MITSUBISHI ELECTRIC RESEARCH LABORATORIES
http://www.merl.com

Constraint-Enforcing Controller for Both Autonomous and Assisted Steering

Kalabić, U.; Berntorp, K.; Di Cairano, S.

TR2017-135 August 27, 2017

Abstract

This paper considers the design of a controller and a constraint-enforcement scheme for application to dualmode, autonomous and manual steering systems. A tracking controller is designed to track a desired pinion angle during autonomous operation, and to provide assistive torque during manual operation. The tracking controller is designed using H-infinity synthesis with tracking made possible via the solution to a full-information output regulation problem. A reference governor scheme is implemented in order to enforce constraints. Numerical simulations are presented corresponding to an aggressive step-steer maneuver in autonomous mode and show strict constraint enforcement.

Conference on Control Technology and Applications (CCTA)

Constraint-Enforcing Controller for Both Autonomous and Assisted Steering

(Invited Paper)

Uroš Kalabić Karl Berntorp Stefano Di Cairano

Abstract— **This paper considers the design of a controller and a constraint-enforcement scheme for application to dual-mode, autonomous and manual steering systems. A tracking controller is designed to track a desired pinion angle during autonomous operation, and to provide assistive torque during manual operation. The tracking controller is designed using H_∞ synthesis with tracking made possible via the solution to a full-information output regulation problem. A reference governor scheme is implemented in order to enforce constraints. Numerical simulations are presented corresponding to an aggressive step-steer maneuver in autonomous mode and show strict constraint enforcement.**

I. Introduction

Assisted steering systems [1], [2], [3] are designed to provide assistance to a driver in turning the vehicle steering wheel. When a driver turns the steering wheel, the assistive steering system provides additional torque that assists the driver in handling the vehicle. These systems commonly consist of a motor which actuates a pinion shaft in proportion to the driver input torque. In reality, the motor does not require any manual input from the driver in order to perform its primary function, implying that it is able to steer the vehicle on its own. For vehicles that operate in both autonomous and manual modes, this means that the same actuation mechanism can be used for both steering (in autonomous mode) and steering assistance (in manual mode).

We focus on vehicles that operate in both autonomous and manual modes; appropriate modifications can be made for vehicles having only one mode of operation. The controller consists of two main components: a tracking and stabilizing control component, and a constraint-enforcement component. The tracking controller is designed to track a desired pinion angle, *i.e.*, angle of the vehicle wheels, or to track the manual torque input from the driver, where the first tracking task is performed when the vehicle operates in autonomous mode and the second task is performed in manual mode. The system is constrained, *e.g.*, in the achievable steering and pinion shaft angles as well as in their allowable rates of change. The scheme that we use for the enforcement of these constraints is the reference governor [4]. The reference governor is an add-on scheme placed before a closed-loop system in order to enforce system constraints and thereby ensure safe and reliable operation. The design of the overall steering controller is therefore modular: the tracking and stabilizing control is designed without regard to constraints, and the reference governor scheme is then designed to modify the tracking reference input in order to enforce constraints.

The design of the tracking and stabilizing controller is described as follows. After introducing an unaggressive low-pass filter for the reference input signal, we solve the full information output regulation problem [5], [6] to obtain feedforward gains with which we can modify the feedback control design to ensure tracking. This step is useful in that it allows us to design, in the next step, a control system using H_∞ synthesis [7], [8] and to take advantage of the robustness properties that it provides. Typically, it is difficult to implement H_∞ controllers for tracking since this involves the introduction of an integrator and it is difficult to apply H_∞ controllers to systems with eigenvalues on the imaginary axis. Previous work in which the authors utilized H_∞ for application to assisted steering [9] did not consider the tracking performance because the system did not operate in autonomous mode, and hence did not have to track a desired pinion angle.

After the design of the closed-loop tracking controller, we introduce the reference governor constraint-enforcement scheme. The reference governor has been applied to various automotive problems [4] but, to the authors' knowledge, not to assisted steering systems. This may be due to the fact that reference governors are typically applied to closed-loop systems and assisted steering systems have a human driver in the loop. As it is impossible to apply the reference governor to a human driver's internal reference input, reference governors do not ordinarily apply to this problem. In this work, we treat the assisted steering system as a closed-loop system with the driver torque as an external tracking input; that is, the input to be tracked is either the desired pinion angle (in autonomous mode) or an assistive torque (in manual mode). The driver torque is an external input that may lead to constraint violation but, since the driver should be given the ability to override the controller if he so wishes, constraint violations in such a case amount to turning off the assistive component and letting the driver steer under his own power. Thus, the system is designed to give the driver control of the vehicle under any circumstance, without help in instances where the behavior is deemed to have potential for violation of constraints, which have been formulated to ensure safe operation of the vehicle.

