC is called coded modulation (CM), which dates back to G. Ungerboeck's trellis coded modulation[6]. CM is a key technique for designing high rate coherent receivers[7], or to achieve gains via probabilistic and geometric shaping[8,9].

While early generations of fiber optical systems were based on HD-FEC, modern SD-FEC such as low-density parity-check (LDPC) codes are very popular. Although SD-FEC codes in general outperform HD-FEC codes in terms of post-FEC BER, SD-FEC decoders are typically power hungry. SD-FEC decoders also suffer from high latency as they are typically based on iterative decoding. The never-ending increase in line rates and the power consumption and latency issues of SD-FEC make the use of HD-FEC very attractive in practice . One particularly popular family of HD-FEC codes for fiber optical communications are staircase codes (SCCs), introduced in[10]. Very recently, the combination of SCCs and probabilistic shaping has been studied[11].

In this paper, we analyze CM based on HD-FEC from an achievable information rate point of view. We quantify the penalties caused by using a HD-FEC both from information-theoretic and coding points of view. We first show that as the constellation cardinality increases, the theoretical rate loss caused by using HD-FEC instead of SD-FEC increases. We then argue that the correct metric to study the relative rate loss. This rate loss is then shown to decrease as the cardinality increases. Finally, we show that this effect is also observed if the state-of-the-art HD-FEC are used. The relative losses caused by using SCCs with less than

System Model, AIRs and Staircase Codes

The most popular AIR for binary SD-FEC is the generalized mutual information (GMI)[1,12]. The GMI is defined as

$$\text{AIR}^{\text{SD}} = \text{GMI} \triangleq \sum_{k=1}^{m} I(B_k;Y), \qquad (1)$$

where $I(B_k;Y)$ is the mutual information between the code bits and the received symbols and $M = 2^m$ is the number of constellation points in the constellation. When the binary FEC under consideration is HD, an AIR is given by

$$\text{AIR}^{\text{HD}} = m(1 - H_{\text{b}}(\text{BER})), \qquad (2)$$

where BER is the average bit error rate across all the m bit positions and $H_{\text{b}}(\cdot)$ is the binary entropy function.

Throughout this paper we consider a four-dimensional (4D) real additive white Gaussian noise (AWGN) channel and pulse amplitude modulation (PAM) labeled by the binary reflected Gray code. The 4D constellation with cardinality M is formed by the product of four PAM constel-

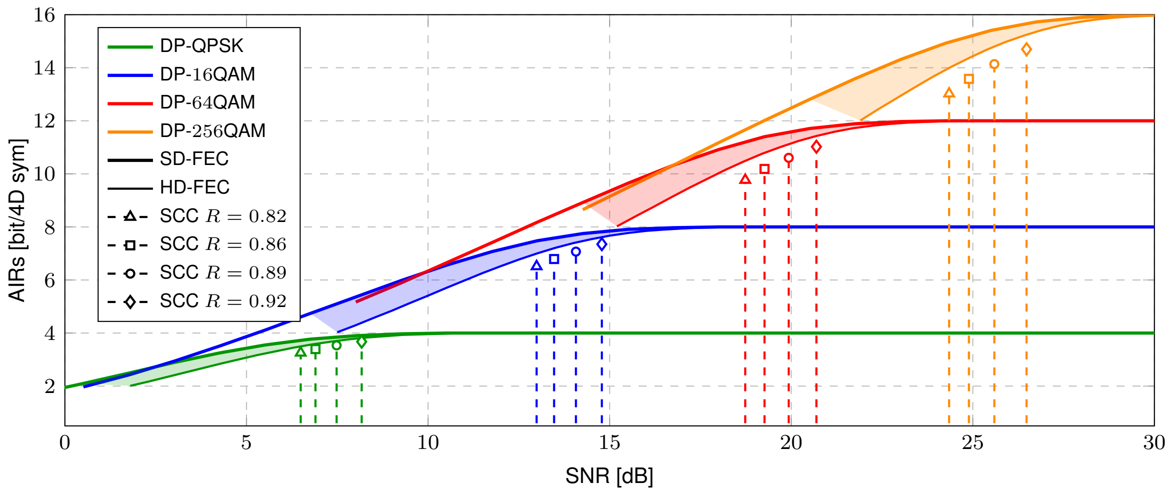

Fig. 1: AIRs for SD (thick) and HD (thin) for different constellation sizes (colors). Markers show the results for SCCs.

