Performance Analysis of Distributed Single Carrier Systems with Distributed Cyclic Delay Diversity

Kim, K.J.; Liu, H.; Renzo, M.D.; Orlik, P.V.; Poor, H.V.
TR2017-207 August 2017

Abstract
This paper investigates a distributed cyclic delay diversity (CDD) transmission scheme for cyclic-prefixed single carrier systems in non-identically and identically distributed frequency selective fading channels. The distinguishable feature of the proposed scheme lies in providing a transmit diversity gain while reducing the burden of estimating the channel state information (CSI), which is a challenging task in distributed and cooperative systems. To effectively use the distributed CDD scheme at the transmitters, two sufficient conditions are derived to eliminate the intersymbol interference at the receiver and leveraged to convert the multi-input single-output channel into a single-input single-output channel. These conditions allow the system to achieve the maximum diversity for frequency selective fading channels at a full rate. To achieve this maximum diversity, a fixed number of CDD transmitters is selected based on the channel conditions, symbol block size, and maximum time dispersion of the channel, and a new two-stage transmission mode is proposed. Based on the distributed CDD and the proposed selection schemes, a new expression for the signal-to-noise ratio at the receiver is obtained with the aid of order statistics, and then closed-form expressions for the outage probability and average symbol error rate (ASER) are derived. As far as the identically-distributed frequency selective fading channel model is concerned, the achievable maximum diversity gain is proved, with the aid of asymptotic analysis, to be equal to the product of the total number of transmitters in the system and the number of multipath components. Link-level simulations are also conducted to validate the mathematical expressions of outage probability, ASER, and maximum achievable diversity gain.

IEEE Transactions on Communications
Performance Analysis of Distributed Single Carrier Systems with Distributed Cyclic Delay Diversity

Kyeong Jin Kim, Senior Member, IEEE, Marco Di Renzo, Senior Member, IEEE, Hongwu Liu, Member, IEEE, Philip V. Orlik Senior Member, IEEE, and H. Vincent Poor Fellow, IEEE

Abstract—This paper investigates a distributed cyclic delay diversity (CDD) transmission scheme for cyclic-prefixed single carrier systems in non-identically and identically distributed frequency selective fading channels. The distinguishable feature of the proposed scheme lies in providing a transmit diversity gain while reducing the burden of estimating the channel state information (CSI), which is a challenging task in distributed and cooperative systems. To effectively use the distributed CDD scheme at the transmitters, two sufficient conditions are derived to eliminate the intersymbol interference at the receiver and leveraged to convert the multi-input single-output channel into a single-input single-output channel. These conditions allow the system to achieve the maximum diversity for frequency selective fading channels at a full rate. To achieve this maximum diversity, a fixed number of CDD transmitters is selected based on the channel conditions, symbol block size, and maximum time dispersion of the channel, and a new two-stage transmission mode is proposed. Based on the distributed CDD and the proposed selection schemes, a new expression for the signal-to-noise ratio at the receiver is obtained with the aid of order statistics, and then closed-form expressions for the outage probability and average symbol error rate (ASER) are derived. As far as the identically-distributed frequency selective fading channel model is concerned, the achievable maximum diversity gain is proved, with the aid of asymptotic analysis, to be equal to the product of the total number of transmitters in the system and the number of multipath components. Link-level simulations are also conducted to validate the mathematical expressions of outage probability, ASER, and maximum achievable diversity gain.

Index Terms—Distributed single carrier system, cyclic delay diversity, diversity order, transmitter selection, frequency selective fading.

I. INTRODUCTION

UNDER the assumption that exact channel state information (CSI) is available at the transmitter, the maximum ratio transmission (MRT) scheme [1], [2] has been proposed for exploiting the availability of multiple transmit antennas at each transmitter. In particular, by applying a transmit weight vector that maximizes the signal-to-noise ratio (SNR) [1], [2], better receiver performance can be achieved by virtue of a diversity gain proportional to the number of transmit antennas. Under the same conditions, MRT has been applied among distributed transmitters as well [3], in order to achieve a diversity order proportional to the number of cooperating transmitters. A distributed space-time-coded (STC) cooperative diversity scheme has been proposed in [4] and [5]. However, full rate orthogonal space-time block codes (STBCs) do not exist for a general number of distributed transmitters.

Since acquiring CSI is a challenging task in distributed cooperative systems, we consider, in the present paper, a more practical transmit diversity scheme, which is referred to as distributed cyclic delay diversity (CDD) [6]–[10]. Owing to its compatibility with the Orthogonal Frequency Division Multiplexing (OFDM) transmission scheme and thanks to its reduced hardware complexity [8], CDD has been adopted in several wireless communication systems that are based on the 802.11ac [11], 802.11n [12], and Long-Term Evolution (LTE) protocols [13]. As for OFDM transmission, it is usually required to use forward error correction (FEC) codes in order to convert spatial diversity into frequency diversity.

Cyclic prefixed single-carrier (CP-SC) transmission [14] has been proposed as a good candidate scheme for several wireless systems [15]–[20], including cooperative relaying [15]–[18], spectrum sharing systems [19] and physical layer security [20]. In contrast to OFDM transmission, CP-SC transmission exhibits a reduced sensitivity to frequency offset errors, a lower peak-to-average power ratio (PAPR), and a reduced power-backing off. In addition, it alleviates the dynamic range requirements of the linear amplifiers [6], [14], [15].

Recently, several works [6], [7], [9], [10] have attempted to exploit CDD transmission for application to CP-SC systems. Notably, the block iterative generalized decision feedback equalizer (BI-GDFE) was proposed as an effective means for cancelling the interference [6]. Its maximum achievable diversity order, however, was not studied. In [7], on the other hand, the authors proved that the BI-GDFE system is

1Since we apply CDD between cooperating transmitters, we call the proposed CDD as the distributed CDD.
capable of achieving the maximum diversity gain only if the transmission rate is no greater than a given threshold. Other works have, however, proved that CDD-based CP-SC systems can achieve the maximum diversity gain at full rate [9], [10]. In [9], in particular, the authors proposed a method forming an equivalent channel matrix for CDD with a proper choice of the delay. In [10], in addition, the CDD scheme was combined with relay selection for attaining the maximum diversity gain in Rayleigh fading channels.

Without the need of channel equalization [6], [21], research works in [15] and [22] have shown that the maximum diversity of cooperative CP-SC systems over frequency selective fading channels is jointly determined by multiuser diversity and multipath diversity. For two-hop cooperative relaying systems, best relaying selection and best terminal selection are respectively proposed in [15] and [22] for the independent and identically distributed (i.i.d.) fading channel. To achieve the maximum diversity, they both assume perfectly known CSI in the system. However, since either a single relay or terminal is selected for cooperation, its achievable coding gain is limited. Based on the above state of the art of research on CDD-based CP-SC systems, it can be concluded that the existing studies are applicable to the analysis of non-cooperative transmitters equipped with multiple transmit antennas. In the present paper, on the other hand, we focus our attention on systems with cooperative (or distributed) single-antenna, and hence low-complexity, transmitters. A major objective of our research work, more specifically, is to propose a CDD-based CP-SC system that is capable of achieving the maximum diversity without necessitating CSI either at the control unit (CU) or at the transmitters. In our system model, the CU employs the distributed CDD scheme among the transmitters. In light of this, the channels among the transmitters and the receiver of interest can be assumed to be independent but non-identically distributed (i.n.i.d.). In the present paper, as a consequence, an i.n.i.d. frequency selective fading channel model is assumed, which makes the performance evaluation of CDD-based CP-SC systems a challenging mathematical problem. To the best of the authors’ knowledge, the mathematical analysis of this system model is not available in the open technical literature. More specifically, the novel contributions of the present paper can be summarized as follows:

1) We propose a new cooperative CP-SC system that employs the distributed CDD scheme with a systematic delay assignment. In particular, we assume a general system model where only a subset of the available transmitters cooperatively apply the distributed CDD scheme. The selected transmitters are identified based on the maximum time dispersion of the channel and size of the block symbol for CP-SC transmission. A two-stage selection process is proposed in order to select the collaborative transmitters.

2) Inspired by the work in [9] and [10], we derive two sufficient conditions for achieving the maximum diversity order at full rate that is offered by CP-SC transmission.

