QoS-Constrained Relay Control for Full-Duplex Relaying with SWIPT

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Abstract

This study investigates relay control for simultaneous wireless information and power transfer in full-duplex relay networks under Nakagami-m fading channels. Unlike previous work, harvest-transmit (HT) and general harvest-transmit-store (HTS) models are respectively considered to maximize average throughput subject to quality of service (QoS) constraints. The end-to-end outage probability of the network in an HT model is presented in an exact integral-form. To prevent outage performance degradation in an HT model, time switching (TS) is designed to maximize average throughput subject to QoS constraints of minimizing outage probability and maintaining a target outage probability, respectively. The optimal TS factors subject to QoS constraints are presented for an HT model. In general, in an HTS model, energy scheduling is performed across different transmission blocks and TS is performed within each block. Compared with the block-based HTS model without TS, the proposed general HTS model can greatly improve outage performance via greedy search (GS). By modeling the relay’s energy levels as a Markov chain with a two-stage state transition, the outage probability for GS implementation of the general HTS model is derived. To demonstrate the practical significance of QoS-constrained relay control, numerical results are presented showing that the proposed relay control achieves substantial improvement of outage performance and successful rate.

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Index Terms—Energy harvesting, wireless power transfer, amplify-and-forward relay, full-duplex relay, relay control.

I. INTRODUCTION

With capability to harvest energy from ambient radio-frequency (RF) signals, simultaneous wireless information and power transfer (SWIPT) techniques provide a more promising way for wireless communications to function in environments with physical or other limitations [1]–[3]. Based on two practical receiver architectures, namely, time switching (TS) and power splitting (PS) receivers [1], SWIPT techniques have been widely applied in wireless networks [3, and references therein]. One line of research that has emerged is relay-assisted SWIPT [4]–[11]. It has shown that relay-assisted SWIPT not only enables wireless communications over long distances or across barriers, but also keeps energy-constrained relays active through RF energy harvesting (EH). In [4] and [5], TS and PS relaying protocols have been designed for amplify-and-forward (AF) and decode-and-forward (DF) relay networks, respectively. Several power allocation schemes for relay-assisted SWIPT networks with multiple source-destination pairs were studied in [6]. Outage probability and diversity of relay-assisted SWIPT networks with spatial randomly located relays were investigated in [10] and distributed PS-based SWIPT was investigated for interference relay networks in [11]. Moreover, multi-antenna technologies have been applied in relay-assisted SWIPT networks in [7]–[9]. Nevertheless, all these studies are limited to half-duplex relay (HDR) mode. Since the source-to-relay and relay-to-destination channels are kept orthogonal by either frequency division or time division multiplexing, about 50% spectral efficiency (SE) loss occurs in HDR mode. As a key technology for future relay networks, full-duplex relay (FDR) systems have drawn considerable attention [12]–[18]. Since FDR mode realizes an end-to-end (e2e) transmission via one channel utilization, significant improvement of SE over HDR mode can be achieved.

A few studies have been conducted for FDR-assisted SWIPT networks. In [14] and [15], outage probability, throughput, and ergodic capacity have been analyzed for FDR-assisted SWIPT networks, in which a TS-based relay is operated cooperatively. In practice, since an FDR node suffers severe self-interference from its own transmit signal, FDR transmission is difficult to implement. To suppress self-interference, MIMO antennas have been employed at the relay to aid FDR-assisted SWIPT [19]. For conventional two-phase AF HDR networks, a self-interference immunized FDR node was proposed by employing EH in the second time phase [20], so that the relay can transmit information and extract energy simultaneously via separated transmit and receive antennas. Note that all the above studies of FDR-assisted SWIPT are conducted to maximize network throughput without further considering quality of service (QoS) constraints, whereas it is envisioned that SWIPT techniques will be required to support various types of traffic having different QoS requirements [21]–[23]. For point-to-multipoint PS-based SWIPT networks, QoS constraints based on signal-to-interference-plus-noise ra-
tio (SINR) and mean-square-error (MSE) have been applied to minimize transmission power in [21] and [24], respectively. A game-theoretic approach has been applied to optimize multiple-pair SWIPT communications subject to SINR and EH constraints in [25]. For TS-based SWIPT, [23] studies joint TS and power control to maintain a given level of throughput. Notably, all the above control schemes are based on estimation of instantaneous channel state information (CSI), which requires dedicated reverse-link training from the EH receiver [26].

Motivated by these previous studies, in this paper we consider the maximization of average throughput for a QoS-constrained FDR network with SWIPT. In the considered FDR network, the source has a reliable power supply, whereas the relay has to harvest energy from the source-emitted RF signal via TS operation. Since TS affects both SE and QoS, the relaying mode and corresponding TS have a complicated relationship in achieving the allowable maximum SE subject to QoS constraints. Compared with existing works, some distinct features of our study are highlighted here.

- In [20], the effective information transmission time is the same as that of HDR networks, so that the SE improvement is achieved in HDR mode rather than FDR mode. In our study, we consider maximization of average throughput of FDR transmission, i.e., the relay receives and forwards the source information to the destination simultaneously.
- Zhong et al. [14] and [15] investigated FDR-assisted SWIPT in the harvest-transmit (HT) model to improve average throughput with optimized TS. Unfortunately, outage performance seriously degrades in delay-limited transmissions when TS is optimized without a QoS constraint. In our study, both outage probability minimization and target outage probability are adopted as QoS constraints, under which TS is optimized to achieve the allowable maximum average throughput. Thus, serious outage performance degradation can be prevented and successful rate can be maximized [27].
- The analysis of outage probability and throughput in [14] and [15] are conducted by modeling dual-hop and residual self-interference (RSI) channels as Rayleigh fading. However, SWIPT operates most efficiently within a relatively short range. In this situation, a line of sight (LoS) path exists with high probability and Nakagami-m fading can provide a better model [9], [28]. Furthermore, although a Rician fading model is appropriate for the RSI channel in the RF-domain [29], the exact behavior of the RSI channel in the digital-domain is still unknown due to several complicated stages of active interference cancellation (IC) [30], [31]. Therefore, this study investigates outage probability conditioned on the RSI channel power without modeling the RSI channel gain via a specific distribution and considers Nakagami-m fading for dual-hop channels, so that our analytical results can be applied to a wide range of FDR-assisted SWIPT networks employing different IC schemes.
- Due to propagation loss and channel fluctuations, the relay-harvested energy within a transmission block can be very limited and energy accumulation is needed to improve the reliability of information transmission [32], [33]. Different from the block-based harvest-transmit-store (HTS) model [32], this study considers a general HTS model by employing TS within each transmission block. With the aid of in-block TS, the general HTS model can greatly improve outage performance and includes the block-based HTS model as a special case.