Numerical simulation results are presented, considering the the operation of the controller in autonomous mode, and show successful tracking and constraint enforcement of our

The authors are with Mitsubishi Electric Research Laboratories (MERL), Cambridge, MA 02139 {kalabic,berntorp,dicairano}@merl.com

control scheme. Simulations considering operation in manual mode have been omitted due to space constraints and are a subject of future work.

The paper is structured as follows. Section II is the description of the tracking and stabilizing controller design. Section III describes the constrained control scheme. Section IV presents and discusses the results of the numerical simulation. Section V is the conclusion.

II. Feedback Control System Design

We begin by describing the assisted steering controller design. The dynamics of vehicle steering are governed by the following equations of motion [10],

$$J_h\ddot{\theta}_h = T_m - K_t(\theta_h - \theta_p) - C_t(\dot{\theta}_h - \dot{\theta}_p), \tag{1a}$$

$$J_p\ddot{\theta}_p = K_t(\theta_h - \theta_p) - C_t(\dot{\theta}_h - \dot{\theta}_p) - C_p\dot{\theta}_p + T_a - T_{\text{align}}, \tag{1b}$$

where θ_h is the steering wheel angle, θ_p is the pinion shaft angle, J_h is the steering wheel inertia, J_p is the pinion shaft inertia, K_t is the torque sensor spring stiffness, C_t is the torque sensor damping coefficient, and C_p is the pinion shaft damping coefficient. The torques T_a and T_m are the mechanical and manual control torques, respectively, and T_{align} is the road alignment torque which can be approximated as proportional to the steering wheel and pinion shaft angles according to,

$$T_{\text{align}} = K_{\text{align},p}\theta_p - K_{\text{align},h}\theta_h, \tag{1c}$$

for some positive constants $K_{\text{align},p}$ and $K_{\text{align},h}$.

The dynamics (1) can be represented in state-space form by,

$$\dot{x}_s = A_s x_s + B_a T_a + B_m T_m, \tag{2a}$$

$$y_s = C_s x_s, \tag{2b}$$

where $x_s = (\theta_h, \dot{\theta}_h, \theta_p, \dot{\theta}_p)$ is the state, $u_s = (T_a, T_m)$ is the control input, and $y_s = (\hat{T}_m, x_s)$ is the measured output where $\hat{T}_m$ is the measured value of the manual torque input T_m.

The system (2) is controlled by both an assistive and a manual torque input, T_a and T_m. In the remainder of this section, we describe the design of a control system that provides the assistive torque T_a, *e.g.*, by an electric motor, in order to serve one of two functions depending on whether the mechanical system is steering the vehicle by itself or assisting the driver in doing so.

A. Controller for Both Autonomous and Manual Modes

The steering control system operates in one of two operating modes, an autonomous mode or a manual mode. The control system that determines the assistive torque T_a is designed to function independent of the mode of operation.

In autonomous mode, the steering controller must track a desired pinion angle θ_d, which is received from a higher-level system controller, *e.g.*, the path-planner of [11]. In manual mode, the driver steers the vehicle using the steering wheel, providing a torque T_m, and this torque is accompanied by a torque T_a, which provides assistance to the driver by a factor of K_{asst}, *i.e.*, $T_a \approx K_{\text{asst}}T_m$. Therefore, the controller must perform a tracking function when the vehicle is operating in autonomous mode, and an amplifying function when the vehicle operates in manual mode.

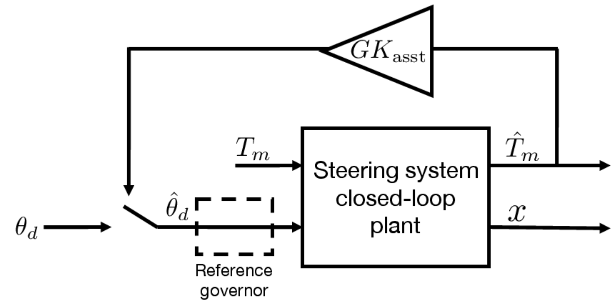

Fig. 1. Schematic of closed-loop steering system operating in both autonomous and manual modes

The overall design that we present in this work is that of a tracking controller, whose input is an augmented desired pinion angle $\hat{\theta}_d$. When the vehicle is operating in autonomous mode, the input is set to $\hat{\theta}_d = \theta_d$ while, when the vehicle is in manual mode, $\hat{\theta}_d$ is set to a multiple of the manual torque T_m. To relate the torque T_m to the desired angle $\hat{\theta}_d$, we set $\hat{\theta}_d$ to a value that will deliver the required assistive torque in steady-state. This approach is relatively simplistic, but it serves us well for the purpose of justifying the overall constrained control design that is presented in this paper, and more advanced methods can be used in the future following the lines of [12].