3) In the general i.n.i.d. frequency selective fading channel, we derive closed-form expressions of the outage probability and average symbol error rate (ASER) when either one or two CDD transmitters are available. As far as the analysis of system models with a larger number of CDD transmitters is concerned, we provide closed-form expressions of the same performance metrics in the i.i.d. frequency selective fading channel. This is due to the mathematical intractability of i.n.i.d. frequency selective fading channels if more than two CDD transmitters are considered. Based on the proposed mathematical frameworks, we prove that the maximum achievable diversity order is equal to the product of the number of available transmitters and multipath components.

The rest of the present paper is organized as follows. In Section II, the system and channel models are summarized. The distributed CDD-based CP-SC system model is introduced as well. In Section III and Section IV, the outage probability and ASER are computed in i.n.i.d. and i.i.d. frequency selective fading channels, respectively. Simulation results are presented in Section V and conclusions are drawn in Section VI.

Notation: The superscript $(\cdot)^H$ denotes complex conjugate transposition; $<\cdot>$ denotes the modulo operation with base $Q$; $I_N$ denotes an $N \times N$ identity matrix; $0$ denotes an all-zero matrix of appropriate dimensions; $CN(\mu, \sigma^2)$ denotes the complex Gaussian distribution with mean $\mu$ and variance $\sigma^2$; $C^{m \times n}$ denotes the vector space of all $m \times n$ complex matrices; $F_{\varphi}(\cdot)$ denotes the cumulative distribution function (CDF) of the random variable (RV) $\varphi$, whose probability density function (PDF) is denoted by $f_{\varphi}(\cdot)$; $\binom{n}{k}$ denotes the binomial coefficient; $a(l)$ denotes the $l$th element of vector $a$ and $A(k, l)$ denotes the $(k, l)$ element of matrix $A$.

II. SYSTEM AND CHANNEL MODEL

A block diagram of the considered cooperative system is provided in Fig. 1. The CU provides perfect backhaul connections $\{b_m\}_{m=1}^M$ to $M$ single-antenna transmitters $\{TX\}_{m=1}^M$. This assumption originates from the fact that remote radio head (RRH) types of transmitters are assumed. Likewise, the receiver, $R$, is equipped with a single receive antenna. We assume two types of channel models: (1) i.n.i.d. frequency selective fading channels, which, in general, are comprised of a different number of multipath components. Since the single antenna equipped transmitters can be distributed at random in the region of interest, different path losses and different fading severity are assumed. A distance-dependent path loss component is also used to model large scale fading; (2) i.i.d. frequency selective fading channels, which are made of the same number of multipath components. In this case, the transmitters are located at the same distance from the receiver. This channel model, despite being simplified, is often considered for getting some insight for system design and optimization and it is widely used in the literature.

Since there are $M \geq K$ transmitters in the system, the CU needs to select those that will take part to the CDD processing. To this end, we propose the transmitter selection process discussed in the next sections.

As for the use of a baseband unit (BBU) instead of the CU, perfect fronthaul links are assumed from the BBU to RRHs.
Fig. 1. Block diagram of the proposed distributed CDD-based cooperative CP-SC system. All the single antenna equipped transmitters are connected to the CU via perfect backhaul links \((b_m)_{m=1}^M\) and communicate with the receiver \(R\) through independent frequency selective fading channels \((h_m)_{m=1}^M\). Out of \(M \geq K\) transmitters, only \(K\) transmitters take part in the data transmission with the aid of CDD-aided processing. So, \(M-K\) non-CDD transmitters do not participate to data transmission.

A. Pilot Transmission for Initialization

Due to the presence of a larger number of distributed transmitters compared with the number of transmitters that employ CDD processing, two questions need to be answered:

\(Q_1\): How to choose only \(K\) CDD transmitters out of \(M\) \((M \geq K)\) available transmitters?

\(Q_2\): How to assign a CDD delay \(\Delta_k\) to the CDD transmitter TX\(_k\)?

To answer these questions, we assume that pilot symbols can be used at the transmitter and that they are known at the receiver. The signal received at the receiver and transmitted from the \(k\)th transmitter can be written as follows:

\[
p_k = \sqrt{P_T} \alpha_k h_k p + z_R
\]

(1)

where \(P_T\) is the transmission power of each transmitter, \(\alpha_k\) is the path loss component of the independent channel \(h_k\), \(H_k \in \mathbb{C}^{Q \times Q}\) is a right circulant matrix whose \((j, l)\)th element is \(H_k(j,l) = h_k(j-l > q)\), and \(z_R\) is the receiver noise \(z_R \sim \mathcal{CN}(0, \sigma_z^2 I_Q)\). A common pilot symbol block is denoted by \(p \in \mathbb{C}^{Q \times 1}\) with \(E[p] = 0\), \(E[pp^H] = I_Q\).

The block size of \(p\) is denoted by \(Q\). Since known pilot symbols are used, no detection is necessary at the receiver. In addition, by employing appropriate channel sounding schemes, the receiver is assumed to have exact knowledge of the number of multipath components of each channel \(h_k\).

From (1), the SNR at the receiver is as follows [15]:

\[
\gamma_k = \frac{\Delta h_k}{\sigma_z^2} = \frac{\alpha_k^2 ||h_k||^2}{\sigma_z^2}
\]

(2)

where \(\alpha_k = \frac{P_T \alpha_k}{\sigma_z^2}\).

The \(M\) available SNRs are arranged in ascending order of magnitude as follows [23], [24]:

\[
0 \leq \gamma_1 \leq \gamma_2 \leq \cdots \leq \gamma_M
\]

(3)

and their corresponding indices are denoted by \(X_i \equiv \{1, 2, \ldots, M\}\). To reduce the feedback overhead from the receiver to the CU, the receiver feeds back \(X_I\) and the maximum number of multipath components estimated from channel sounding, namely, \(N_h = \max(N_1, \ldots, N_K)\), to the CU.

The CU is assumed to be aware of \(N_h\) and the CP length, \(N_p\). Thus, the CDD delay length, \(\Delta_i\), can be determined from the following two conditions:

\[
C_1 : N_p = N_h, \quad C_2 : \Delta_i = (i-1)N_p
\]

(4)

(5)

where \(C_1\) is needed to remove the intersymbol interference (ISI) caused by the CP-SC transmission [15], and \(C_2\) is required to form a non-overlapping equivalent channel vector that allows us to convert the multi-input single-output (MISO) channel into a single-input single-output (SISO) channel [9]. More precisely, the ISI can be removed if \(N_p \geq N_h\). Since it is preferable to keep the CP length as small as possible compared to the symbol block size \(Q\), we consider \(N_p = N_h\).

Based on \(C_1\) and \(C_2\), we propose to determine the number of CDD transmitters, \(K\), as a function of the symbol block size, \(Q\), and the maximum number of multipath components, as follows:

\[
K = 1 + \left\lfloor \frac{Q}{N_p} \right\rfloor
\]

(6)

where \(\lfloor \cdot \rfloor\) denotes the floor function.

Since we assume \(M \geq K\), the CU needs to select the \(K\) CDD transmitters that are specified by the last \(K\) elements of \(X_I\) and form a table of CDD delays, \(X_\Delta \equiv \{\Delta_1, \ldots, \Delta_{K-1}, \Delta_K\}\), which is used for assigning the CDD delays to the CDD transmitters. The main objective is, in fact, uniquely assigning one out of the \(K\) delays in \(X_\Delta\) to a given CDD transmitter. To this end, consider the \(K\) chosen CDD transmitters. Assume that \(Q\) transmission symbols, \(\{s_1, \ldots, s_Q\}\), are transmitted sequentially from the CU or BBU to all the transmitters. Each CDD transmitter collects them to form a transmission symbol block \(s = [s_1, \ldots, s_Q]^T \in \mathbb{C}^{Q \times 1}\), where we assume that \(E[s] = 0\) and \(E[ss^H] = I_Q\). Let \(\Delta_k\) be the unique CDD delay assigned to the \(k\)th CDD transmitter. The exact value of \(\Delta_k\) is discussed in Corollary 1 below. The \(k\)th CDD transmitter applies circular shifting operations by using its assigned CDD delay \(\Delta_k\), which can be expressed by applying the permutation shifting matrix \(P_{Q=4}^{\Delta_k}\). In particular, the matrix \(P_{Q=4}^{\Delta_k}\) is obtained by circularly shifting down the identity matrix \(I_Q\) by \(\Delta_k\). For instance, \(P_{Q=4}^{\Delta_k=1}\) is given by

\[
P_{Q=4}^{\Delta_k=1} = \begin{bmatrix}
0 & 1 & 0 & 0 \\
0 & 0 & 1 & 0 \\
0 & 0 & 0 & 1 \\
1 & 0 & 0 & 0
\end{bmatrix}.
\]

