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Terahertz Imaging of Multi-Level Pseudo-Random Reflectance

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Abstract—This paper introduces a terahertz (THz)-based absolute positioning system with a *single* THz transceiver as the read head and a *multi-level pseudo-random* reflectance pattern (e.g., multi-level m -sequences) as the high-resolution scale in a compressed scanning mode. One of key technical challenges here is to computationally recover the multi-level pseudo-random reflectance pattern from compressed measurements. To this end, we develop a variational Bayesian approach to exploit the finite alphabet of reflectance levels and enable a pixel-wise iterative inference for fast recovery. Numerical results confirm the effectiveness of the proposed method.

I. INTRODUCTION

Over the past years, there has been an increased interest in the use of terahertz (THz) wave for sensing, detection and imaging. THz sensing can operate in a *raster* or *compressed* scanning mode [1]–[3].

In the raster scanning mode, as shown in Fig. 1 (a), the sample under inspection is illuminated by a THz point source with a time-compact source pulse and a small spot size. The THz emitter sends a focused beam to inspect a small area of the sample, and a programmable mechanical raster moves the sample in order to measure the two-dimensional surface of the sample. In the compressed scanning mode, as shown in Fig. 1 (b), the THz pulse is first collimated to a broad beam and then spatially encoded with a random mask with the help of a spatial light modulator [3]. At the receiver side, the spatially encoded beam is re-focused by a focusing lens and received by a single-pixel photoconductive detector. The sample image can then be recovered by sparsity-driven minimization methods. Compared with the raster scanning mode, the compressed scanning mode has a much shorter acquisition period without a mechanical raster move.

Here we are particularly interested in THz-based absolute positioning systems where pseudo-random sequences (e.g., M -sequences) are used for high-resolution position encoding [4]–[6]. Fig. 1 (c) shows a THz absolute positioning system which uses a single THz transceiver, along with random masks and collimating/focusing lenses, to scan an area of the scale encoded by a multi-layer, multi-track, multi-level pseudo-random code pattern which is mapped into a unique position (hence absolute positioning). An example of the multi-level scale is shown in Fig. 3 (a), where 4 different levels are arranged into a pseudo-random code pattern in order to uniquely encode a position. The multi-level encoding at the scale can be realized by a metamaterial plate designed to reflect energy proportional to the polarization direction of the

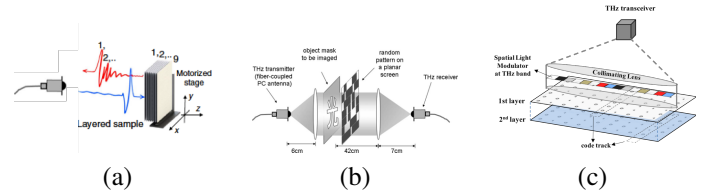


Fig. 1. THz sensing with a) a raster scanning (from [2]), b) a compressed scanning (from [3]), and c) a multi-layer THz encoder system.

incident THz wave [6]. In this paper, we aim to address the remaining technical challenge: how to recover the multi-level pseudo-random code pattern with compressed measurements received at the single THz transceiver for real-time positioning.

II. PROPOSED SCHEME

In this paper, we exploit the *non-negative, finite alphabet* features of the code pattern to recover the reflectance pattern from compressed THz measurements. To this end, we use a variational Bayesian framework to impose a hierarchical prior model for enforcing the two features and to develop a decoupled element-wise iterative algorithm to estimate the pseudo-random pattern in a computationally efficient way.

A. Compressed Measurements

Let $\mathbf{x} = [x_1, \dots, x_N]^T$, $x_n \in \{\mu_1, \dots, \mu_K\}$ denote the pseudo-random code pattern to be estimated with μ_k specifying the non-negative reflectance from a finite set of K unknown levels. The compressed scanning generates the following measurements

$$\mathbf{y} = \mathbf{A}\mathbf{x} + \mathbf{v}, \quad (1)$$

where each row of \mathbf{A} represents a random mask at the THz band, $\mathbf{v} = [v_1, \dots, v_M]^T$ is the Gaussian noise with zero mean and variance β^{-1} , and $\mathbf{y} = [y_1, \dots, y_M]^T$ collects M compressed measurements.