We begin by letting G be the steady-state gain from T_m to θ_p, *i.e.*, $\lim_{t\to\infty}\theta_p(t) = GT_m$ whenever $T_m(t) = T_m$ and $T_a(t) = 0$ for all t, at some constant vehicle speed. This gain is explicitly given by $G = -\begin{bmatrix}0 & 0 & 1 & 0\end{bmatrix} A_s^{-1}B_m$. Assuming that θ_p tracks $\hat{\theta}_d$ when $T_m \equiv 0$, in order to achieve the desired amount of assistive torque, we set the $\hat{\theta}_d$ to be a K_{asst} multiple of $\hat{T}_m$ as in,

$$\hat{\theta}_d = GK_{\text{asst}}\hat{T}_m. \tag{3}$$

This implies that, under constant inputs and in steady state, the effective torque applied is $(1 + K_{\text{asst}})T_m$ since $\lim_{t\to\infty}\theta_p(t) = GT_m + \lim_{t\to\infty}\hat{\theta}_d(t) = GT_m + \lim_{t\to\infty}K_{\text{asst}}\hat{T}_m(t) = G(1 + K_{\text{asst}})T_m$.

The resulting switched-system design is given in Fig. 1. The figure includes a dashed box showing the placement of a constraint-enforcing reference governor. The reference governor, and constraint enforcement in general, will be discussed in the following section; for now, we continue to design the tracking controller without regard to constraints.

B. Tracking Controller for Power Steering

In this section, we detail the design of the steering controller, which consists of three main parts: (i) a low-pass filter of the reference signal, (ii) a tracking gain, and (iii) a stabilizing controller. We describe all three in the following.

1) Low-Pass Filter for the Reference Signal: When in autonomous mode, the command θ_d is received as an input from a higher-level control function, such as a lane-tracking scheme. To guard against instantaneous changes in $\hat{\theta}_d = \theta_d$, such as an abrupt switch from manual to autonomous mode, we introduce a low-pass second-order filter,

$$\hat{\Theta}_p(s) = \frac{\omega_0^2}{(s+\omega_0)^2}\hat{\Theta}_d(s), \tag{4}$$

whose output $\hat{\theta}_p$ is then passed to the tracking component of the controller. The filter cut-off frequency ω_0 is related to the desired response time of the pinion angle θ_p.

2) Tracking Gain: We design an H_∞-controller to control and stabilize the steering system. Ordinarily, it is difficult to design a tracking controller with H_∞ because the addition of an integrator to the system dynamics would place a pole on the imaginary axis of the open-loop system, resulting in difficulty in performing H_∞ synthesis as the underlying Riccati equation cannot be solved. Our solution around this is to solve the full-information output regulation problem [5], [6] and use the resulting feedforward gains in H_∞ synthesis. The problem is to compute gains which ensure that the tracking error e_p of following system tends to 0,

$$\dot{x}_s = A_s x_s + B_a T_a, \tag{5a}$$
$$\dot{w} = Sw, \tag{5b}$$
$$e_p = C_x x_s - C_w w, \tag{5c}$$

where $w = (\hat{\theta}_p, \dot{\hat{\theta}}_p, \hat{\theta}_d)$ and $e_p = \theta_p - \hat{\theta}_p$, so that,

$$S = \begin{bmatrix} 0 & 1 & 0 \\ -\omega_0^2 & -2\omega_0 & \omega_0^2 \\ 0 & 0 & 0 \end{bmatrix},$$
$$C_x = \begin{bmatrix} 0 & 0 & 1 & 0 \end{bmatrix},$$
$$C_w = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix}.$$

Note that, due to the absence of a preview of the reference input $\hat{\theta}_d$, in the above it is assumed that $\hat{\theta}_d$ is kept constant for future times. As shown in [5], determining the pair of matrices P and F solving the linear matrix equations,

$$PS = A_s P + B_a F, \tag{6a}$$
$$C_x P = C_w, \tag{6b}$$

guarantees that the control input,

$$T_a = K(x_s - Pw) + Fw, \tag{7}$$

drives the pinion-angle tracking error e_p to 0, for any stabilizing feedback gain K. In the next section, we design an appropriate feedback controller K.