t_c	m_c	n_c	k_c	R_c	R	OH%
2	9	512	493	0.96	0.93	8
3	9	512	484	0.95	0.89	11
4	9	512	475	0.93	0.86	14
5	9	512	466	0.91	0.82	18

Tab. 1: Parameters for SCCs considered in this paper

lations with $\sqrt[4]{M}$ constellation points. The constellation sizes under consideration are $M = 16, 256, 4096, 65536$, which corresponds to constellations with $1, 2, 3$ and 4 bits per real dimension. This is equivalent to consider the following dual-polarization (DP) constellations: DP-QPSK, DP-16QAM, DP-64QAM, DP-256QAM.

To make the results in this paper more practically relevant, we also consider SCCs with variable FEC overhead. Table 1 shows the parameters of the SCCs under consideration. The component codes of these SCCs are rate $R_c = k_c/n_c$ binary extended Bose-Chaudhuri-Hocquenghem (BCH) codes with 1 extra parity bit, $t_c = 2, 3, 4, 5$ and $m_c = 8$. The codeword length is $n_c = 2^{m_c}$, which contains $k_c = n_c - m_c t_c - 1$ information bits. As shown in Table 1, the resulting SCCs code rates are $R = \{0.82, 0.86, 0.89, 0.93\}$. Decoding is done using iterative bounded-distance decoding with a window length of 9 and 7 iterations per window. These parameters were chosen because they lead to multiple FEC code rates, but more importantly, to the same SCC block length, namely all these codes have a block length of $65, 536$ code bits. We decided to have this parameter fixed as it gives a rough indication of decoding complexity and delay.

Numerical Results: AIRs

The SD- and HD-AIRs in (1) and (2) are shown in Fig. 1 (thick and thin lines, resp.). These results show the theoretical loss caused by using HD-FEC instead of HD-FEC. These results also seem to indicate that as the constellation size M increases, the loss increases. Equivalently, the SNR penalty caused by using HD-FEC instead of SD-FEC increases as M increases. This effect is schematically shown in Fig. 1 using shaded areas.

Fig. 1 also shows (with markers) the SNR required for SCCs to achieve a post-FEC BER of $5 \cdot 10^{-5}$. Here we assume a concatenation with an outer BCH code with rate $R_o = 0.9922$, which will bring the post-SCC BER down to 10^{-15}. The "height" of these markers corresponds to the throughput the code under consideration can achieves and is calculated as $\Theta = R_o R m$ [bit/4D sym], where R is the rate of the SCC and m is the number of bits per symbol of the constellation under consideration.

Numerical Results: Absolute Gains

Fig. 2 shows the absolute AIR losses (AL) caused by HD-FEC with respect to SD-FEC (solid lines), which is defined as AL $=$ AIR$^{\text{SD}}$ $-$ AIR$^{\text{HD}}$. This figure shows that indeed the rate losses increase as the modulation format increases.

Fig. 2 also shows the results obtained by SCCs (markers). In this case, the AIR loss is given by AL $=$ AIR$^{\text{HD}}$ $- \Theta$ and it is shown for the SNR values where the SCC under consideration achieves a post-FEC BER of $5 \cdot 10^{-5}$ (see thresholds in Fig. 1). The results in Fig. 2 show that this absolute loss caused by using the (suboptimal) SCC instead of considering AIR$^{\text{HD}}$ is also increasing as the constellation size increasing. These results highlight the fact that for a given SCC (a given marker), the AIR loss grows approximately linearly with constellation size. The slope of this increasing AIR loss decrease as the code rate of