In this paper, the HT model and the general HTS are investigated subject to QoS constraints. The corresponding control schemes are developed and analytical results are presented to verify the QoS metrics. The contributions of the paper are summarized as follows:

- By modeling dual-hop channels as Nakagami-m fading, we present the analytical e2e outage probability conditioned on the RSI channel power for the HT model. QoS constraints of minimizing outage probability and maintaining a target outage probability are respectively considered in optimizing TS to achieve the allowable maximum average throughput. The optimal TS factor that maximizes average throughput subject to minimizing outage probability is presented in closed form. The optimal TS factor that maximizes average throughput subject to a target outage probability is also derived. Employing the obtained optimal TS factors, the successful rate is achieved with the guaranteed outage performance.
- To accumulate energy for optimizing FDR transmission, a general HTS model is designed by employing TS within each block. In the general HTS model, the first phase of each block is dedicated for EH. In the second phase of each block, the relay node can switch to EH or FDR transmission depending on the relay’s residual energy level and CSI. A greedy search (GS) policy is implemented to realize the proposed general HTS model. By allocating a small portion of time for EH within each block, the GS policy improves outage performance significantly over that of the block-based HTS model [32].
- The proposed general HTS model degenerates to the block-based HTS model by setting a zero TS factor. Thus, the general HTS model and the block-based HTS model can be analyzed under a uniform framework. With respect to the time-switched two operational phases in each block, the relay’s residual energy levels are modeled as a Markov chain (MC) with a two-stage state transition. Then, the e2e outage probability is derived for the GS implementation of the general HTS mode including the block-based HTS model as a special case.

The rest of this paper is organized as follows. Section II describes the HT model and develops its e2e outage probability. The optimal TS that maximizes average throughput subject to QoS constraints is also derived in Section II. Section III
presents the general HTS model and its GS implementation. The analytical e2e outage probability for the GS policy is also derived in Section III. Section IV presents numerical results and discusses the system performances of the proposed control schemes. Finally, Section V summarizes the contributions of our study.

Notation: \( \lbrack \cdot \rbrack \) is the floor function, \( F_X(\cdot) \) and \( \bar{F}_X(\cdot) \) denote the cumulative distribution function (CDF) and the complementary CDF (CCDF) of the random variable \( X \), respectively, \( \Gamma(\cdot) \) denotes the gamma function, \( \Gamma_n(\cdot) \) denotes the upper incomplete gamma function, and \( K_n(\cdot) \) is the \( n \)-th order modified Bessel function of the second kind [34, Eq. 8.432].

II. Harvest-Transmit Model

In this paper, we consider a wireless dual-hop FDR network, in which a source node intends to transfer its information to the destination node. Due to physical isolation or environmental limitations between the source and destination, a cooperative relay is employed to assist information transmission from the source to the destination. The relay is assumed to be an energy-selfish or energy-constrained device such that it needs to harvest energy from the source-emitted RF signal to forward the source information to the destination. For simplicity of implementation, the AF relaying scheme and TS architecture are chosen at the relay. The channels of the source-to-relay and relay-to-destination links are denoted by \( h_1 = \sqrt{L_1} h_1 \) and \( h_2 = \sqrt{L_2} h_2 \), respectively, where \( L_i \) and \( h_i \) \((i = 1, 2)\) are large-scale fading and small-scale fading of the dual-hop channels, respectively. The large-scale fading is comprised of the distance-dependent path loss as well as shadowing attenuation, i.e.,

\[
\mathcal{L}_i \triangleq \frac{A_i L_0 L_s}{(d_i/d_0)^\varphi},
\]

where \( A_i \) is the transmit antenna gain of the \( i \)-th hop link, \( L_0 \) is the measured path loss at the reference distance \( d_0 \), \( d_i \) is the distance between the transmitter and receiver of the \( i \)-th hop link, \( \varphi \) is the path loss exponent, and \( L_s \) is the shadowing attenuation. To account for the LoS communication setting, the shadowing attenuation is set to \( L_s = 1 \) in this study. For the sake of exposition, the channel gain of \( h_i \) is denoted by \( g_i \triangleq |h_i|^2 \) for \( i \in \{1, 2\} \). The small-scale channel magnitude of the dual-hop links, \(|h_i|\), is modeled as Nakagami-\( m \) fading with unit mean such that \( g_i \) is distributed according to the gamma distribution with shape factor \( m_i \) and scale factor \( \theta_i \triangleq \frac{\bar{g}_i}{m_i} \), where \( \bar{g}_i \triangleq \mathbb{E}(|h_i|^2) \) is the average channel gain for \( i \in \{1, 2\} \). Then, the CDF and CCDF of \( g_i \) \((i = 1, 2)\) can be respectively expressed as

\[
F_{g_i}(x) = 1 - \frac{\Gamma(m_i, x/\theta_i)}{\Gamma(m_i)}, \quad \bar{F}_{g_i}(x) = \frac{\Gamma(m_i, x/\theta_i)}{\Gamma(m_i)}. \tag{2}
\]

For energy-constrained networks, the CSI can be estimated via dedicated reverse-link training from the EH receiver by employing a two-phase training-transmission protocol [26] and hence, we assume that the relay can access perfect dual-hop CSI in this study.

According to the experimental results of [29], the RSI channel incident on the receive antenna at a full-duplex node can be characterized as Rician. Nevertheless, RSI cannot be eliminated completely because of RF impairments [35] and the e2e detection performance of FDR networks is still limited by RSI [36]. To the best of our knowledge, after several complicated stages of active IC, the distribution of the RSI observed in the digital-domain is not known in practice [30], [31], [36]. In this study, the RSI channel and RSI channel power in the digital-domain after active IC are denoted by \( h_r \) and \( g_e \triangleq |h_r|^2 \), respectively. Furthermore, the normalized transmitted signals of the source and relay are denoted by \( x_s(t) \) and \( x_r(t) \), respectively, and the transmission powers at the source and relay are denoted by \( p_s \) and \( p_r \), respectively.