3) Stabilizing Controller: We design the feedback controller by H_∞ synthesis [7], [8], where the feedback output corresponds to the form given in (7) in order to ensure the stabilization of the tracking error e_p. In general, H_∞ synthesis is applied to systems of the form,

$$\dot{x} = Ax + B_w u_w + B_u u, \tag{8a}$$
$$z = C_z x + D_{zu} u, \tag{8b}$$
$$y = C_y x + D_{yw} u_w, \tag{8c}$$

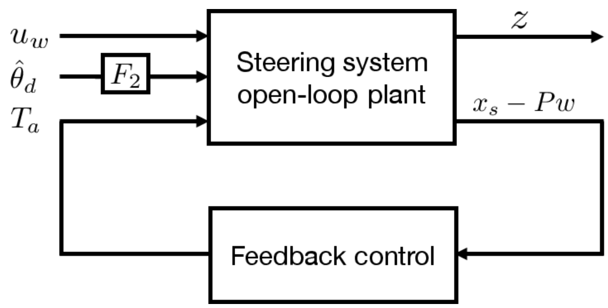

Fig. 2. Schematic of open-loop steering system in closed-loop with feedback controller including modifications made to enable tracking of the input $\hat{\theta}_d$

where x is the system state, u_w is the disturbance input, u is the control input, z is the performance output, and y is the measured feedback output. The goal of H_∞ control is to minimize the infinity norm of the transfer function T_{zw} from the disturbance input u_w to the performance output z, *i.e.*, to minimize $\|T_{zw}\|_\infty = \sup_\omega \bar{\sigma}(T_{zw}(j\omega))$.

Previous work that has considered H_∞-synthesis for steering control includes [9]. In our design of a steering controller, we set the state to be $x = (x_s, \hat{\theta}_p, \dot{\hat{\theta}}_p)$, the vector of disturbance to be equal to small perturbations of x_s, and the control input to be equal to T_a. Two tracking inputs are introduced, θ_d and T_m, whose deviation from the nominal point are treated as disturbance inputs. The feedback output is set to the tracking feedback $x_s - Pw$ from (7). The performance input is a linear combination of the state x_s and the input T_a.

The modified system (8) can be compactly represented by,

$$\begin{bmatrix} \dot{x} \\ z \\ y \end{bmatrix} = \left[\begin{array}{c|c:c:c:c} A & B_\theta & B_T & B_w & B_u \\ \hline C_z & 0 & 0 & 0 & D_{zu} \\ \hdashline C_y & D_{y\theta} & D_{yT} & D_{yw} & 0 \end{array}\right] \begin{bmatrix} x \\ \theta_d \\ T_m \\ u_w \\ T_a \end{bmatrix}, \tag{9}$$

where the block matrix introduced in (9) is given by,

$$\left[\begin{array}{cc|c:c:cc:c} A_s & B_a F_1 & B_a F_2 & B_a F_2 G K_{\text{asst}} + B_m & W & 0 & B_a \\ 0 & S_{11} & S_{12} & S_{12} G K_{\text{asst}} & 0 & 0 & 0 \\ \hline Q & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & R \\ \hdashline I_4 & -P_1 & -P_2 & -P_2 G K_{\text{asst}} & 0 & V & 0 \end{array}\right],$$

and the matrices $S_{11} \in M_{2,2}$, $S_{12} \in M_{2,1}$, $P_1 \in M_{4,2}$, $P_2 \in M_{4,1}$, $F_1 \in M_{1,2}$, and $F_2 \in M_{1,1}$ are related to S, P, and F according to,

$$S = \begin{bmatrix} S_{11} & S_{12} \\ 0 & 0 \end{bmatrix}, \; P = \begin{bmatrix} P_1 & P_2 \end{bmatrix}, \; F = \begin{bmatrix} F_1 & F_2 \end{bmatrix}.$$

The matrices Q and R are gains for the performance variables, *i.e.*, the state x and the mechanical control input T_a, which will be chosen to ensure adequate response time and overshoot. The matrices W and V are noise gains and will be chosen to reflect the requirement that the system be robust to measurement noise and jittery driver handling.

Note that the third column is a linear combination of the second and fourth columns and therefore the design does not require it. It has been included to allow for consistency between the two modes of operation. Specifically, this ensures that the weight on tracking θ_d is the same as the weight on amplifying T_m. This has not been done because these two functions will never be performed simultaneously, but because the relative ratios between the disturbance inputs u_w and θ_d and the relative ratios between u_w and T_m ought to be the same.

C. Controller Performance

Here we choose the design parameters for our controller and provide simulation results corresponding to a step-input response in autonomous mode and a modified J-turn [13] in manual mode.

For the performance variables, we choose the weighting matrices,

$$Q = \mathrm{diag}(1, 10^3, 1, 10^3), \ R = 6 \cdot 10^3.$$

For the noise gains, we choose the weighting matrices,

$$W = I_4, \ V = \mathrm{diag}(1, 10^{-3}, 1, 10^{-3}).$$

We perform two simulations. The first is in autonomous mode and corresponds to a step-request of $300°/G_{\text{gear}}$ in $\hat{\theta}_d = \theta_d$, where G_{gear} is the gear ratio from the steering shaft to the pinion shaft. The second is in manual mode and corresponds to an aggressive J-turn maneuver where the torque input T_m is made such that the corresponding steady-state value of $\hat{\theta}_d$ ramps up from 0 to $300°/G_{\text{gear}}$ at a constant rate of $1200°/\text{s}/G_{\text{gear}}$. The results of the first and second simulation are presented in Figs. 3 and 4, respectively. The results show a fast unconstrained response with less than a 0.2s rise time in the response to a step change in the desired angle θ_d, and about a 0.3s rise time in the response to a J-turn maneuver.