(7)
Let us apply the QR decomposition (QRD) to the right
circulant matrices $H_{c,i}$ and $H_{c,i}^{\Delta_k} = H_{c,i} P_{Q}^{\Delta_k}$. We obtain

\[ H_{c,i}^{\Delta_k} = Q^{\Delta_k} R^{\Delta_k}, \quad \text{where} \]

\[ Q^{\Delta_k} = P_{Q}^{\Delta_k} Q, \quad \text{and} \quad R^{\Delta_k} = R \]

(8)

which shows that the upper triangular matrix, $R^{\Delta_k}$, obtained
from the QRD of the column permuted circulant matrix is
independent of the column permutation, whereas the unitary
matrix, $Q^{\Delta_k}$, is obtained by pre-multiplying the permutation
matrix by $Q$3. With these prerequisites, the following corollary holds.

**Corollary 1**: Let the delays of the $K$ CDD transmitters
satisfy the conditions $C_1$ and $C_2$. Then, provided that each
transmitter is assigned a different delay, different assignments of
the cyclic delays to the CDD transmitters result in the same
performance if a maximum likelihood detector (MLD) [15] is
used at the receiver.

**Proof**: See Appendix A.

Corollary 1 implies that the system performance, which
depends on trace $\left((H_{c,i}^{\Delta_k})^H H_{c,i}^{\Delta_k}\right)$, is independent of the selection
priority of the delays. For example, the CU has the freedom of assigning the delay $\Delta_k$ to the CDD transmitter $TX_k$
without any performance loss. In the sequel, this assumption
is retained for simplicity but without loss of generality. Based on
Corollary 1, as a result, the CU needs only $N_k$ and $\mathcal{X}_I$
for applying the proposed CDD-based CP-SC transmission
scheme.

**B. Information Data Transmission via Distributed CDD**

Let us apply the permutation shifting matrix to the $k$th CDD
transmitter. The corresponding symbol $\tilde{s}_k$ can be formulated as: $
\tilde{s}_k = P_{Q}^{\Delta_k} s$, where $s \in \mathbb{C}^{K \times 1}$. Before transmission, a CP that
contains the last $N_p$ symbols of $\tilde{s}_k$, is added to the front of $\tilde{s}_k$.
The obtained symbol, $s_k$, is sent through a frequency-selective fading
channel that is denoted by $h_k$ and is assumed to have
$N_k$ multipath components.

At the receiver, after removing the CP, the signal can be formulated as

\[ r = \sum_{k=1}^{K} \sqrt{P_T \alpha_k} H_k P_{Q}^{\Delta_k} s + z_R \]  

(9)

where the additive noise is $z_R \sim \mathcal{CN}(0, \sigma^2 I_Q)$. Since the product of two right circulant matrices, $H_k$ and $P_{Q}^{\Delta_k}$, is another right circulant matrix, with the aid of (5), (9) can be expressed as follows:

\[ r = H^{\text{CDD}} s + z_R \]

(10)

where $H^{\text{CDD}}$ is an equivalent channel matrix comprising the frequency fading channels from the $K$ CDD transmitters to the receiver. Its first column vector is as follows:

\[ h^{\text{CDD}} \Delta [\sqrt{P_T \alpha_1 (h_1)^T}, 0_{1 \times (N_p-N_1)}, ..., \sqrt{P_T \alpha_K (h_K)^T}, 0_{1 \times (N_p-N_K)}]^T \in \mathbb{C}^{K \times 1}. \]  

(11)  

3If the diagonal components of the matrix $R^{\Delta_k}$ are all positive, these two prerequisites are true.

Since right circulant matrices are determined by their first
column vector, then $h^{\text{CDD}}$ completely specifies the channel matrix $H^{\text{CDD}}$.

From the equivalent expression of the received signal $r$, we can observe the following facts:

1) The received signal does not include interference from
other CDD transmitters. This is obtained by virtue of the properly designed CDD delays $\Delta_k$. As a result, the MISO channel is converted into a SISO channel for distributed CP-SC transmission. Since each channel vector comprises $N_p$ elements, additional zeros are required in forming $h^{\text{CDD}}$.

2) Maximum transmit diversity can be achieved by employing the proposed distributed CDD scheme which specifies the CDD delay according to two sufficient conditions specified by Eqs. (4) and (5). This is proved mathematically in the following sections.

**III. PERFORMANCE ANALYSIS IN I.N.I.D. FREQUENCY
SELECTIVE FADING CHANNELS**

To investigate the performance of the proposed distributed
CDD-based CP-SC transmission scheme, the distribution of the SNR at the receiver needs to be computed.

**A. SNR at the Receiver**

From (9), the SNR [15] over the channel from the $k$th CDD transmitter to the receiver can be formulated as follows:

\[ \gamma_k = \frac{P_T \alpha_k \| h_k \|^2}{\sigma^2} = \alpha_k \| h_k \|^2 \]

(12)

which coincides with (2). The CDF and PDF of $\gamma_k$ are, respectively, given by

\[ F_k(x) = 1 - e^{-\frac{\pi_k}{\alpha_k} \sum_{l=0}^{N_k-1} \frac{1}{l! \left( \alpha_k \right)^l}} \]

and

\[ f_k(x) = \frac{x^{N_k-1}}{\Gamma(N_k) \left( \alpha_k \right)^{N_k}} e^{-\frac{\pi_k}{\alpha_k}} \]

(13)

where $\Gamma(\cdot)$ denotes the gamma function. Based on (9), the aggregated SNR from the $K$ CDD transmitters is given by

\[ S^K = \sum_{k=1}^{K} \tilde{\alpha}_k \sum_{l=1}^{N(M-K+k)} \| h_{(M-K+k)}(l) \|^2 \]

\[ = \sum_{k=1}^{K} \gamma(M-K+k). \]

(14)

It is important to mention that the selected $K$ CDD transmitters
provide the largest $K$ SNRs to the receiver. This implies that the analysis of (14) requires the mathematical tool of order statistics. In other words, $\gamma(M)$ is the largest SNR, $\gamma(M-1)$ is the second largest SNR, etc. Thus, $\sum_{k=1}^{K} \gamma(M-K+k)$ is the sum of the $K$ largest SNRs. This implies that the SNRs in (14) are correlated and, thus, the mathematical analysis of (14) is a non-trivial problem.

Let us arrange the SNRs in increasing order of magnitude,
i.e., $\gamma(M+1) < \gamma(M+2) < \ldots < \gamma(M)$. The joint
PDF of $\gamma_1 \triangleq \gamma(M-K+1)$, $\gamma_2 \triangleq \gamma(M-K+2)$, \ldots, $\gamma_K \triangleq \gamma(M)$ can be written as [24]:

$$f_{r_1, r_2, \ldots, r_K}(x_1, x_2, \ldots, x_K) = \frac{1}{(M-K)!} \text{Per} \mathbf{A}_K \quad (15)$$

where

$$\mathbf{A}_K \triangleq \begin{bmatrix}
F_1(x_1) & f_1(x_1) & \ldots & f_1(x_K) \\
F_2(x_1) & f_2(x_1) & \ldots & f_2(x_K) \\
\vdots & \vdots & \ddots & \vdots \\
F_M(x_1) & f_M(x_1) & \ldots & f_M(x_K) \\
M-K & \mathbf{1} & \mathbf{1} & \mathbf{1}
\end{bmatrix} \quad (16)$$

and $F_k(\cdot)$ and $f_k(\cdot)$ are the CDF and PDF of $\gamma_k$, i.e., the $k$th SNR without CDD operation. Their expressions are provided in (13). Also, let us define the matrix

$$\mathbf{a} = \begin{bmatrix}
a_1 & a_2 \\
\vdots & \vdots \\
a_{M1} & a_{M2}
\end{bmatrix}$$

containing $i$ copies of the first column vector and $j$ copies of the second column vector. The permanent of a square matrix $\mathbf{A}$, denoted by $\text{Per} \mathbf{A}$, is defined similar to the matrix determinant except for the fact that all signs are positive [23], [24]. If a square matrix $\mathbf{A}$ is considered, for example, $\mathbf{A} = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$, we have $\text{Per} \mathbf{A} = ad + bc$.