To account for the *non-negative, finite alphabet* features of x_n , we impose a hierarchical prior model on x_n

$$\mathbb{P}(x_n | \boldsymbol{\alpha}_n, \mathbf{C}_n; \mathbf{u}) = \prod_{i=1}^K \mathcal{N}_+(x_n; \mu_i, \alpha_{n,i}^{-1})^{C_{n,i}}, \quad (2)$$

where $\mathbf{C}_n = [C_{n,1}, \dots, C_{n,K}]$ is a label vector with only one non-zero element assigning one of the K truncated Gaussian

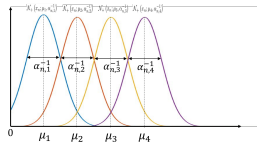


Fig. 2. The truncated Gaussian mixture distribution for the n -th reflectance x_n with 4 components.

components to x_n and

$$\mathcal{N}_+(x_n; \mu, \alpha^{-1}) = \begin{cases} \eta^{-1} \sqrt{\frac{\alpha}{2\pi}} e^{-\frac{\alpha(x-\mu)^2}{2}} & x \geq 0, \\ 0, & x < 0, \end{cases} \quad (3)$$

with $\eta = 1 - \Phi(-\mu\sqrt{\alpha})$ denoting the normalization factor and $\Phi(\cdot)$ denoting the cumulative distribution function of the standard normal distribution. Moreover, we assume that the label variable C_n follows the categorical distribution or generalized Bernoulli distribution $\mathbb{P}(C_n; \boldsymbol{\pi}) = \prod_{i=1}^K \pi_i^{C_{n,i}}$, with event probabilities $\boldsymbol{\pi} = [\pi_1, \dots, \pi_K]$ where $\sum_{i=1}^K \pi_i = 1$. It is easy to see that

$$\mathbb{P}(x_n | \boldsymbol{\alpha}_n; \mathbf{u}) = \sum_{i=1}^K \pi_i \cdot \mathcal{N}_+(x_n; \mu_i, \alpha_{n,i}^{-1}), \quad (4)$$

results in the truncated Gaussian mixture distribution for x_n which is illustrated in Fig. 2 for the case of $K = 4$. We further assume the Gamma distribution for $\alpha_{n,i}$, i.e., $\mathbb{P}(\alpha | a; b) = \prod_{i=1}^K \prod_{n=1}^N \text{Gamma}(\alpha_{n,i} | a, b)$ with $a = b = 10^{-6}$.

B. Proposed Code-Pattern Recovery Algorithm

1) *Decoupled element-wise likelihood function*: To enable an element-wise recovery algorithm, we first decouple the original likelihood function of \mathbf{y} into a decoupled approximate likelihood function of $\{x_n\}_{n=1}^N$

$$\mathbb{P}(\mathbf{y} | \mathbf{x}; \beta) \approx \prod_{n=1}^N \frac{1}{\sqrt{2\pi\hat{\tau}_n}} e^{-\frac{(x_n - \hat{r}_n)^2}{2\hat{\tau}_n}}. \quad (5)$$

where the approximated element-wise mean \hat{r}_n and variance $\hat{\tau}_n$ can be found in a similar way of [5].

2) *Posterior distributions of hidden variables $\{\mathbf{x}, \boldsymbol{\alpha}, \mathbf{C}\}$* : Next, we derive the posterior distributions for hidden variables $\{\mathbf{x}, \boldsymbol{\alpha}, \mathbf{C}\}$. The *element-wise reflectance* $\{x_n\}_{n=1}^N$ follows an independent truncated Gaussian posterior distribution,

$$q(x_n) = \begin{cases} \phi_n^{-1} \frac{1}{\sqrt{2\pi\tilde{\sigma}_n}} \exp\left(-\frac{(x_n - \tilde{\mu}_n)^2}{2\tilde{\sigma}_n^2}\right) & x_n > 0 \\ 0 & x_n \leq 0 \end{cases}, \quad (6)$$

where $\phi_n = 1 - \Phi(-\tilde{\mu}_n/\tilde{\sigma}_n)$ is the normalization factor. The *label vector \mathbf{C}* has the categorical posterior distribution as

$$q(C_{n,i}) = \prod_{i=1}^K (\tilde{\pi}_{n,i})^{C_{n,i}} \quad (7)$$

with $\tilde{\pi}_{n,i} = \exp(\gamma_{n,i} - \ln(\sum_{i=1}^K \exp(\gamma_{n,i})))$ and $\gamma_{n,i} = -0.5\langle \alpha_{n,i} \rangle \langle (x_n - \mu_i)^2 \rangle - \langle \ln \eta_{n,i} \rangle + \ln \pi_i$. The variable $\boldsymbol{\alpha}$ has the Gamma posterior distribution, i.e.,