In the HT model, each transmission block with a duration of \( T \) is divided into two phases for EH and FDR transmission, respectively. Denoting the TS factor by \( \alpha \left(0 < \alpha < 1\right)\), the first phase assigned with a duration of \( \alpha T \) is applied for power transfer from the source to the relay. The second phase assigned with the remaining duration of \( (1 - \alpha)T \) is used for FDR transmission. The relay-received RF signals in the two phases are sent to the EH receiver and information processing (IP) receiver, respectively, as illustrated in Fig. 1. With respect to the EH receivers’ sensitivity \( S_{\text{min}} \) [37], a piecewise behavior is assumed in the HT model, i.e., the EH and IP receivers at the relay are activated only when

\[
p_s g_1 \geq S_{\text{min}},
\]

otherwise the EH and IP receivers keep silent. When (3) is satisfied, the harvested energy at the relay can be expressed as

\[
E_h = \eta_h p_s g_1 \alpha T,
\]

where \( \eta_h \) is the energy conversion efficiency depending on the rectifier circuit [37]. By utilizing the harvested energy in the first phase, the relay transmission power in the second phase is given by

\[
p_r = \frac{\eta_h E_h}{(1 - \alpha)T} \equiv \kappa p_s g_1,
\]

where \( \eta_h \in (0, 1) \) is the energy utilization efficiency and \( \kappa \triangleq \frac{\eta_h}{\gamma - 1} \). For the sake of exposition, we assume a normalized block duration in the sequel and hence, we can use the terms power and energy interchangeably.

In the FDR mode, the relay concurrently receives the signal \( y_r(t) \) and transmits the signal \( x_r(t) \) on the same frequency, as depicted in Fig. 1, where \( \tilde{n}_a^r(t) \) and \( \tilde{n}_u^r(t) \) are narrow-band Gaussian noises introduced by the receive and transmit.

Fig. 1. Block diagram of the considered FDR node.
antennas, respectively. In addition, \( \tilde{n}_c^{[r]}(t) \) and \( \tilde{n}_r^{[r]}(t) \) are baseband additive white Gaussian noises (AWGNs) caused by down-conversion and up-conversion, respectively [1]. For simplicity, the equivalent baseband noise comprising both \( \tilde{n}_c^{[r]}(t) \) and \( \tilde{n}_r^{[r]}(t) \) is modeled by the zero mean AWGN \( n_c^{[r]}(t) \) with variance \( \sigma_c^2 \), and the equivalent baseband noise comprising both \( \tilde{n}_c^{[r]}(t) \) and \( \tilde{n}_r^{[r]}(t) \) is modeled by the zero mean AWGN \( n_r^{[r]}(t) \) with variance \( \sigma_r^2 \). Then, the overall AWGN at the relay can be modeled as the zero mean AWGN \( n_r(k) \equiv n_c^{[r]}(k) + n_r^{[r]}(k) \) with variance \( \sigma_r^2 \) + \( \sigma_c^2 \). The sampled baseband signal after some stages of IC is given by

\[
y_r(k) = \sqrt{p_r}x_r(k) + n_r(k),
\]

where \( k \) denotes the symbol index, \( x_r(k) \) and \( x_r(t) \) are the sampled signals of \( x_r(t) \) and \( x_r(t) \), respectively. The transmitted signal in (6) can be expressed as

\[
x_r(t) = \sqrt{\beta}y_r(t - \tau),
\]

where \( \tau \geq 1 \) is the processing delay at the relay and \( \beta = (p_g1 + p_gr + \sigma_r^2)^{-1} \) is the normalization coefficient. The sampled received signal at the destination is given by

\[
y(d) = \sqrt{p_r}h_2x_r(k) + n_d(k),
\]

where \( n_d(k) \) is the additive noise at the destination with zero mean and variance \( \sigma_d^2 \). In this network, the e2e SINR can be expressed as

\[
\gamma_{e2e} = \frac{\gamma_h \gamma_d}{\gamma_d + \gamma_h + 1},
\]

where \( \gamma_h \equiv \frac{p_g1}{p_gr + \sigma_r^2} \) and \( \gamma_d \equiv \frac{p_g2}{\sigma_d^2} \) are the SINRs at the relay and destination, respectively.

In each transmission block, an outage event occurs when the power of the ambient RF signal at the relay is less than the EH receiver sensitivity or when the e2e SINR is less than the required threshold for correct data detection given that EH is successful. Therefore, the e2e outage probability in the HT model can be expressed as

\[
P_{out} \triangleq \Pr \{ p_s g_1 < S_{min} \}
+ \Pr \{ (p_s g_1 \geq S_{min}) \cap (\gamma_{e2e} < \gamma_{th}) \},
\]

where \( \gamma_{th} \) is the e2e SINR threshold for correct data detection at the destination. Note that in conventional AF relay networks without EH, the information outage probability is defined as

\[
P_{out}^I \triangleq \Pr \{ \gamma_{e2e} < \gamma_{th} \}.
\]

Then, the average throughput can be expressed as

\[
R_{HT} = (1 - \alpha)(1 - P_{out})R,
\]

where \( R \triangleq \log_2(1 + \gamma_{th}) \) is the fixed transmission rate. The design goal is to maximize the average throughput by optimizing TS subject to QoS constraints. The optimal \( \alpha^* \) can be obtained by solving the following maximization problem:

\[
\alpha^* = \arg \max_\alpha R_{HT}(\alpha), \quad \text{s.t. } 0 < \alpha < 1 \quad \text{and} \quad Q_i,
\]

where \( Q_i \in \{ Q_1, Q_2 \} \) is a QoS constraint as we will explain later. Notably, without considering a QoS constraint, the achievable maximum average throughput of (13) may be obtained by a large \( 1 - \alpha \) at the cost of a large \( P_{out}^I \), which greatly degrades the system performance due to a large amount of repetition transmissions [27], [38]. Therefore, the QoS requirements of decreasing outage probability [38] and maintaining a target outage probability [27] are respectively considered, i.e.,

\[
Q_1 := \{ P_{out}^I \text{ is minimized} \} \quad \text{and} \quad Q_2 := \{ P_{out}^I \leq \varepsilon \},
\]

where \( \varepsilon \) is a given target information outage probability [27]. To obtain \( \alpha^* \), the immediate task is to characterize the e2e outage probability of the system. For the sake of mathematical tractability, we focus on the RSI dominated scenario which is of practical interest [12], [14].