With the aid of some algebraic manipulations, a desired compact expression for $\text{Per} \mathbf{A}_K \triangleq \text{Per} \mathbf{A}_K^{(M-K)}$ can be shown to be (17) at the next page. For ease of analysis, we introduce the notation $\mathbf{X}_M \triangleq \{1, \ldots, M\}$ and $\mathbf{X}_p \triangleq \mathbf{X}_M - \{i_1, \ldots, i_{M-K}\}$. Also, the list of all possible permutations of the elements of $\mathbf{X}_p$ is denoted by $\mathbb{P}_p \triangleq \text{Perms}(\mathbf{X}_p)$, where $q$ denotes the $q$th permutation of $\mathbb{P}_p$. In addition, $k_{l,q}$ denotes the $l$th element of the CDD-based CP-SC transmission scheme.

**Theorem 1**: The CDF of the aggregated received SNR from two CDD transmitters in i.i.d. frequency selective fading channels with $N_h = N_k, \forall k$ is given by (20) at the next page. In (20), we have defined $D_2 \mathcal{M}_{\gamma, \alpha_{K+2}}$ and $\gamma(\cdot, \cdot)$ denotes the lower-incomplete gamma function.

**Proof**: See Appendix B.

If $K = 1$, i.e., a single CDD transmitter is considered, $\text{Per} \mathbf{A}_K$ is given by (21) at the next page. Note that (21) is the PDF of $\gamma(M)$ and $S^{K=1}$. Different but equivalent expressions for $\gamma(M)$ are derived in [26]. From (21), the CDF of $S^{K=1}$ can be formulated as the expression in (22) provided at the next two pages.

**B. Outage Probability**

From the CDF, the outage probability can be readily formulated in closed-form. For a given outage threshold, $\gamma_{th}$, the outage probability is as follows:

$$O_{\text{out}}(\gamma_{th}) = \begin{cases}
F_{S^{K=1}}(\gamma_{th}), & \text{for } K = 1, \\
F_{S^{K=2}}(\gamma_{th}), & \text{for } K = 2.
\end{cases} \quad (23)$$

It is worth noting that $F_{S^{K=1}}(\gamma_{th})$ is the outage probability corresponding to the worst-case scenario for the proposed CDD-based CP-SC transmission scheme.

**C. Average Bit Error Rate**

According to [27], the ASER can be expressed, as a function of the CDF of the received SNR, as follows:

$$P_e = \frac{m_a \sqrt{m_b}}{2 \sqrt{\pi}} \int_0^\infty x^{-1/2} F_{S^{K}}(x) e^{-x m_b} dx \quad (24)$$

where $m_a$ and $m_b$ are specified by the modulation scheme being used.

With the aid of the closed-form expressions of $F_{S^{K=1}}(x)$ and $F_{S^{K=2}}(x)$, an explicit expression of the ASER is provided in the following theorem.

**Theorem 2**: The closed-form expression of the worst ASER of the CDD-based CP-SC transmission scheme is given by (25) at the next two pages.

**Proof**: The computation of $P_e^{K=1}$ follows from the following notable integral:

$$P_e^{K=1} = \frac{m_a \sqrt{m_b}}{2 \sqrt{\pi}} \int_0^\infty x^{-1/2} \gamma(\tilde{m}_1, D_1 x) e^{-x m_b} dx \quad (26)$$
(17) \[
\text{Per} \tilde{A}_K = \sum \left( q_{1,j} \prod_{j=1}^{M-K} (1 - e^{-x_{1,j}}) \sum \left( x_1 \right)^{N_{k_1,q} - 1} \frac{1}{\Gamma(\tilde{\alpha}_{k_1,q})} \prod_{l=2}^{K} \left( x_1 \right)^{N_{k_l,q} - 1} e^{-x_{1,l}} \frac{1}{\Gamma(\tilde{\alpha}_{k_l,q})} \right).
\]

(18) \[
\text{Per} \tilde{A}_K = \sum \left( q_{1,j} \prod_{j=1}^{M-K} \left( \frac{q_j}{q_{j,1}! \cdots q_{j,N_h}!} \right) \prod_{j=1}^{N_{k_1,q}} \prod_{j=1}^{N_{k_{M,q}}} \left( q_{j,1} \right)^{N_{k_1,q}} e^{-x_1 D_1 x_1 \tilde{m}_1 - 1} \frac{1}{\Gamma(\tilde{\alpha}_{k_1,q})} \prod_{l=2}^{K} \left( x_1 \right)^{N_{k_l,q} - 1} e^{-x_{1,l}} \frac{1}{\Gamma(\tilde{\alpha}_{k_l,q})} \right).
\]

F_{SK2}(x) = \sum \left( q_{1,j} \prod_{j=1}^{M-K} \left( \frac{q_j}{q_{j,1}! \cdots q_{j,N_h}!} \right) \prod_{j=1}^{N_{k_1,q}} \prod_{j=1}^{N_{k_{M,q}}} \left( q_{j,1} \right)^{N_{k_1,q}} e^{-x_1 D_1 x_1 \tilde{m}_1 - 1} \frac{1}{\Gamma(\tilde{\alpha}_{k_1,q})} \prod_{l=2}^{K} \left( x_1 \right)^{N_{k_l,q} - 1} e^{-x_{1,l}} \frac{1}{\Gamma(\tilde{\alpha}_{k_l,q})} \right).

\]

(20) \[
\sum \left( q_{1,j} \prod_{j=1}^{M-K} \left( \frac{q_j}{q_{j,1}! \cdots q_{j,N_h}!} \right) \prod_{j=1}^{N_{k_1,q}} \prod_{j=1}^{N_{k_{M,q}}} \left( q_{j,1} \right)^{N_{k_1,q}} e^{-x_1 D_1 x_1 \tilde{m}_1 - 1} \frac{1}{\Gamma(\tilde{\alpha}_{k_1,q})} \prod_{l=2}^{K} \left( x_1 \right)^{N_{k_l,q} - 1} e^{-x_{1,l}} \frac{1}{\Gamma(\tilde{\alpha}_{k_l,q})} \right).
\]

(21) \[
\sum \left( q_{1,j} \prod_{j=1}^{M-K} \left( \frac{q_j}{q_{j,1}! \cdots q_{j,N_h}!} \right) \prod_{j=1}^{N_{k_1,q}} \prod_{j=1}^{N_{k_{M,q}}} \left( q_{j,1} \right)^{N_{k_1,q}} e^{-x_1 D_1 x_1 \tilde{m}_1 - 1} \frac{1}{\Gamma(\tilde{\alpha}_{k_1,q})} \prod_{l=2}^{K} \left( x_1 \right)^{N_{k_l,q} - 1} e^{-x_{1,l}} \frac{1}{\Gamma(\tilde{\alpha}_{k_l,q})} \right).
\]
\[
F_{S^K_1}(x) = \sum_{1 \leq n_1 < n_2 < \ldots < n_1 + q_{M-1} - 1 \leq M} \sum_{q \in \mathbb{F}_p} \sum_{q_1=0}^{1} \ldots \sum_{q_{M-1}=0}^{1} \frac{1}{q_1} \ldots \frac{1}{q_{M-1}} \left( -1 \right)^{q_1 + \ldots + q_{M-1}}
\]
\[
\sum_{q_1, \ldots, q_{M-1}, N_h} \sum_{q_{M-1}, \ldots, q_1, N_h} \cdots \sum_{q_1, \ldots, q_{M-1}, N_h} \prod_{j=1}^{M-1} \left( \frac{q_j}{q_jj_1 \ldots q_jj_N_h} \right)
\]
\[
= \frac{1}{\Gamma(N_h)(D_1)^{m_h}} \gamma(m_1, D_1 x) \ .
\]

where \(G_{p,q}^{m,n} (t \mid a_1, \ldots, a_m, a_{m+1}, \ldots, a_p, b_1, \ldots, b_{m-1}, \ldots, b_q)\) denotes the Meijer G-function [25, eq. (9.301)]. In the derivation of (26), we use [28, eq.(06.06.26.0004.01)] in (a) and [29, eq. (2.24.3.1)] in (b). Replacing \(K_1\) in (22) with (26), the final result in (25) follows.