III. Constrained Control Design

The tracking controller is designed without taking state or input constraints into account. The steering system, however, is subject to constraints on the angular positions and their rates of change. To handle these constraints, as shown in Fig. 1, we introduce a reference governor constraint-enforcement scheme to modify the reference input $\hat{\theta}_d$. The reference governor and its method of operation are explained in the following.

A. Reference Governor Overview

The reference governor [4], [14], [15] is an add-on constraint-enforcement scheme which modifies a reference input in order to enforce constraints on system states or inputs. The linear-systems variant of the reference governor is applied to linear discrete-time systems of the form,

$$x(t+1) = Ax(t) + Bv(t), \tag{10a}$$

$$y(t) = Cx(t) + Dv(t) \in Y, \tag{10b}$$

where $x(t)$ is the system state, $v(t)$ is the reference input, $y(t)$ is the constrained output, and Y is an n-dimensional compact, convex, and often polytopic set.

Given a desired reference $r(t)$, the reference governor modifies the reference $v(t)$ in order to ensure that the constraint (10b) is satisfied for all present and future time-instants $t = 0, 1, 2, \ldots$. The constraint-admissible reference $v(t)$ is set to a convex sum of the previously constraint-admissible reference $v(t-1)$ and the desired reference $r(t)$,

$$v(t) = \kappa(t)r(t) + (1 - \kappa(t))v(t-1), \tag{11}$$

where $\kappa(t)$ is a scalar parameter that is maximized subject to constraints, *i.e.*,

$$\kappa(t) = \max_{\kappa\in[0,1]} \{\kappa : (x(t), \kappa r(t) + (1-\kappa)v(t-1)) \in P\}, \tag{12}$$

where the set P is a set of initial-condition/constant-reference pairs satisfying constraints. Note that when $\kappa(t) = 1$, $v(t) = r(t)$ as desired, and when $\kappa(t) = 0$, $v(t) = v(t-1)$; this latter case is the worst-case scenario, as the input $v(t-1)$ has been guaranteed to be constraint-admissible by the operation of the reference governor at the previous time instant $t-1$.

The constraint set P is an approximation of the maximum output admissible set O_∞ [16]. The latter is the set of all constraint-admissible initial-condition/constant-reference pairs and is defined as,

$$\begin{aligned} O_\infty = \{(x, v) : x(0) = x, v(t) \equiv v, \\ \text{(10) is satisfied for all } t = 0, 1, 2, \ldots\}. \end{aligned} \tag{13}$$

Under certain conditions O_∞, or an arbitrarily close approximation, is polytopic and can be expressed as a finite set of linear inequalities.

Specifically, when the constraint set Y is polytopic, it can be expressed as a set of linear inequalities,

$$Y = \{y : Hy \le h\}, \tag{14}$$

for some matrices H and h. Given an initial state $x(0)$ and a constant reference input $v(t) \equiv v$, according to (10),

$$y(t) = CA^t(x(0) - \Gamma v) + (C\Gamma + D)v, \tag{15}$$

where $\Gamma = (I_n - A)^{-1}B$. In this case, the set O_∞ can also be expressed as a set of linear inequalities. It is computed as the limit of the recursion [14], [17], [15],

$$O_{t+1} = O_t \cap X_t, \tag{16}$$

where $O_0 = X_0$ and,

$$\begin{aligned} X_t &= \{(x, v) : x(0) = x, \ v(t) \equiv v, \ y(t) \in Y\}, \\ &= \{(x, v) : HCA^t x + H(C(I_n - A^t)\Gamma + D)v \le h\}, \end{aligned}$$

The operation (16) corresponds to appending the linear inequalities defining X_t to those defining O_t. In practice, spurious linear inequalities are identified and removed by the algorithm computing O_{t+1}. Under certain additional conditions, there exists a time t^* for which $O_{t^*} = O_{t^*+1}$, implying that $O_{t^*} = O_{t^*+k}$ for all $k = 0, 1, 2$. The set O_{t^*} is an approximation[1] to O_∞, so we set,

$$P := O_{t^*} = \{(x, v) : H'_x x + H'_r v \le h'\}. \tag{17}$$

[1] Generally, the approximation can be made arbitrarily tight at the expense of increasing t^* and therefore the time to compute O_{t^*}.