**Proposition 1.** Conditioned on \( g_r \), the e2e outage probability of the system in the HT model is given by

\[
P_{out} = 1 - (1 - P_{out}^I)F_{g_1}(\frac{S_{min}}{p_s}),
\]

where

\[
p_{out}^I = \left\{ \frac{2^{2-m_1-m_2}}{\Gamma(m_1)\Gamma(m_2)} D_{\mu,\nu}(2\sqrt{\xi}), \quad 0 < \alpha < \frac{1}{1+\eta g_r \gamma_{th}}, \quad 1, \right. \]

\[
\xi \triangleq \frac{\gamma_h \sigma_d^2 (1 + \kappa g_r)}{\eta p_s \theta_1 \gamma_d (1 - \kappa g_r) g_r},
\]

\[
\mu = m_1 + m_2 - 1, \quad \nu = m_1 - m_2, \quad D_{\mu,\nu}(y) = \int_0^y x^\mu \nu_\nu(x)dx, \quad \text{and} \quad F_{g_1}(x) \triangleq \Gamma_u(\frac{m_1 + \gamma/d_1}{\gamma/d_2}) \text{ is the CCDF of } g_1.
\]

**Proof.** See Appendix A. \( \square \)

Proposition 1 shows that when \( \frac{1}{1+\eta g_r \gamma_{th}} \leq \alpha < 1, \) \( P_{out} = 1. \) To avoid \( P_{out} = 1, \) it is required to set \( 0 < \alpha < \frac{1}{1+\eta g_r \gamma_{th}} \) or equivalently to eliminate the RSI by \( g_r < \frac{1}{\kappa g_r}. \) Moreover, as \( p_s \rightarrow \infty, \) we have \( \bar{F}_{g_1}(\frac{S_{min}}{p_s}) \rightarrow 1 \) and

\[
P_{out}^I \rightarrow \left\{ \begin{array}{ll}
0, & 0 < \alpha < \frac{1}{1+\eta g_r \gamma_{th}} \\
1, & \frac{1}{1+\eta g_r \gamma_{th}} \leq \alpha < 1.
\end{array} \right.
\]

Thus, \( P_{out}^I \rightarrow 0 \) and \( P_{out} \rightarrow 0 \) can be achieved by setting \( \alpha \in (0, \frac{1}{1+\eta g_r \gamma_{th}}) \) as \( p_s \rightarrow \infty. \) Furthermore, as \( p_s \rightarrow \infty \) and \( g_r \rightarrow 0, \) we can achieve \( P_{out}^I \rightarrow 0 \) and \( P_{out} \rightarrow 0 \) by setting \( \alpha \in (0, 1). \) Recalling that the effective FDR transmission time is \( 1 - \alpha, \) \( \alpha \) should be set as small as possible to achieve the allowable maximum average throughput. Thus, it can be shown that the optimal solution of (13) with the QoS constraint \( Q_1 \) is \( \alpha^* \rightarrow 0 \) as \( p_s \rightarrow \infty. \)

With the obtained \( P_{out}^I \) of (16), maximizing \( 1 - P_{out}^I \) is equivalent to maximizing \( 1 - P_{out}^I \). Then, the maximization problem (13) can be simplified as

\[
\alpha^* = \arg \max_\alpha (1 - \alpha)(1 - P_{out}^I),
\]

\[
s.t. \quad 0 < \alpha < \frac{1}{1+\eta g_r \gamma_{th}} \quad \text{and} \quad Q_i.
\]

Since outage probability and successful rate are two important metrics in delay-limited transmissions [27], [39], we also consider the successful rate for the FDR-assisted SWIPT network. According to [27], the successful rate is defined as
the product of the fixed transmission rate and the success probability. With respect to TS operation, we define the successful rate for the considered FDR-assisted SWIPT network as

$$ R_{\text{HT}}^\alpha \triangleq \begin{cases} (1 - \alpha)(1 - \varepsilon)R, & P_{\text{out}}^\alpha \leq \varepsilon, \\ 0, & \text{otherwise,} \end{cases} \quad (20) $$

where we have assumed that the physical-layer outage is fixed to the target outage probability [27]. Obviously, the successful rate is a strictly QoS-dependent metric, which represents the average throughput subject to an explicitly defined outage constraint. To achieve a non-zero successful rate, the obtained $P_{\text{out}}^\alpha$ subject to the outage constraint should be less than or equal to $\varepsilon$ when $p_s \geq S_{\text{min}}$. In the following, the optimal TS factors of (19) subject to $Q_1$ and $Q_2$ are respectively presented.

**Proposition 2.** The optimal TS factor that maximizes the average throughput subject to $Q_1$ in the HT model is given by

$$ \hat{\alpha} = \frac{1}{1 + \eta g_r(\sqrt{\gamma_{\text{th}}(\gamma_{\text{th}} + 1)} + \gamma_{\text{th}})} \quad (21) $$

and the corresponding information outage probability is given by

$$ P_{\text{out}}^\hat{\alpha} = \frac{2^{2-m_1-2m_2}}{\Gamma(m_1)\Gamma(m_2)} D_{m,\nu} \left(2\sqrt{\hat{\xi}}\right), \quad (22) $$

where

$$ \hat{\xi} \triangleq \frac{g_r\gamma_{\text{th}}\sigma_{\text{th}}^2}{p_s\theta_1\theta_2} \left(2\left(\gamma_{\text{th}} + \sqrt{\gamma_{\text{th}}(\gamma_{\text{th}} + 1)}\right) + 1\right). \quad (23) $$