Finally, we note that \(F_{S^K_2}(x)\) can be expressed in terms of the summation of a finite number of lower-complete gamma functions. This implies that the same approach as for the computation of \(F_{S^K_1}(x)\) can be used. The resulting closed-form expression is not provided due to space limitations.

In the next section, simplified expressions of outage probability and ASER in i.i.d. frequency selective fading channels are provided.

IV. PERFORMANCE ANALYSIS IN I.I.D. FREQUENCY SELECTIVE FADING CHANNELS

Let us assume a frequency selective fading channel, where each channel has the same number of multipath components. A closed-form expression of the CDF of \(S^K\) is provided in the following theorem.

**Theorem 3:** In i.i.d. frequency selective fading channels, the CDF of the aggregate received SNR from \(K\) CDD transmitters, is, for \(K < M\) and \(K = M\), respectively, given by (27) at the next page. In (27), we have defined \(\beta = \frac{K}{1 + \frac{p}{r}}\), \(m_1 = N_h K - \hat{i}\), \(m_2 = \hat{i} + \hat{q} + N_h\), \(\hat{q} = \sum_{t=0}^{N_h-1} t q_{t+1}\) for a non-negative integer set \(\{q_1, q_2, \ldots, q_{N_h}\}\) satisfying the condition \(\sum_{k=1}^{N_h} q_k = p\) and \(\hat{i} = \sum_{t=0}^{N_h-1} t h_{t+1}\) for another non-negative integer set \(\{l_1, l_2, \ldots, l_{N_h}\}\) satisfying the condition \(\sum_{k=1}^{N_h} l_k = K\).

**Proof:** See Appendix C.

A. Outage Probability and Average Symbol Error Rate

With the aid of the CDF of \(S^K\), the outage probability of the CDD-based CP-SC system can be formulated as follows:

\[
\hat{O}_{\text{out}}(\gamma_{\text{th}}) = \hat{F}_{S^K}(\gamma_{\text{th}}).
\]

Similar to the derivation of the ASER in i.i.d. frequency selective fading channels, the ASER in i.i.d. frequency selective fading channels is provided in the following theorem.

**Theorem 4:** In i.i.d. frequency selective fading channels, the ASER of the proposed CDD-based CP-SC system is given by (29) at the next page.

The details of the proof are omitted because it directly follows by applying the notable integral in (26).

B. Asymptotic Analysis of Outage Probability and Average Symbol Error Rate

To better understand the performance of the proposed scheme, we analyze the behavior of the CDF of \(S^K\) in the high-SNR regime. This is useful for identifying the diversity order of the system.

**Proposition 1:** In the high-SNR regime, the CDF of \(S^K\) can be simplified as follows:

\[
\hat{F}_{S^K}(x) = K \left( \frac{M}{K} \right) \frac{\Gamma(M N_h - K N_h + N_h)}{\Gamma(N_h + 1)^{M - K}} \frac{\Gamma(N_h)}{\Gamma(M N_h)}.
\]

**Proof:** See Appendix D.

From Proposition 1, high-SNR expressions of outage probability and ASER can be obtained as follows:

\[
\hat{O}_{\text{out}}(\gamma_{\text{th}}) = \hat{F}_{S^K}(\gamma_{\text{th}}),
\]
where \( C \) is the total number of transmitters available in the system and \( N \) achieve a diversity order equal to 

\[
\text{transmission schemes which specify the CDD delay according}
\]

**Proof:** 

\[
\begin{align*}
\hat{F}_{S_G < M}(x) &= \frac{M}{\Gamma(N_h)} \left( \frac{M-1}{K} \right) \sum_{p=0}^{M-K-1} \left( \frac{M-K-1}{p} \right) (-1)^p \sum_{q_1, \ldots, q_N_h = p}^{p!} q_1! q_2! \ldots q_N_h ! \\
& \sum_{i_1, \ldots, i_{N_h}} \sum_{l_1, l_2 = 0}^{K!} \frac{N_h-1}{l_1 ! l_2 !} l_1 + \sum_{i_1, l_2} l_2 + 1 \Gamma(\hat{\omega} + \hat{\eta} + N_h)(1 + p + K)^{-l - q - N_h} \\
& \left[ \sum_{j=1}^{m_1} (-1)^{m_1-f} \beta^{m_1-f} (1-\beta)^{-1} - \sum_{j=1}^{m_2} (-1)^{m_2-f} \beta^{m_2-f} (1-\beta)^{-1} - \sum_{j=1}^{m_2} \right] \\
& \hat{F}_{S_G = M}(x) = \frac{\gamma(M N_h, x/\hat{d})}{\Gamma(M N_h)}. 
\end{align*}
\]

(27)

\[
\begin{align*}
\hat{P}_{e < M} &= \frac{M}{\Gamma(N_h)} \left( \frac{M-1}{K} \right) \sum_{p=0}^{M-K-1} \left( \frac{M-K-1}{p} \right) (-1)^p \sum_{q_1, \ldots, q_N_h = p}^{p!} q_1! q_2! \ldots q_N_h ! \\
& \sum_{i_1, \ldots, i_{N_h}} \sum_{l_1, l_2 = 0}^{K!} \frac{N_h-1}{l_1 ! l_2 !} l_1 + \sum_{i_1, l_2} l_2 + 1 \Gamma(\hat{\omega} + \hat{\eta} + N_h)(1 + p + K)^{-l - q - N_h} \\
& \left[ \sum_{j=1}^{m_1} (-1)^{m_1-f} \beta^{m_1-f} (1-\beta)^{-1} - \sum_{j=1}^{m_2} (-1)^{m_2-f} \beta^{m_2-f} (1-\beta)^{-1} - \sum_{j=1}^{m_2} \right] \\
& \hat{P}_{e = M} = \frac{m_a}{2 \sqrt{\pi} \Gamma(M N_h)} G^{1,2}_{2,2} \left( \frac{1}{m_a \hat{d}} \Gamma(M N_h, 0) \right). 
\end{align*}
\]

(29)

\[
\hat{P}_{e_{\text{as}}} = K \frac{M}{K} \frac{\Gamma(M N_h - K N_h + N_h)}{\Gamma(N_h + 1)^{M-K} \Gamma(N_h)} \frac{m_a}{2 \sqrt{\pi} \Gamma(M N_h)} G^{1,2}_{2,2} \left( \frac{1}{m_a \hat{d}} \Gamma(M N_h, 0) \right). 
\]

(32)

Finally, from the asymptotic expressions of the outage probability and ASER, the achievable diversity order of the proposed CDD-based CP-SC transmission scheme is provided in the following theorem.

**Theorem 5:** The proposed distributed CDD-based CP-SC transmission schemes which specify the CDD delay according to two sufficient conditions provided by Eqs. (4) and (5) achieve a diversity order equal to \( G_d = M N_h \), where \( M \) is the total number of transmitters available in the system and \( N_h \) is the number of multipath components of the channel.

**Proof:** We first approximate (31) as:

\[
\hat{O}_{\text{out}}(\gamma_{\text{th}}) \approx C_{\alpha} K \frac{M}{K} \frac{\Gamma(M N_h - K N_h + N_h)}{\Gamma(N_h + 1)^{M-K} \Gamma(N_h)} \left( \frac{\gamma_{\text{th}}/\hat{d}}{\Gamma(M N_h + 1)^{M-K} \Gamma(N_h)} \right) P_T \left( \frac{P_T}{\sigma^2} \right)^{-M N_h} 
\]

(33)

where \( C_{\alpha} \) is an approximation constant.

