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Tri-tone freeforms

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Abstract

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Laser Display and Lighting Conference

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Tri-tone freeforms

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ABSTRACT

We find a closed-form solution for the shape of the refractive surface that uniformly irradiates a disk from a Lambertian point light source, then algebraically tailor this surface to project tri-tone graphics.

1. TAILORING WITHOUT ITERATIVE OPTIMIZATION

From a numerical perspective, the problem of tailoring a freeform optical surface to produce an arbitrary irradiance pattern is a very well-behaved optimization problem. Many iterative improvement algorithms have been proposed; most will yield good approximate solutions, provided that the mapping from the optical surface to the projection surface is smooth, Lipshitz, and nonsingular. This note considers a class of target irradiance patterns where none of these conditions hold: "Tri-tone" graphics consisting of a uniformly lit foreground figure and completely unlit background, separated by a brightly lit border. We show that the desired freeform surface can be obtained exactly in a purely algebraic construction, i.e., no iterations or numerical approximations. Figure 1 summarizes the approach: We first solve for the refractive surface that provides uniform illumination of a disk from a Lambertian light source, then "blacken" parts of this disk by redirecting rays to the graphic's boundary.

2. UNIFORM ILLUMINATION FROM A LAMBERTIAN POINT SOURCE

We begin with the well-known relationship between the emittance of a Lambertian light source and the surface area of a disk: A Lambertian source radiates along inclination ϕ with intensity proportional to $\cos\phi$, and therefore the total flux exiting a cone of half-angle ϕ is

$$\mathrm{Flux}(\phi) = \int_0^{\phi}\int_{-\pi}^{\pi} \cos\alpha \sin\alpha \, d\theta \, d\alpha = \pi \sin^2\phi = \mathrm{DiskArea}(r = q\sin\phi,\ q = 1) \tag{1}$$

To uniformly irradiate a disk of radius p, we seek an optic that maintains this relationship, in some fixed ratio q, for all cone/disk pairs (ϕ, r) up to some limit $0 < \phi \leq \beta$, $0 < r \leq p$. It follows that a suitably shaped lens will bend rays emitted at inclination angle ϕ to hit a projection plane $q\sin\phi$ units from the optical axis, with $q = p\csc\beta$. There is an active literature[1,2] on numerical approximations for the geometry of this surface, but apparently it has not been noticed that the problem can be solved in closed form.

Addressing the lens surface in spherical coordinates and the projection plane in polar coordinates, we can write the correspondence between lens points and plane points as

$$(R(\phi), \phi, \theta) \longleftrightarrow (p\csc\beta\sin\phi, \theta) \tag{2}$$

with $R(\phi)$ the radial extent of the lens surface at inclination ϕ from the optical axis, and θ the azimuthal angle on the lens and polar angle on the projection surface (see fig. 1).

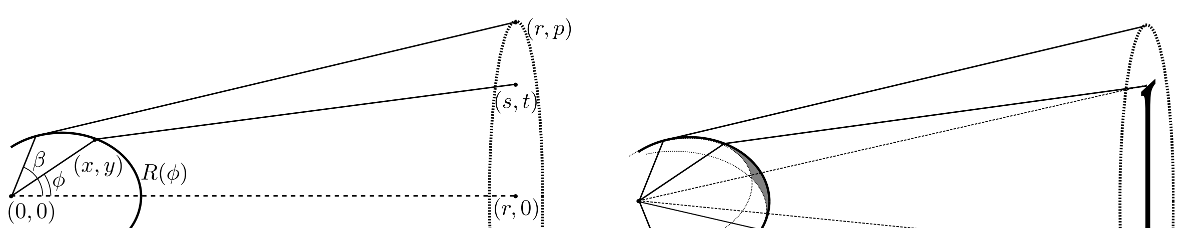

Figure 1. Summary of this paper. LEFT: Refractor surface $R(\phi)$ converts a Lambertian point source at origin to a uniformly irradiated disk of radius p at distance r. RIGHT: Sinking parts of the surface produces black (unlit) regions whose rays are redirected to the figure/ground boundary, producing a tri-tone graphic.