**Proof.** See Appendix B.

Note that $\hat{\alpha}$ can be easily computed since it is independent of the CSI of the dual-hop channels. With the obtained $\hat{\alpha}$ and $P_{\text{out}}^\hat{\alpha}$, the average throughput can be expressed as

$$ \hat{R}_{\text{HT}} = \frac{\eta g_r(\sqrt{\gamma_{\text{th}}(\gamma_{\text{th}} + 1)} + \gamma_{\text{th}}) \left(1 - P_{\text{out}}^\hat{\alpha}\right) F_{g_1} \left(\frac{\tilde{S}_{\text{min}}}{P_s}\right) R}{1 + \eta g_r(\sqrt{\gamma_{\text{th}}(\gamma_{\text{th}} + 1)} + \gamma_{\text{th}})} \quad (24) $$

As $p_s \to \infty$, we have $P_{\text{out}}^\hat{\alpha} \to 0$, $F_{g_1} \left(\frac{\tilde{S}_{\text{min}}}{P_s}\right) \to 1$, and the asymptotic average throughput

$$ \tilde{R}_{\text{HT}} \to \frac{\eta g_r\left(\sqrt{\gamma_{\text{th}}(\gamma_{\text{th}} + 1)} + \gamma_{\text{th}}\right) R}{1 + \eta g_r\left(\sqrt{\gamma_{\text{th}}(\gamma_{\text{th}} + 1)} + \gamma_{\text{th}}\right)}. \quad (25) $$

In such a case, the asymptotic average throughput is independent of the CSI of the dual-hop channels and is a monotonically increasing function of $g_r$. Thus, a larger asymptotic average throughput is obtained with a larger $g_r$, which can alleviate the IC burden in the high $p_s$ region if $\hat{\alpha}$ is applied.

When the QoS constraint $Q_2$ is considered, we denote by $F_y^{-1}(\varepsilon)$ the solution $2\sqrt{\tilde{\xi}}$ to the equation $F_y \left(2\sqrt{\tilde{\xi}}\right) = \varepsilon$ [27], so that $\tilde{\xi} = (F_y^{-1}(\varepsilon))^2/4$. Note that we have applied the fact $P_{\text{out}}^\alpha = F_y \left(2\sqrt{\tilde{\xi}}\right)$ from (A.6) to obtain $\tilde{\xi}$; now we begin to characterize the optimal TS of (19) subject to $Q_2$.

**Proposition 3.** For a given target information outage probability $\varepsilon$, the optimal TS factor that maximizes the average throughput subject to $Q_2$ in the HT model is given by

$$ \hat{\alpha} = \begin{cases} \frac{1}{\sqrt{\eta g_r}} \sqrt{\frac{p_s\theta_1\theta_2 F_y^{-1}(\varepsilon)^2}{4\gamma_{\text{th}}\sigma_{\text{th}}^2 \left(2\gamma_{\text{th}} + \sqrt{\gamma_{\text{th}}(\gamma_{\text{th}} + 1)}\right) + 1}} \hat{\xi}, & \text{if } g_r > \hat{g}_r, \\ \hat{\xi}, & \text{if } g_r \leq \hat{g}_r, \end{cases} \quad (26) $$

where $\hat{g}_r \triangleq p_s\theta_1\theta_2 F_y^{-1}(\varepsilon)^2/4\gamma_{\text{th}}\sigma_{\text{th}}^2 \left(2\gamma_{\text{th}} + \sqrt{\gamma_{\text{th}}(\gamma_{\text{th}} + 1)}\right) + 1$ and $\hat{\xi}$ is given by (27).

**Proof.** See Appendix C.

Proposition 3 shows that $\hat{\alpha}$ does not exist and the target information outage probability cannot be achieved when $g_r > \hat{g}_r$. In such a case, the corresponding FDR transmission cannot be realized along with an e2e outage probability of 1. As can be seen from Appendix C, the information outage probability achieved by $\hat{\alpha}$ is $P_{\text{out}}^\hat{\alpha} = \varepsilon$ when $g_r \leq \hat{g}_r$. With the obtained $\hat{\alpha}$ and $P_{\text{out}}^\hat{\alpha}$, the average throughput given that $g_r \leq \hat{g}_r$ can be expressed as

$$ \tilde{R}_{\text{HT}} = \frac{(1 - \varepsilon)\eta R F_{g_1} \left(\frac{\tilde{S}_{\text{min}}}{P_s}\right)}{\eta + \hat{\xi}}. \quad (28) $$

As $p_s \to \infty$, we have $\hat{\xi} \to 0$, $F_{g_1} \left(\frac{\tilde{S}_{\text{min}}}{P_s}\right) \to 1$, $\hat{g}_r \to \infty$, and $g_r < \hat{g}_r$. Thus, the asymptotic average throughput can be expressed as

$$ \tilde{R}_{\text{HT}}^\infty \to (1 - \varepsilon)R. \quad (29) $$

The expression in (29) implicitly shows that $\hat{\alpha} \to 0$ as $p_s \to \infty$. In other words, $\hat{\alpha}$ achieves an effective FDR transmission time of a whole block as $p_s \to \infty$, so that the asymptotic average throughput of (29) is the same as that of a corresponding conventional FDR network.

**Corollary 1.** As $p_s \to \infty$, when $g_r < \frac{1 - \varepsilon}{\eta g_r(\sqrt{\gamma_{\text{th}}(\gamma_{\text{th}} + 1)} + \gamma_{\text{th}})}$, $\tilde{R}_{\text{HT}}^\infty < \tilde{R}_{\text{HT}}^\infty$; otherwise, $\tilde{R}_{\text{HT}}^\infty \geq \tilde{R}_{\text{HT}}^\infty$.

**Proof.** Comparing (25) and (29), it can be shown that $\tilde{R}_{\text{HT}}^\infty < \tilde{R}_{\text{HT}}^\infty$ has an equivalence $g_r < \frac{1 - \varepsilon}{\eta g_r(\sqrt{\gamma_{\text{th}}(\gamma_{\text{th}} + 1)} + \gamma_{\text{th}})}$. Furthermore, it can be shown that $\tilde{R}_{\text{HT}}^\infty \geq \tilde{R}_{\text{HT}}^\infty$ has an equivalence $g_r \geq \frac{1 - \varepsilon}{\eta g_r(\sqrt{\gamma_{\text{th}}(\gamma_{\text{th}} + 1)} + \gamma_{\text{th}})}$. This proves Corollary 1.