Therefore, the reference governor scheme amounts to solving the optimization,

$$\kappa(t) = \max_{\kappa\in[0,1]} \{\kappa : \kappa H_r'(r(t) - v(t-1)) \leq h' - H_x' x(t)\},$$

and setting $v(t)$ according to (11). This optimization can be performed as a finite number of scalar divisions and logical comparisons (see [14], [15] for details).

B. Application of Constrained Control to Assisted Steering

As discussed previously, the steering system designed in Section II-B is both state- and input-constrained. The specific constraints imposed on the system are given by,

$$|\theta_p| \leq 0.5 \text{ rad}, \qquad |\theta_h| \leq 0.5 G_{\text{gear}} \text{ rad}, \tag{18a}$$
$$|\dot{\theta}_p| \leq \pi/4 \text{ rad/s}, \qquad |\dot{\theta}_h| \leq \pi/4 G_{\text{gear}} \text{ rad/s}, \tag{18b}$$
$$|\ddot{\theta}_p| \leq 6 \text{ rad/s}^2. \tag{18c}$$

Constraints (18a) correspond to the limit on the range of wheel turn. Constraints (18b) correspond to the speed at which the wheels may turn to ensure safe and comfortable operation for the driver. Constraint (18c) corresponds to the allowable angular acceleration of the wheels, which is derived from desired road response characteristics: as explained in [18], [19], since $J_p\ddot{\theta}_p \approx T_a - T_{\text{align}}$, a large mechanically-delivered torque T_a will result in the road torque T_{align} being perceived to be much smaller than reality; as such, it is desirable to keep the difference between T_a and T_{align} constrained.

IV. Numerical Simulation

The simulation corresponds to the one performed in Section II-C, which considered a response to a step change in the desired pinion angle when operating in autonomous mode, and a response to a modified J-turn maneuver in manual mode. The same inputs are used in this simulation as in the baseline simulation.

The results are presented in Figs. 3-8. The reference governor is able to enforce constraints during autonomous operation and is mostly able to handle constraints during manual operation. An example of a brief constraint violation is shown in the bottom subplot of Fig. 7 at the 65ms mark. This is due to the fact that, as shown schematically in Fig. 1, the reference governor has not been designed to modify the driver torque input T_m; the result is that T_a is set to a very small value, as shown in the bottom subplot of Fig. 8, in order to mitigate constraint violation as much as possible. This is the limit of the reference governor control authority; the driver has full control authority and the reference governor cannot backtrack the reference according to (11).

V. Conclusion

This work considered a controller for application to dual-mode, autonomous and manual steering systems. In autonomous mode, the controller tracks a desired pinion angle, which is modified by a reference governor in order to enforce constraints. In manual mode, the controller provides assistance to the driver utilizing the same tracking

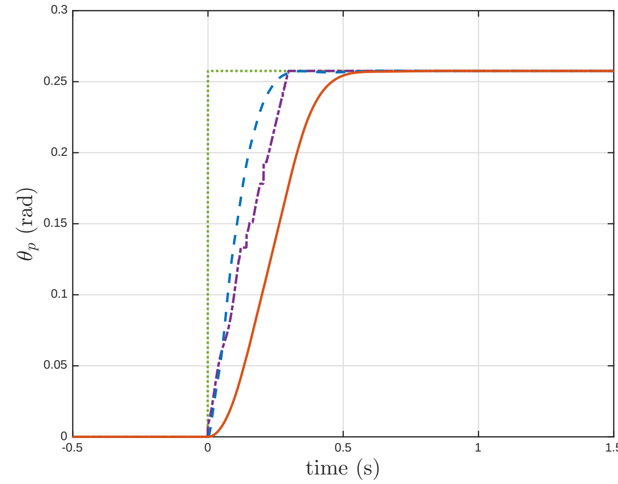

Fig. 3. Autonomous mode simulation results corresponding to a step input in θ_d with the desired θ_d (dotted, green), unconstrained response of θ_p (dashed, blue), constrained $\hat{\theta}_d$ (dot-dashed, purple), and constrained response of θ_p (solid,red)

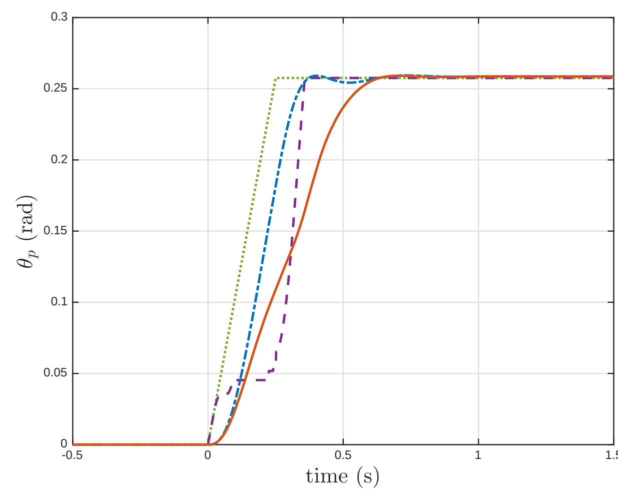

Fig. 4. Manual mode simulation results corresponding to a ramp input in T_m with the equivalent steady-state desired θ_d (dotted, green), unconstrained response of θ_p (dashed, blue), constrained $\hat{\theta}_d$ (dot-dashed, purple), and constrained response of θ_p (solid,red)