We observe that \( G_{p,q}^n \left( z \mid a_1, \ldots, a_n, a_{n+1}, \ldots, a_p \right) \propto z^\beta \) as \( z \to 0 \), where \( \beta = \min(b_1, \ldots, b_m) \) [30, Section 5.4.1]. Based on this, we can approximate (32) as follows:

\[
\hat{P}_{e_{\text{as}}} \approx C_{p,K} \frac{M}{K} \frac{\Gamma(M N_h - K N_h + N_h)}{\Gamma(N_h + 1)^{M-K} \Gamma(N_h)} \left( \frac{m_a}{2 \sqrt{\pi} \Gamma(M N_h)} \right) \frac{1}{M N_h} \left( \frac{P_T}{\sigma^2} \right)^{-M N_h} 
\]

(34)

where \( C_p \) is an approximation constant. The proof follows by direct inspection of (33) and (34).

It is worth nothing that the constants \( C_{\alpha} \) in (33) and \( C_p \) in (34) affect the accuracy of proposed asymptotic approximations, i.e., the coding gain, however they do not affect the diversity order.

Finally, we note that the number of cooperating CDD transmitters, \( K \), does not affect the diversity order of the system. This is a novel finding with respect to past research works, such as [31]–[33]. In [33], the difference between the total number of transmitters, \( M \), and the number of selected transmitters, \( K \), determines the maximum diversity order [31], [32]. Our proposed system, on the other hand, is more similar.
to cooperative relaying, where the diversity order is a function of the total number of relays [15], [34].

V. SIMULATION RESULTS

In this section, link-level simulations are conducted to validate analysis and findings. For simplicity, Binary Phase Shift Keying (BPSK) modulation is used. The curves obtained via link-level simulations are denoted by Ex. Analytical performance curves are denoted by An. High-SNR curves are denoted by As. The transmission block size for CP-SC transmission is $Q = 64$ with $N_p = 16$. The transmission power is assumed to be $P_T = 1$ for all transmitters. The SNR threshold causing an outage is $\gamma_{th} = 3$ dB. Note that we consider the i.n.i.d. frequency selective fading channel and the i.i.d. frequency selective fading channel in order. Taking into account of transmitter cooperation, we compare the performance of this work with that of selection combining which was proposed by [20] and [35]. We can see that this selection combining is a special case of the proposed CDD scheme with $K = 1$.

A. Independent but non-identically distributed (i.n.i.d.) frequency selective fading channel

We choose a particular location of the receiver and six transmitters at the most, that is, $M = 6$. The pathloss components over the channels from the transmitters to receiver are given by $\alpha = \{0.12, 0.13, 0.14, 0.15, 0.16, 0.143\}$; that is, $\alpha_1 = 0.12, \ldots, \alpha_6 = 0.143$. The same number of multipath components for each channel is assumed.

1) Outage Probability Analysis: For this particular set of pathloss components, Figs. 2 and 3 show the accuracy of the derived outage probability obtained by using (23), when compared with the exact outage probability from simulations.

In Fig. 2, we investigate the effect of the number of multipath components and the number of CDD transmitters on the outage probability. This figure shows that the derived outage probability for various scenarios is very tight to that obtained via link-level simulations. For a fixed number of four transmitters and two CDD transmitters, a different number of multipath components results in a different outage probability. As the number of multipath components increases, for instance, $N_h = 3$ vs. $N_h = 1$, a steeper slope can be observed. Thus, we can infer from this figure that the number of multipath components is one of the key factors that determine the diversity gain. For a fixed number of four transmitters and two multipath components, this figure shows that a lower outage probability is obtained if more CDD transmitters are chosen. This is due to an increased aggregated signal power at the receiver. However, we can observe that the same slope is obtained, while the curves move to a lower outage probability region. This indicates that the number of CDD transmitters, $K$, influence the coding gain rather than the diversity gain. An example is given by the curves corresponding to the setups $(K = 3, N_h = 2)$ vs. $(K = 3, N_h = 1)$. Since the distributed CDD scheme can aggregate more signal power at the receiver as the number of CDD transmitters increases, the setup with a single CDD transmitter results in the worst outage probability. Note that the system proposed by [15] and [22] is somewhat similar to the set up of a single CDD transmitter, so that the distributed CDD scheme can provide advantages on the aggregate signal power at

Fig. 2. Outage probability as a function of the number of multipath components and CDD transmitters. When $K = 1$, the outage probability corresponds to the CP-SC system with selection combining.

Fig. 3. Outage probability for several system setups.

Fig. 3, we investigate the effect of the number of transmitters on the outage probability. We assume two CDD transmitters and two multipath components. This figure shows that, as the number of transmitters increases, the distributed CDD scheme provides a smaller outage probability and a steeper curve’s slope. An example is given by the setups $M = 6$ vs. $M = 2$. As the number of transmitters increases, it is more likely to get relative large channel gains, so that the distributed CDD scheme provides advantages on the aggregate signal power at
the receiver. Thus, the number of transmitters in the system is also a key factor in determining the slope of the outage probability, which corresponds to the diversity gain.

2) Average Symbol Error Rate Analysis: To validate our mathematical derivation of the ASER, we compare the derived ASER with that obtained by the QRD-M detector\cite{15}, \cite{36}.

Fig. 4: ASER for several system setups. When $K = 1$, the ASER corresponds to the CP-SC system with selection combining.

Fig. 4 shows good agreement between the simulated ASER and the mathematical expression of the ASER for various values of $K$ and $N_h$. This figure shows that as either the number of transmitters or the number of multipath components increases, a better ASER is obtained. Since using more CDD transmitters yields a higher aggregated signal power at the receiver, a better ASER is obtained as well.

In Fig. 5, we investigate the coding gain of the system by assuming a single CDD transmitter. Under the assumption of three multipath components, we observe that the ASEP gets better as the number of transmitters increases. An example if given by the setups $(M = 5, K = 1, N_h = 3)$ vs. $(M = 3, K = 1, N_h = 3)$. The case study $(M = 3, K = 1, N_h = 1)$, among those studied, provides the worst ASER. For a given slope (diversity order), we study the individual impact of $K$ and $N_h$. From the figure, we note that the impact of multipath is more pronounced. Two setups showing these trends are $(M = 4, K = 1, N_h = 3)$ vs. $(M = 3, K = 1, N_h = 4)$, and $(M = 5, K = 1, N_h = 3)$ vs. $(M = 3, K = 1, N_h = 5)$.

B. Independent and identically distributed (i.i.d.) frequency selective fading channel

In this case, we assume $\alpha = 0.14$ for all path-losses.

1) Outage Probability Analysis: Fig. 6 compares the outage probability in (29) with simulations and show a good matching

\cite{36}.

4Interested readers can find relevant information about the QRD-M detector

between them. Given the number of CDD transmitters and the number of multipath components, we note that the slope of the curves (diversity order) does not change. In particular, two different slopes are shown in the figure: the setups $(M = 3, K = 1, N_h = 1)$, $(M = 3, K = 2, N_h = 1)$, and $(M = 3, K = 3, N_h = 1)$ have the same slope, whereas the setups $(M = 3, K = 1, N_h = 2)$, $(M = 3, K = 2, N_h = 2)$, $(M = 3, K = 3, N_h = 2)$, and $(M = 2, K = 1, N_h = 3)$ have a steeper slope than the other case studies. Once again, these numerical results confirm that the number of CDD transmitters do not affect the diversity order.

2) Average Symbol Error Rate Analysis: Similar to the i.n.i.d. frequency selective fading channel model, we compare
the ASER of the proposed scheme against that obtained by using the QRD-M detector. The results are reported in Fig. 7. For the considered case studies, e.g., $(M = 3, K = 1, N_h = 1)$, $(M = 3, K = 3, N_h = 1)$, $(M = 3, K = 3, N_h = 2)$, and $(M = 4, K = 3, N_h = 1)$, a good accuracy between modeling and simulations is obtained. In addition, this figure is obtained by using the same parameters used for the QRD-M demodulator in the i.n.i.d. frequency selective fading channel. This shows that the same diversity order is obtained.