Being consistent with contemporary wireless systems where an outage level near 1% is typical [27], [39], it can be shown that $\frac{1 - \varepsilon}{\eta g_r(\sqrt{\gamma_{\text{th}}(\gamma_{\text{th}} + 1)} + \gamma_{\text{th}})} \gg 1$ with the substitution of practical $\eta$ and $\gamma_{\text{th}}$, whereas $g_r$ is less than 1 due to active/passive IC. Thus, we have $g_r < \frac{1 - \varepsilon}{\eta g_r(\sqrt{\gamma_{\text{th}}(\gamma_{\text{th}} + 1)} + \gamma_{\text{th}})}$ in practice and the asymptotic average throughput achieved by $\hat{\alpha}$ is larger than that of $\hat{\alpha}$.

### III. HARVEST-TRANSIT-STORE MODEL

In the HT model, all the harvested energy has been fully utilized for FDR transmission within each block, without considering energy accumulation and scheduling across channel realizations. Although the HT model is easy to implement, it would perform better if energy accumulation and scheduling were allowed to store a part of the harvested energy for future usage. Therefore, we propose a general HTS model with its GS implementation for the considered FDR-assisted SWIPT network. Note that a block-based HTS model has been proposed for HDR-assisted SWIPT networks in [32]. In contrast, our general HTS model achieves superior outage performance over that of the block-based HTS model. Furthermore, under
\[
K = \frac{p_s \theta_1 \theta_2 (F_y^{-1}(\varepsilon))^2 - 4g_r \gamma_\text{th} \sigma_d^2}{2p_s \theta_1 \theta_2 g_r \gamma_\text{th} (F_y^{-1}(\varepsilon))^2 - 16p_s \theta_1 \theta_2 g_r \gamma_\text{th}^2 \sigma_d^2 (F_y^{-1}(\varepsilon))^2}.
\]

(27)

Based on the considered discretized battery model, the energy that can be harvested during the first phase is defined as \( \varphi_{h1} \triangleq \varphi \gamma \), where

\[
i_{h1}^* = \arg \max_{i \in \{0, \ldots, L+1\}} \{ \varphi_i : \varphi_i < \varphi_{h1} \}
\]

and \( \varphi_{h2} \triangleq \alpha \eta_i p_s g_1 \). If the relay is operated in the EH mode \( \mu_h \) in the second phase, the energy that can be harvested is defined as \( \varphi_{h2} \triangleq \varphi_i \), where

\[
i_{h2}^* = \arg \max_{i \in \{0, \ldots, L+1\}} \{ \varphi_i : \varphi_i < \varphi_{h2} \}
\]

and \( \varphi_{h2} \triangleq (1 - \alpha) \eta_i p_s g_1 \). When the relay is operated in the FDR transmission mode \( \mu_r \) in the second phase, the relay uses the stored energy to power its transmission. Corresponding to \( p_r \) of (31), the required energy level for transmission is given by

\[
\varphi_r \triangleq \begin{cases} 
\varphi^*, & \text{if } \frac{(1-\alpha)p_r}{\eta_i} \leq p_h \\
\infty, & \text{otherwise}
\end{cases}
\]

(34)

For intermediate and high SINRs, the e2e SINR of (9) can be approximated as [40]:

\[
\gamma_{e2e} \approx \min\{\gamma_r, \gamma_\text{th}\},
\]

(30)

where \( \gamma_r = \frac{p_r \gamma_\text{th} \sigma_d^2}{\sigma_2^2} \) is the SINR at the relay in the considered RSI dominated scenario. In order to decode the relaying data received at the destination, it is required that the e2e SINR at least equals the target value \( \gamma_\text{th} \). Based on the above approximation, the required relay transmission power that ensures signal detection can be simplified to

\[
p_r = \begin{cases} 
\gamma_r \geq \gamma_\text{th}, & \text{if } \gamma_\text{r}^* \text{ does not exist,} \\
\gamma_\text{r}^* \geq \gamma_\text{th}, & \text{otherwise}
\end{cases}
\]

(31)

where \( \gamma_\text{r}^* \triangleq \frac{p_s \theta_1 \theta_2 \gamma_\text{th} \sigma_d^2}{\sigma_2^2} \) denotes the SINR at the relay given that \( p_r = \gamma_r \gamma_\text{th} \sigma_d^2 \). Notably, the consumed energy for the relay transmission is \((1 - \alpha)p_r \). In the following, we again assume the time normalization of each block, so that we can consider energy and power interchangeably. Furthermore, we assume that a rechargeable battery has been employed at the relay with the battery size \( p_h = \rho p_s \) (\( \rho > 0 \) ). The battery is discretized into \( L + 2 \) energy levels \( \varphi_i \triangleq \epsilon p_h / (L + 1) \), where \( i = 0, 1, \ldots, L + 1 \) [32], [41]. We define \( s_t, i = 0, 1, \ldots, L + 1 \) as \( L + 2 \) energy states of the battery, so that the battery is in the state \( s_t \) when its stored energy equals to \( \varphi_i \).

![Frame structure of the general HTS model](image)

(a): \( 0 < \alpha < 1 \)

(b): \( \alpha = 0 \)

For intermediate and high SINRs, the e2e SINR of (9) can be approximated as [40]:

\[
\gamma_{e2e} \approx \min\{\gamma_r, \gamma_\text{th}\},
\]

(30)

where \( \gamma_r = \frac{p_r \gamma_\text{th} \sigma_d^2}{\sigma_2^2} \) is the SINR at the relay in the considered RSI dominated scenario. In order to decode the relaying data received at the destination, it is required that the e2e SINR at least equals the target value \( \gamma_\text{th} \). Based on the above approximation, the required relay transmission power that ensures signal detection can be simplified to

\[
p_r = \begin{cases} 
\gamma_r \geq \gamma_\text{th}, & \text{if } \gamma_\text{r}^* \text{ does not exist,} \\
\gamma_\text{r}^* \geq \gamma_\text{th}, & \text{otherwise}
\end{cases}
\]