scheme, by setting the desired pinion angle to the equivalent torque input. Numerical results showed effective tracking and constraint-enforcement in autonomous operation. Numerical results showing controller performance in manual mode were omitted due to space constraints; they will be a subject of future work.

Acknowledgment

The authors thank Prof. Ilya Kolmanovsky of the University of Michigan for technical discussions.

References

[1] T. D. Gillespie, *Fundamentals of Vehicle Dynamics*. Warrendale, PA: SAE, 1992.
[2] R. Rajamani, *Vehicle Dynamics and Control*, 1st ed. New York: Springer, 2006.
[3] R. N. Jazar, *Vehicle Dynamics: Theory and Applications*. New York: Springer, 2008.

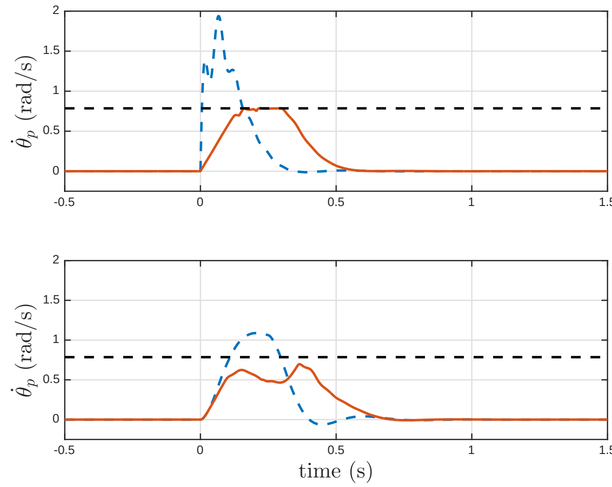

Fig. 5. Autonomous (top) and manual (bottom) mode simulations showing constrained (solid, red) and unconstrained (dashed, blue) responses in $\dot{\theta}_p$ plotted against constraints (dashed, black)

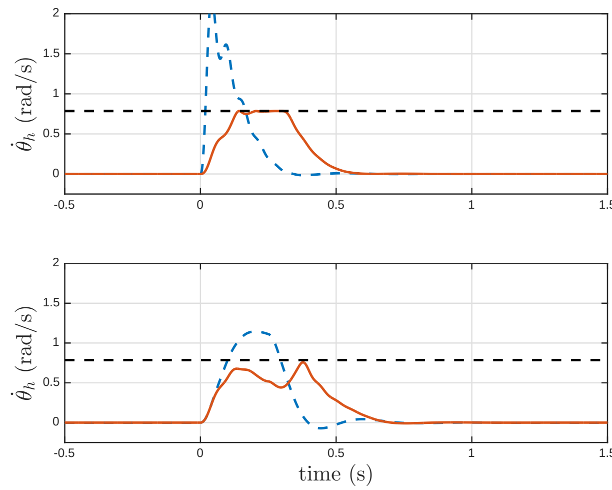

Fig. 6. Autonomous (top) and manual (bottom) mode simulations showing constrained (solid, red) and unconstrained (dashed, blue) responses in $\dot{\theta}_h$ plotted against constraints (dashed, black)

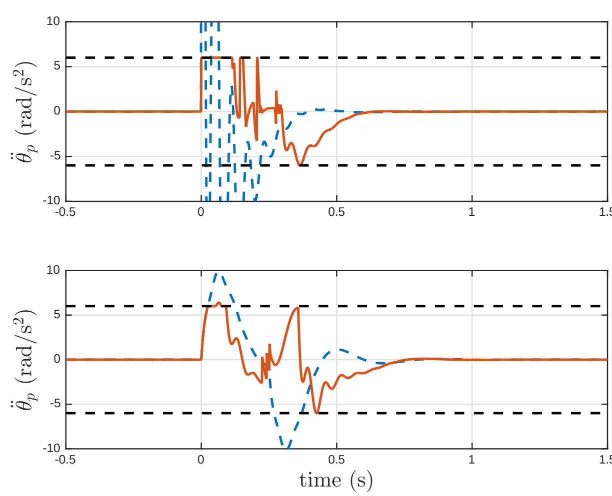

Fig. 7. Autonomous (top) and manual (bottom) mode simulations showing constrained (solid, red) and unconstrained (dashed, blue) responses in $\ddot{\theta}_p$ plotted against constraints (dashed, black)