C. Asymptotic Performance Analysis on Outage Probability and ASER

In Figs. 8 and 9, we compare the outage probability and ASER against their high-SNR asymptotic approximations. These two figures allow us to validate Theorem 5 and then to extract the maximum achievable diversity from the outage probability and ASER. As far as the approximations are concerned, we use the following constants: $C_o = 0.8$ for $(M = 4, K = 1, N_h = 1)$, $C_o = 0.25$ for $(M = 3, K = 2, N_h = 1)$, $C_o = 0.9$ for $(M = 3, K = 1, N_h = 1)$, $C_o = 0.15$ for $(M = 3, K = 2, N_h = 2)$, and $C_o = 0.65$ for $(M = 2, K = 2, N_h = 4)$. By using these values, we obtain a tight approximation and note, as expected, that the slope of the curves does not change. By direct inspection of the curves, we note that the slope of the curves of the high-SNR asymptotic approximation of the outage probability is equal to $G_d = MN_h$. In particular, the setups $(M = 4, K = 1, N_h = 1)$, $(M = 2, K = 2, N_h = 2)$, and $(M = 2, K = 2, N_h = 4)$ have a diversity order equal to $G_d = 4$, $G_d = 3$, $G_d = 3$, $G_d = 6$, and $G_d = 8$, respectively.

To produce the curves of the ASER in the high-SNR regime, we use the following constants: $C_p = 0.3$ for $(M = 4, K = 3, N_h = 1)$, $C_p = 25$ for $(M = 3, K = 2, N_h = 1)$, and $C_p = 0.4$ for $(M = 6, K = 3, N_h = 1)$. In this case as well, a good approximation is obtained in the high-SNR regime. Similar to the outage probability, the diversity gain is $G_d = MN_h$ and, in particular, the setups $(M = 3, K = 1, N_h = 1)$, $(M = 4, K = 3, N_h = 1)$, $(M = 6, K = 3, N_h = 1)$ provide the largest diversity order.

VI. CONCLUSIONS

In this paper, we have proposed a new distributed CDD-based CP-SC transmission scheme. Two conditions have been derived to achieve the maximum diversity at full rate, which allow us to suppress the interference caused by allowing
multiple transmitters to be active and by the time dispersion introduced by the channel. The outage probability and the ASER of the proposed scheme have been analyzed in i.i.d and i.i.d. frequency selective fading channels. It has been proved that the maximum diversity order of the system is equal to the product of the number of available transmitters and of the number of multipath components. With the aid of simulations, it has been shown that the number of CDD transmitters, on the other hand, affects the coding gain but it does not affect the diversity order.

**APPENDIX A: DERIVATION OF PROPOSITION 1**

It is known that the performance of a MLD depends on \( (H_{cir}^*)^H H_{cir}^* \), which is given by

\[
\text{trace} \left( (H_{cir}^*)^H H_{cir}^* \right) = (R^*(1, 1))^2
= \sum_{i=1}^{N} |h_{cir}^*(i)|^2 = \sum_{i=1}^{N} |h_{cir}(i)|^2
\]

where \( h_{cir}^* \) and \( h_{cir} \) are the first column vectors of \( H_{cir}^* \) and \( H_{cir} \), respectively, whose \( l \)th elements are denoted by \( h_{cir}^*(l) \) and \( h_{cir}(l) \). Eq. (A.1) shows that the trace \( (H_{cir}^*)^H H_{cir}^* \) is independent of the column permutations. This implies that the MLD provides the same performance for different assignments of the cyclic delays to the CDD transmitters, provided that each transmitter is assigned a different (unique) delay.

**APPENDIX B: DERIVATION OF THEOREM 1**

If \( K = 2 \), the MGF simplifies to:

\[
\Phi_{S^2}(s) = \int_{0}^{\infty} \int_{0}^{\infty} e^{-s(x_1+x_2)} \text{Per}_K A dx_1 dx_2
\]

which is evaluated as (B.2) at the next page. To compute (B.2), we have used the series expansion of the lower incomplete gamma function \([25, \text{eq. (8.352.1)}]\). The following equivalent expressions of \( J_2 \) and \( J_3 \) can be obtained:

\[
J_2 = \sum_{f=1}^{m_1} (-1)^{m_1-f} (D_2 - D_1)^{-(m_1+N_h-f)} \left( \frac{m_1 + N_h - f - 1}{m_1 - f} \right) (s + D_1)^{-f} + \sum_{f=1}^{N_h} (-1)^{N_h-f} (D_1 - D_2)^{-(m_1+N_h-f)} \left( \frac{m_1 + N_h - f - 1}{N_h - f} \right) (s + D_2)^{-f}
\]

and

\[
J_3 = \sum_{f=1}^{m_1-b} (-1)^{m_1-b-f} \left( \frac{D_2}{2} - \frac{D_1}{2} \right)^{-(m_1+N_h-f)} \left( \frac{m_1 + N_h - f - 1}{m_1 - b - f} \right) (s + D_1)^{-f} + \sum_{f=1}^{N_h+b} (-1)^{N_h+b-f} \left( \frac{D_1}{2} - \frac{D_2}{2} \right)^{-(m_1+N_h-f)} \left( \frac{m_1 + N_h - f - 1}{N_h + b - f} \right) (s + D_2)^{-f}
\]

By applying the inverse MGF to \( J_2/s \) and \( J_3/s \), the CDF can be expressed as the summations of the following two terms:

\[
F_{J_2} = \sum_{f=1}^{m_1} (-1)^{m_1-f} (D_2 - D_1)^{-(m_1+N_h-f)} \left( \frac{m_1 + N_h - f - 1}{m_1 - f} \right) \gamma(f, D_1 x) + \sum_{f=1}^{N_h} (-1)^{N_h-f} (D_1 - D_2)^{-(m_1+N_h-f)} \left( \frac{m_1 + N_h - f - 1}{N_h - f} \right) \frac{\gamma(f, D_2 x)}{\Gamma(f)(D_2)^f}
\]

and

\[
F_{J_3} = \sum_{f=1}^{m_1-b} (-1)^{m_1-b-f} \left( \frac{D_2}{2} - \frac{D_1}{2} \right)^{-(m_1+N_h-f)} \left( \frac{m_1 + N_h - f - 1}{m_1 - b - f} \right) \gamma(f, D_1 x) + \sum_{f=1}^{N_h+b} (-1)^{N_h+b-f} \left( \frac{D_1}{2} - \frac{D_2}{2} \right)^{-(m_1+N_h-f)} \left( \frac{m_1 + N_h - f - 1}{N_h + b - f} \right) \frac{\gamma(f, D_2 x)}{\Gamma(f)(D_2)^f}
\]

Replacing \( J_2 \) and \( J_3 \) in (B.2) by \( F_{J_2} \) and \( F_{J_3} \), we can readily obtain (20).

**APPENDIX C: DERIVATION OF THEOREM 3**

According to [37], conditioned on \( \gamma_{(M-K)} \) and \( \alpha = 1 \), \( S^K \) can be written as a summation of \( K \) i.i.d. random variables as follows:

\[
S^K | \gamma_{(M-K)} = \sum_{s=1}^{K} \gamma^s
\]

where \( \gamma_1, \ldots, \gamma^K \) are i.i.d. random variables whose PDF is

\[
f_{\gamma^s}(y) = \frac{f_1(y)}{1-F_1(x)} \text{ for } y > x
\]

with \( F_1(\cdot) \) and \( f_1(\cdot) \) denoting, respectively, the CDF and PDF of \( \gamma_1 \). From (C.1), the PDF of \( S^K \) and its corresponding MGF can be formulated as follows:

\[
f_{S^K}(y) = \int_{0}^{y} f_{S^K | \gamma_{(M-K)} = x}(x) f_{\gamma_{(M-K)}}(x) dx \quad \text{and} \quad \Phi_{S^K}(s) = \int_{0}^{\infty} \Phi_{\gamma^s}(s) f_{\gamma_{(M-K)}}(x) dx
\]

where \( \Phi_{\gamma^s}(s) \) is the MGF of \( \gamma^s \). From (C.2), \( \Phi_{\gamma^s}(s) \) is given by

\[
\Phi_{\gamma^s}(s) = \frac{1}{(1+s)^N} \left( 1 - F_1((1+s)x) \right)^{-1}.
\]

Applying the binomial and multinomial theorems \([25, \text{eq. (1.111)}]\), \( \Phi_{S^K}(s) \) is computed as in (C.5) at the next page.
expressions [33] as:

\[
\Phi_{SK}(s) = \sum_{q_1, \ldots, q_{C-2} \geq q_1, \ldots, q_{C-2} = q_1 \geq 0} \frac{1}{q_1!} \cdots \frac{1}{q_{C-2}!} \left( \frac{1}{q_{C-2}} \right) (-1)^{q_1 + \cdots + q_{C-2}} 
\]

\[
\left( \frac{1}{q_{C-2}} \right) (-1)^{q_1 + \cdots + q_{C-2}} 
\]

Applying the partial fraction (PF) to \( J_4 \) w.r.t. \( s \), (C.5) can be expressed as (C.6). By applying the inverse MGF of \( \Phi_{SK}(s) \) w.r.t. \( s \), the CDF of \( S^K \) can be derived.