(31)

where \( \gamma_\text{r}^* \triangleq \frac{p_s \theta_1 \theta_2 \gamma_\text{th} \sigma_d^2}{\sigma_2^2} \) denotes the SINR at the relay given that \( p_r = \gamma_r \gamma_\text{th} \sigma_d^2 \). Notably, the consumed energy for the relay transmission is \((1 - \alpha)p_r \). In the following, we again assume the time normalization of each block, so that we can consider energy and power interchangeably. Furthermore, we assume that a rechargeable battery has been employed at the relay with the battery size \( p_h = \rho p_s \) (\( \rho > 0 \) ). The battery is discretized into \( L + 2 \) energy levels \( \varphi_i \triangleq \epsilon p_h / (L + 1) \), where \( i = 0, 1, \ldots, L + 1 \) [32], [41]. We define \( s_t, i = 0, 1, \ldots, L + 1 \) as \( L + 2 \) energy states of the battery, so that the battery is in the state \( s_t \) when its stored energy equals to \( \varphi_i \).
in the transition probabilities of the MC are determined in the
are the transition probabilities corresponding to an outage event
then, the transition probability for this case is given by $P_{0,0} = P_{0,0} + \tilde{P}_{0,0}$.
2) The Empty Battery Is Partially Charged ($s_0 \rightarrow s_k \rightarrow s_j$: $0 < j < L + 1$): In this case, an outage event occurs when $0 \leq k \leq j$ and a non-outage event occurs when $j + 1 \leq k \leq L + 1$. The corresponding transition probabilities are respectively given by $P_{0,j} = \sum_{k=0}^{L+1} P_{0,0}^{(1)} \tilde{P}_{k,j}^{(2)}$ and $P_{0,j} = \sum_{k=0}^{L+1} P_{0,0}^{(1)} \tilde{P}_{k,j}^{(2)}$, where $P_{0,0}^{(1)}$ is given by (40),

$$
\tilde{P}_{k,j}^{(2)} = \Pr\{(\varphi_k - \varphi_j) \leq \tilde{\varphi}_h \rightarrow \varphi_{k+1} \} = \tilde{F}_g \left( \frac{g_{\varphi_k - \varphi_j, \gamma} \sigma^2_{\varphi_j}}{p_{\varphi_k, \gamma} \gamma} \right),
$$

$$
\tilde{P}_{k,j}^{(2)} = \Pr\{(\varphi_k - \varphi_j) \leq \tilde{\varphi}_h \rightarrow \varphi_{k+1} \} = \tilde{F}_g \left( \frac{g_{\varphi_k - \varphi_j, \gamma} \sigma^2_{\varphi_j}}{p_{\varphi_k, \gamma} \gamma} \right),
$$

and

$$
P_{L+1,0} = \sum_{k=0}^{L+1} F_{k} P_{k,0}^{(1)} \tilde{P}_{k,L+1}^{(2)},
$$

where $P_{k,0}^{(1)}$ is given by (40) and $\tilde{P}_{k,L+1}^{(2)}$ is given by

$$
\tilde{P}_{k,L+1}^{(2)} = \Pr\{(\varphi_k - \varphi_{L+1}) \leq \tilde{\varphi}_h \rightarrow \varphi_{k+1} \} = \tilde{F}_g \left( \frac{g_{\varphi_k - \varphi_{L+1}, \gamma} \sigma^2_{\varphi_{L+1}}}{p_{\varphi_k, \gamma} \gamma} \right).
$$

Furthermore, the transition probability corresponding to a non-outage event is given by $P_{L+1,0} = 0$ due to the absence of discharging in the second phase. Then, the transition probability of this case can be expressed as $P_{L+1,0} = P_{L+1,0}$.

4) The Battery Remains Full ($s_{L+1} \rightarrow s_{L+1} \rightarrow s_{L+1}$): This case corresponds to the scenarios in which the battery is fully charged at the beginning of the first phase, so that the battery cannot harvest more energy. In the second phase, a) the required transmitted energy is higher than the battery size given that $\tilde{\gamma}_h \geq \gamma_{th}$ or b) $\tilde{\gamma}_h < \gamma_{th}$. In such a case, $P_{L+1,L+1} = P_{L+1,L+1}^{(1)}$ and $P_{L+1,L+1}^{(2)}$ can be obtained by substituting $k = L + 1$ and $\varphi_{L+1} = \varphi_0$ into (46). Thus, we have

$$
\tilde{P}_{L+1,L+1}^{(1)} = \tilde{F}_g \left( \frac{g_{\varphi_k - \varphi_j, \gamma} \sigma^2_{\varphi_j}}{p_{\varphi_k, \gamma} \gamma} \right) + \tilde{F}_g \left( \frac{g_{\varphi_k - \varphi_j, \gamma} \sigma^2_{\varphi_j}}{p_{\varphi_k, \gamma} \gamma} \right).
$$

and

$$
P_{L+1,L+1}^{(2)} = \Pr\{(\varphi_k - \varphi_{L+1}) \leq \tilde{\varphi}_h \rightarrow \varphi_{k+1} \} = \tilde{F}_g \left( \frac{g_{\varphi_k - \varphi_{L+1}, \gamma} \sigma^2_{\varphi_{L+1}}}{p_{\varphi_k, \gamma} \gamma} \right).
$$

A. MC for the GS Policy

For the GS policy, a specific harvesting/relaying behavior of the relay’s battery can be modeled as a specific state-transition of a finite-state MC, so that the energy level of the relay’s battery at the beginning of each block represents a specific state of the MC. Since the GS policy switches the relay to the mode $\mu_k$ in the first phase and to the mode $\mu_k$ or $\mu_k$ in the second phase, the MC model has a two-stage transition for each block.