$$q(\alpha_{n,i}) = \text{Gamma}\left(\alpha_{n,i} | \tilde{a}_{n,i}, \tilde{b}_{n,i}\right) \quad (8)$$

with $\tilde{a}_{n,i} = a + 0.5\langle C_{n,i} \rangle$, $\tilde{b}_{n,i} = b + 0.5\langle C_{n,i} \rangle \langle (x_n - \mu_i)^2 \rangle$.

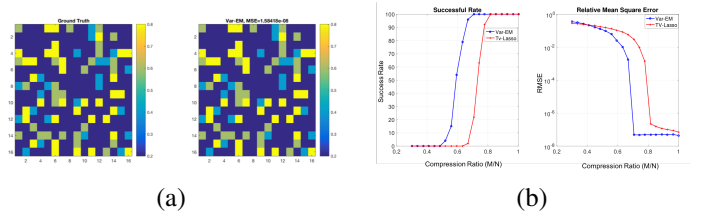


Fig. 3. Numerical validation with a 4-level pseudo-random pattern: (a) Ground truth versus recovered patterns; (b) Success rate and normalized MSE as a function of compression ratio.

3) *Updating for deterministic parameters $\{\beta, \{\mu_i\}_{i=1}^K\}$* : At the t -th iteration, the noise variance β^{-1} can be updated

$$(\beta^{-1})^{t+1} = \sum_{m=1}^M \langle (y_m - w_m)^2 \rangle / M, \quad (9)$$

where w_m is the m -th element of $\mathbf{w} = \mathbf{A}\mathbf{x}$. As we show in [5], there is no closed-form updating rule for the unknown reflectance levels μ_i for the simplest case of $K = 2$, i.e., the binary reflectance. For the generalized multi-level $K \neq 2$ case, we introduce an approximate updating rule

$$\mu_i^{t+1} = \frac{\sum_{n=1}^N \langle C_{n,i} \rangle \langle \alpha_{n,i} \rangle \langle x_n \rangle}{\sum_{n=1}^N \langle C_{n,i} \rangle \langle \alpha_{n,i} \rangle} \quad (10)$$

which turns out to be the weighted average of the posterior mean of x_n (i.e., $\langle x_n \rangle$) in the corresponding class specified by $C_{n,i}$.

III. SIMULATION RESULTS

The proposed method is numerically evaluated with synthetic data and the Monte-Carlo simulation on a sample with a pseudo-random reflectance pattern in Fig. 3 (a) with $K = 4$ levels ($[0.2, 0.4, 0.6, 0.8]$). The recovered reflectance patterns is almost identical to the ground truth. The results in Fig. 3 (b) from the Monte-Carlo simulation suggest that the multi-level pseudo-random pattern can be recovered reliably with compressed measurements.

IV. CONCLUSION

A THz-based encoder system was introduced with a single THz transceiver scanning over a *multi-level pseudo-random* code pattern for high-resolution absolute positioning. This paper proposed an efficient element-wise algorithm to recover the multi-level pseudo-random code pattern with unknown reflectance.

REFERENCES

- [1] N. Horiuchi, "Terahertz technology: Endless applications," *Nature Photonics*, vol. 4, no. 4, pp. 140, Sept. 2010.
- [2] A. Redo-Sanchez, et al., "Terahertz time-gated spectral imaging for content extraction through layered structures," *Nature Communications*, vol. 7, pp. 1–7, Sept. 2016.
- [3] W.L. Chan, et al., "A single-pixel terahertz imaging system based on compressed sensing," *Applied Physics Letters*, vol. 93, Sept. 2008.
- [4] E. M. Petriu, "Absolute position measurement using pseudo-random binary encoding," *IEEE Instrumentation Measurement Magazine*, vol. 1, no. 3, pp. 19–23, Sept. 1998.
- [5] H. Fu, et al., "Terahertz imaging of binary reflectance with variational Bayesian inference," in *The 43rd IEEE ICASSP*, 2018.
- [6] B. Wang, et al., "Metamaterial absorber for THz polarimetric sensing," in *SPIE Photonics West*, 2018.