### APPENDIX D: DERIVATION OF PROPOSITION 1

Consider the following different but equivalent expression for the MGF of \( S^K \):

\[
M_{SK}(s) = \frac{M}{\Gamma(N_h)} \left[ \frac{M - K - 1}{K} \sum_{p=0}^{M-K-1} \frac{M - K - 1}{p} (-1)^p \sum_{q_1, \ldots, q_{N_h} = q_1} \frac{p!}{q_1! \cdots q_{N_h}!} \sum_{l_1, \ldots, l_{N_h}} K^{l_1 \cdots l_{N_h}} \Gamma(l_1 + q_1) \cdots \Gamma(l_{N_h} + q_{N_h}) \right. 
\]

\[
+ \left. \sum_{l_2=0}^{N_h} \frac{K^{l_2+1}}{(l_2)!} \Gamma(l_1 + q_1 + N_h)(1 + p + K)^{l_2} \gamma(l_1 + q_1 + N_h) \left( -1 \right)^{m-i} \beta^{m-i}(1 - \beta)^{m_2 - m_2 + i} \right) 
\]

so that we have the following asymptotic approximation for (D.1):

\[
M_{SK}(s) \approx \frac{K^{M}}{(1 + s)^{M - K}} \int_{0}^{\infty} \left( F_1 \left( \frac{x}{1 + s} \right) \right) \left( 1 - F_1(x) \right)^{K-1} f_1(x) dx 
\]

where we assume \( \alpha = 1 \). In the high SNR region, we can approximate \( 1 - F_1(x) \) and \( F_1(x) \) by their asymptotic expressions [33] as:

\[
1 - F_1(x) \quad x \rightarrow 0 
\]

and \( F_1(x) \quad x \rightarrow 0 \)

so that we have the following asymptotic approximation for (D.1):

\[
M_{SK}(s) \approx \frac{K^{M}}{(1 + s)^{M - K}} \Gamma(N_h + 1) 
\]

\[
\int_{0}^{\infty} x^{MN_h - KN_h + N_h - 1} e^{-x} dx 
\]

\[
= K^{M} \Gamma(MN_h - KN_h + N_h)(1 + s)^{-MN_h}. 
\]

Thus, the high-SNR expression of the CDF of \( S^K \) is as follows:

\[
F_{SK}(x) = K^{M} \Gamma(MN_h - KN_h + N_h)(1 + s)^{-MN_h}. 
\]

\[
\Gamma(MN_h - KN_h + N_h) 
\]

### REFERENCES


Kyeong Jin Kim (SM’11) received the M.S. degree from the Korea Advanced Institute of Science and Technology (KAIST) in 1991 and the M.S. and Ph.D. degrees in electrical and computer engineering from the University of California, Santa Barbara, CA, USA, in 2000. From 1991 to 1995, he was a Research Engineer with the Video Research Center, Daewoo Electronics, Ltd., Korea. In 1997, he joined the Data Transmission and Networking Laboratory, University of California, Santa Barbara. After receiving his degrees, he joined the Nokia Research Center and Nokia Inc., Dallas, TX, USA, as a Senior Research Engineer, where he was an LI Specialist, from 2005 to 2009. During 2010-2011, he was an Invited Professor at Inha University, Incheon, Korea. Since 2012, he has been a Senior Principal Research Staff with the Mitsubishi Electric Research Laboratories, Cambridge, MA, USA. His research include transceiver design, resource management, scheduling in the cooperative wireless communications system, cooperative spectrum sharing system, physical layer secrecy system, and device-to-device communications.

Dr. Kim currently serves as an Editor of the IEEE Transactions on Communications. He also served as an Editor of the IEEE Communications Letters and *International Journal of Antennas and Propagation*. He also served as a Guest Editor of the EURASIP Journal on Wireless Communications and Networking: Special Issue on Co-operative Cognitive Networks and IET Communications: Special Issue on Secure Physical Layer Communications.
Marco Di Renzo (SM’14) received the Laurea (cum laude) and the Ph.D. degrees in electrical engineering from the University of L’Aquila, Italy, in 2003 and in 2007, respectively, and the Doctor of Science degree (HDR) from the University Paris-Sud, France, in 2013. Since 2010, he has been a CNRS Associate Professor (“Chargé de Recherche Titulaire CNRS”) in the Laboratory of Signals and Systems of Paris-Saclay University – CNRS, CentraleSupélec, Univ Paris Sud, Paris, France. He is an Adjunct Professor at the University of Technology Sydney, Australia, a Visiting Professor at the University of L’Aquila, Italy, and a co-founder of the university spin-off company WEST Aquila s.r.l., Italy. He serves as the Associate Editor-in-Chief of IEEE COMMUNICATIONS LETTERS, and as an Editor of IEEE TRANSACTIONS ON COMMUNICATIONS, and IEEE TRANSACTIONS ON WIRELESS COMMUNICATIONS. He is a Distinguished Lecturer of the IEEE Vehicular Technology Society and IEEE Communications Society. He is a recipient of several awards, including the 2013 IEEE-COMSOC Best Young Researcher Award for Europe, Middle East and Africa (EMEA Region), the 2014-2015 Royal Academy of Engineering Distinguished Visiting Fellowship, the 2015 IEEE Jack Neubauer Memorial Best System Paper Award, and the 2015-2018 CNRS Award for Excellence in Research and in Advising Doctoral Students.

Hongwu Liu received the Ph.D. degree from Southwest Jiaotong University in 2008. From 2008 to 2010, he was with Nanchang Hangkong University. From 2010 to 2011, he was a Post-Doctoral Fellow with the Shanghai Institute of Microsystem and Information Technology, Chinese Academy of Science. From 2011 to 2013, he was a Research Fellow with the UWB Wireless Communications Research Center, Inha University, South Korea. From 2014 to 2017, he was an associated professor with Shandong Jiaotong University. He is currently a Research Fellow with the Department of Information and Communication Engineering, Inha University. His research interests include MIMO signal processing, cognitive radios, cooperative communications, wireless secrecy communications, and future IoT.

Philip V. Orlik (M’99) was born in New York, NY in 1972. He received the B.E. degree in 1994 and the M.S. degree in 1997 both from the State University of New York at Stony Brook. In 1999 he earned his Ph. D. in electrical engineering also from SUNY Stony Brook. In 2000 he joined Mitsubishi Electric Research Laboratories Inc. located in Cambridge, MA where he is currently the Manager of the Electronics and Communications Group. His primary research focus is on advanced wireless and wired communications, sensor/IoT networks. Other research interests include vehicular/car-to-car communications, mobility modeling, performance analysis, and queueing theory.

H. Vincent Poor (F’87) received the Ph.D. degree in EECS from Princeton University in 1977. From 1977 until 1990, he was on the faculty of the University of Illinois at Urbana-Champaign. Since 1990 he has been on the faculty at Princeton, where he is currently the Michael Henry Strater University Professor of Electrical Engineering. During 2006 to 2016, he served as Dean of Princeton’s School of Engineering and Applied Science. His research interests are in the areas of information theory and signal processing, and their applications in wireless networks and related fields such as smart grid and social networks. Among his publications in these areas is the book Information Theoretic Security and Privacy of Information Systems (Cambridge University Press, 2017).

Dr. Poor is a member of the National Academy of Engineering and the National Academy of Sciences, and is a foreign member of the Royal Society. He is also a fellow of the American Academy of Arts and Sciences, the National Academy of Inventors, and other national and international academies. He received Marconi and Armstrong Awards of the IEEE Communications Society in 2007 and 2009, respectively. Recent recognition of his work includes the 2016 John Fritz Medal, the 2017 IEEE Alexander Graham Bell Medal, Honorary Professorships at Peking University and Tsinghua University, both conferred in 2016, and a D.Sc. honoris causa from Syracuse University awarded in 2017.