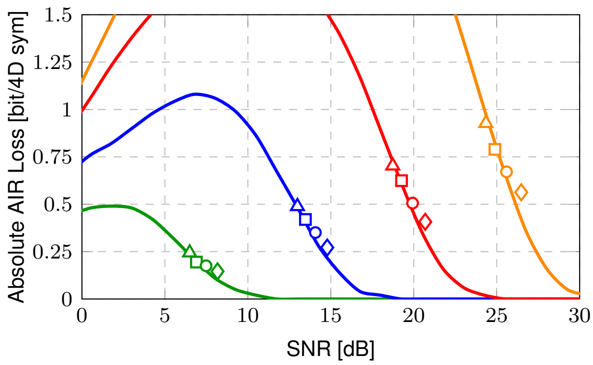

Fig. 2: Absolute AIR losses: SD-FEC vs. HD-FEC (solid lines) and HD-FEC vs. SCCs (markers).

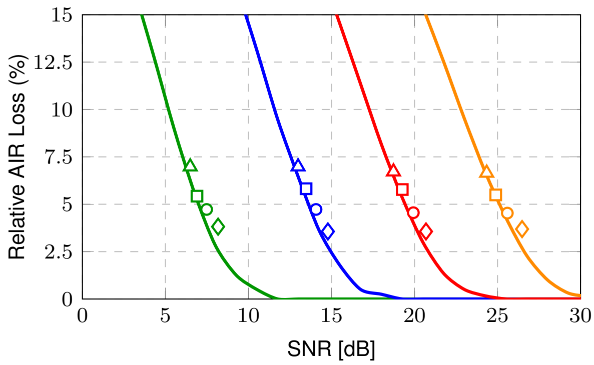

Fig. 3: Relative AIR losses: SD-FEC vs. HD-FEC (solid lines) and HD-FEC vs. SCCs (markers).

the SCC decreases.

Numerical Results: Relative Gains

These absolute AIR loss results in Fig. 2 can be misleading. They indeed show that the AIR loss increases as the constellation size increases. However, for large constellation sizes, large rates are being considered. For a more fair comparison, we propose to study the relative (RL) AIR losses, defined as $\text{RL} = (\text{AIR}^{\text{SD}} - \text{AIR}^{\text{HD}})/\text{AIR}^{\text{SD}}$. The RL are shown in Fig. 3. The results in this figure shows that the relative AIR loses remain approximately constant as the constellation size increases. This is shown by both the AIR results (solid lines) and by the SCC results (markers). Unlike the results in Fig. 2, where for a give SCC (a given marker) the loss increases as the constellation size increases, Fig. 3 shows a constant relative loss. Interestingly, for the codes under consideration, this relative loss is at most 7.5%. This happens for the FEC rate $R = 0.82$, which is a relatively low FEC rate for HD-FEC. For FEC rate $R = 0.93$, the relative loss is only around 3.5%, regardless of the constellation size under consideration.

The results in Fig. 3 show that even if the constellation size increases, the relative loss caused by the demapper making a hard decision on the noisy symbols is constant. This result is slightly counterintuitive as one would expect larger losses when the constellation size increases. Our intuition here is that when the demapper makes a hard-decision at a symbol level, this is equivalent to quantize the received noisy symbol using m bits. For large constellations (e.g., DP-256QAM), the number of symbols is large, and thus the number of quantization bits is large.

Conclusions

In this paper we studied two achievable information rate losses for coded modulation. The first one is the one caused by the use of hard-decision instead of soft-decision FEC. This first analysis was made based on information theoretic quantities and applies to ideal codes. The second achievable information rate loss we studied was the one caused by the use of practical hard-decision FEC. We focused on staircase codes and its loss respect to its information theoretic maximum. In both analyses, it was shown that the absolute rate loss increases as the constellation size increases. However, the relative rate losses were shown to be constant, regardless of the modulation format. The results of this paper give a strong theoretical and practical support for combining high-rate hard-decision FEC and high order modulation formats for future spectrally-efficient fiber optical communications.

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