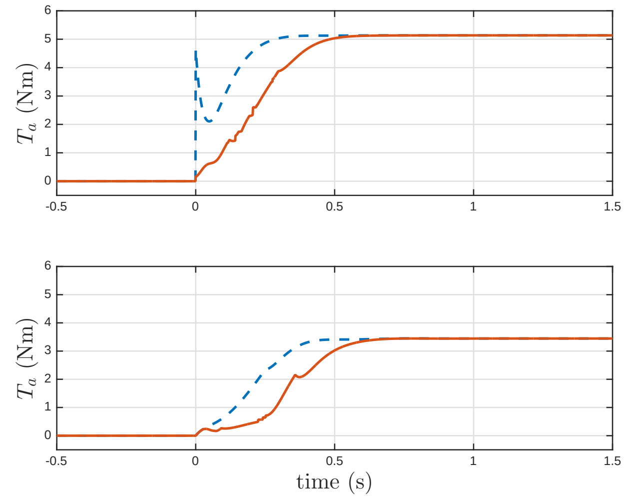

Fig. 8. Autonomous (top) and manual (bottom) mode simulations showing constrained (solid, red) and unconstrained (dashed, blue) responses in T_a

[4] E. Garone, and S. Di Cairano, and I. Kolmanovsky, "Reference and command governors for systems with constraints: A survey on theory and applications," *Automatica*, vol. 75, pp. 306–328, 2017.

[5] B. A. Francis, "The linear multivariable regulator problem," *SIAM J. Control Opt.*, vol. 15, no. 3, pp. 486–505, 1977.

[6] A. Isidori, *Nonlinear Control Systems*, 3rd. ed. London: Springer, 1995.

[7] G. E. Dullerud and F. Paganini, *A Course in Robust Control Theory: A Convex Approach*. New York: Springer, 2000.

[8] S. Skogestad and I. Postlethwaite, *Multivariable Feedback Control: Analysis and Design*. Chichester, England: Wiley, 2005.

[9] X. Chen, T. Yang, X. Chen, and K. Zhou, "A generic model-based advanced control of electric power-assisted steering systems," *IEEE Trans. Control Syst. Technol.*, vol. 16, no. 6, pp. 1289–1300, 2008.

[10] A. Zaremba and R. I. Davis, "Dynamic analysis and stability of a power assist steering system," in *Proc. American Control Conf.*, Seattle, WA, 1995, pp. 4253–4257.

[11] S. Di Cairano, U. V. Kalabić, and K. Berntorp, "Vehicle tracking control on piecewise-clothoidal trajectories by MPC with guaranteed error bounds," in *Proc. Conf. Decision and Control*, Las Vegas, NV, 2016, pp. 709–714.

[12] V. Cerone, M. Milanese, and D. Regruta, "Combined automatic lane-keeping and driver's steering through a 2-DOF control strategy," *IEEE Trans. Control Syst. Technol.*, vol. 17, no. 1, pp. 135–142, 2009.

[13] G. Forkenbrock, W. Garrott, M. Heitz and B. O'Harra, "An experimental examination of J-turn and fishhook maneuvers that may induce on-road, untripped, light vehicle rollover," SAE Tech. Rep. 2003-01-1008, 2003.

[14] E. G. Gilbert, I. Kolmanovsky, and K. T. Tan, "Discrete-time reference governors and the nonlinear control of systems with state and control constraints," *Int. J. Robust Nonlinear Control*, vol. 5, no. 5, pp. 487–504, 1995.

[15] E. G. Gilbert and I. Kolmanovsky, "Fast reference governors for systems with state and control constraints and disturbance inputs," *Int. J. Robust Nonlinear Control*, vol. 9, no. 15, pp. 1117–1141, 1999.

[16] E. G. Gilbert and K. T. Tan, "Linear systems with state and control constraints: The theory and application of maximal output admissible sets," *IEEE Trans. Autom. Control*, vol. 36, no. 9, pp. 1008–1020, 1991.

[17] I. Kolmanovsky and E. G. Gilbert, "Theory and computation of disturbance invariant sets for discrete-time linear systems," *Math. Problems Eng.*, vol. 4, no. 4, pp. 317–367, 1998.

[18] Y.-H. J. Hsu and J. C. Gerdes, "A feel for the road: A method to estimate tire parameters using steering torque," in *Proc. Int. Symp. Adv. Vehicle Control*, Taipei, Taiwan, 2006.

[19] S. Zafeiropoulos and S. Di Cairano, "Vehicle yaw dynamics control by torque-based assist systems enforcing driver's steering feel constraints," in *Proc. American Control Conf.*, Washington, DC, 2013, pp. 6746–6751.