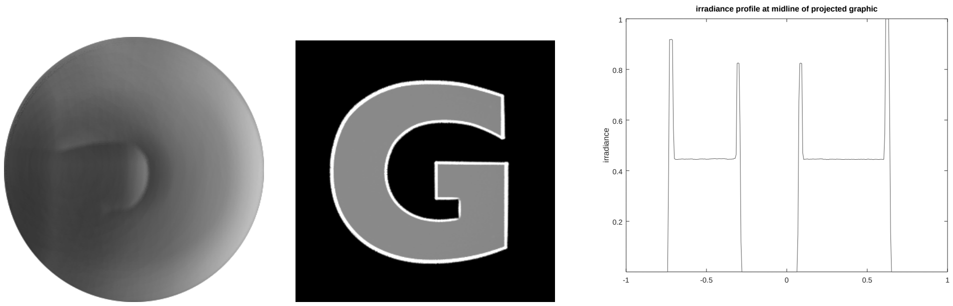

Figure 2. A freeform lens, a ray-traced simulation of its irradiance pattern, and an irradiance profile along a horizontal midline of the image, showing how the extended light source thickens the caustic at the figure/ground boundary.

Using the Cartesian coordinates in fig. 1, the refraction law can be written in cosine-vector form:

$$n\frac{(x,y)^\top \nabla_\phi(x,y)}{\|(x,y)\| \cdot \|\nabla_\phi(x,y)\|} = \frac{((s,t)-(x,y))^\top \nabla_\phi(x,y)}{\|(s,t)-(x,y)\| \cdot \|\nabla_\phi(x,y)\|}\,. \tag{3}$$

where $\nabla_\phi(x,y)$ is the surface tangent vector. Following eqn. (2), we make the identifications $R(\phi) = \|(x,y)\|$, $t = p\csc\beta\sin\phi$. Then, under the assumption that the lens is small relative to the disk radius p and projection distance r, eqn. (3) can be massaged into an ODE of the form $R'(\phi)f(n,p,r,\beta,\phi) = R(\phi)g(n,p,r,\beta,\phi)$, where f and g are algebraic functions. The ODE admits a solution the form $R(\phi) = \frac{2k|\exp(p_2-p_1)|}{1+\cos(\phi)}$, where k determines the lens size and p_1, p_2 are polynomials summed over the roots of quartic equations associated with f and g. The reader may notice that without the exponential term, $R(\phi)$ would be the polar equation for an ellipsoid.

3. TAILORING FOR TRI-TONE GRAPHICS

To tailor this lens to project a graphic, we position the graphic on the projection plane and identify regions on the lens surface that direct unwanted light outside the graphic's boundaries. These regions are then sunk to redirect their flux to the boundaries of the graphic. This approach is akin to the method of piecewise supporting quadrics,[3] except that we use continuous envelopes of quartics.

The correspondence of eqn. (2) gives us the backprojection of the graphic from the projection plane to the lens surface. Let $\mathcal{G}$ be the subset of the projection plane occupied by the graphic and $\mathcal{B}$ be the corresponding set of emission angles as per eqn. (2), i.e. $(\phi,\theta) \in \mathcal{B} \iff (p\csc\beta\sin\phi,\theta) \in \mathcal{G}$. $\mathcal{G}$ has boundary $\partial\mathcal{G}$ and similarly $\mathcal{B}$ has boundary $\partial\mathcal{B}$. Using this, we construct a new sag function that alters the surface of the lens wherever it refracts light outside $\mathcal{G}$:

$$\tilde{R}(\phi,\theta) = \begin{cases} R(\phi), & (p\csc\beta\sin\phi,\theta) \in \mathcal{G} \\ \max_{g\in\partial\mathcal{G}} C_R(g,\phi,\theta) & \text{otherwise} \end{cases} \tag{4}$$

where $C_R(g,\phi,\theta)$ is the radial extent at (ϕ,θ) of a Cartesian oval that focuses light to point g on the graphic boundary and whose surface is tangent to $R(\cdot)$ at the corresponding point $b \in \partial\mathcal{B}$ on the original lens surface $R(\phi)$. Knowing $R(\phi)$ in closed form make this calculation practical. The max operator in eqn. (4) yields a swept surface where every point is contributed by an oval. This replaces any unwanted refractions on the original surface $R(\phi)$ with refractions that transport light to the boundary of the graphic. See, for example, fig. 2 where all rays that $R(\phi)$ directs to the background are redirected to a caustic at the boundary of the "G".

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