Assume that the initial, intermediate, and final states of the battery’s energy level in each block are $s_i$, $s_k$, and $s_j$, respectively, the transitions $s_i \rightarrow s_k$ and $s_k \rightarrow s_j$ occur in the first and second phases, respectively, and the transition $s_i \rightarrow s_k \rightarrow s_j$ occurs throughout the whole block. In the first phase, we have $k \geq i$ due to the EH operation. In the second phase, we have $k > j$ if the relay is operated in the mode $\mu_k$ or $k \leq j$ if the relay is operated in the mode $\mu_k$. It can be shown that an outage event occurs when $k \leq j$ for $s_i \rightarrow s_k \rightarrow s_j$ and a non-outage event occurs when $k > j$ for $s_i \rightarrow s_k \rightarrow s_j$. The transition matrix of the MC can be denoted by $P \in \mathbb{R}^{(L+2) \times (L+2)}$ with its $i$th-row and $j$th-column element $P_{i,j}$ representing the probability of the transition from the state $s_i$ to the state $s_j$ in a transmission block. Similarly, we define $P_{i,k}^{(1)}$ as the transition probability in the first phase and define $\tilde{P}_{i,j}^{(2)}$ and $\tilde{P}_{i,k}^{(2)}$ as the transition probabilities in the second phase for $k \leq j$ and $k > j$, respectively. With respect to the two-stage state transition, $P_{i,j}$ can be written as

$$
P_{i,j} = \sum_{k \leq j} P_{i,k}^{(1)} \tilde{P}_{k,j}^{(2)} + \sum_{k > j} P_{i,k}^{(1)} \tilde{P}_{k,j}^{(2)} = \tilde{P}_{i,j} + \tilde{P}_{i,j},
$$

(39)

where $\tilde{P}_{i,j} = \sum_{k \leq j} P_{i,k}^{(1)} \tilde{P}_{k,j}^{(2)}$ and $\tilde{P}_{i,j} = \sum_{k > j} P_{i,k}^{(1)} \tilde{P}_{k,j}^{(2)}$ are the transition probabilities corresponding to an outage event and a non-outage event, respectively. Based on (39), the transition probabilities of the MC are determined in the following:

1) The Empty Battery Remains Empty ($s_0 \rightarrow s_k \rightarrow s_0$): In this case, an outage event occurs for $s_0 \rightarrow s_0 \rightarrow s_0$ and a non-outage event occurs for $s_0 \rightarrow s_k \rightarrow s_0$ with $1 \leq k \leq L + 1$. Thus, the transition probabilities corresponding to an outage event and a non-outage event are respectively given by $\tilde{P}_{0,0} = P_{0,0}^{(1)} \tilde{P}_{0,0}^{(2)}$ and $\tilde{P}_{0,0} = \sum_{k=1}^{L+1} P_{0,0}^{(1)} \tilde{P}_{k,0}^{(2)}$, where

$$
P_{0,0}^{(1)} = \left\{ \begin{array}{ll}
P_{\varphi_k \leq \varphi_{k+1}}(0), & k = 0 \\
P_{\varphi_k \geq \varphi_{k+1}}(k + 1), & k = L + 1 \\
\frac{\varphi_k - \varphi_{k+1}}{\alpha_0 p_{\varphi_k, \gamma}}, & k = L + 1
\end{array} \right.
$$

(40)

and

$$
\tilde{P}_{0,0}^{(2)} = \Pr\{\varphi_h \geq \varphi_{k+1}\} = \tilde{F}_g \left( \frac{g_{\varphi_k - \varphi_{k+1}, \gamma} \sigma^2_{\varphi_{k+1}}}{p_{\varphi_k, \gamma} \gamma} \right).
$$

(41)
Furthermore, we have $P_{L+1, L+1} = 0$ due to the absence of discharging in the second phase. Therefore, the transition probability for this case can be expressed as $P_{L+1, L+1} = P^{(2)}_{L+1, L+1}$.

5) The Non-Empty and Non-Full Battery Remains Unchanged $(s_i \to s_k \to s_i; 0 < i < L + 1)$: In this case, the transition probability corresponding to an outage event is $\tilde{P}_{i, i} = P^{(1)}_{i, i} + P^{(2)}_{i, i}$, where

$$P_{i, i}^{(1)} = \Pr\{\tilde{\varphi}_{h_i} < \varphi_1\} = F_{g_1}\left(\frac{\varphi_1}{\alpha_{h_i} p_i}\right)$$

and $\tilde{P}_{i, i}^{(2)}$ is obtained from (43) with the substitution $k = j = i$ in it, i.e.,

$$\tilde{P}_{i, i}^{(2)} = F_{g_1}\left(\frac{\varphi_1}{1 - \alpha_{h_i} p_i}\right) \left(\int_{y}^{1} g_2\left(\frac{1 - \alpha_{h_i} p_i}{\alpha_{h_i} p_i}\right) F_{\gamma}\left(\frac{g_2(\cdot) \gamma p_i \sigma_2^2}{p_i \sigma_2^2}\right) \, dy + \int_{0}^{g_2(\cdot)} F_{\gamma}\left(\frac{g_2(\cdot) \gamma p_i \sigma_2^2}{p_i \sigma_2^2}\right) \, dy\right).$$

The transition probability corresponding to a non-outage event is $\tilde{P}_{i, i} = \sum_{k=i+1}^{L+1} P_{i, k}^{(1)} \tilde{P}_{k, i}$, where

$$P_{i, k}^{(1)} = \begin{cases} \Pr\{\tilde{\varphi}_{h_i} \geq \varphi_k - \varphi_1\}, & k = L + 1 \\ \Pr\{\tilde{\varphi}_{k} - \varphi_i \leq \tilde{\varphi}_{h_i} < \varphi_k - \varphi_1\}, & \text{otherwise} \end{cases}$$

and $\tilde{P}_{k, i}$ is obtained from (44) with the substitution $j = i$ in it. Then, the transition probability for this case can be expressed as $P_{i, i} = \tilde{P}_{i, i} + \tilde{P}_{i, i}$.

6) The Non-Empty and Non-Full Battery Is Fully Charged $(s_i \to s_k \to s_{L+1}; 0 < i < L + 1)$: In this case, we have $P_{i, L+1} = 0$ due to the absence of discharging. Then, the transition probability is given by

$$P_{i, L+1} = \tilde{P}_{i, L+1} = \sum_{k=i+1}^{L+1} P_{i, k}^{(1)} \tilde{P}_{k, L+1},$$

where $P_{i, k}^{(1)}$ is given by (50) and $\tilde{P}_{k, L+1}$ is the same as that of the case 3).

7) The Non-Empty and Non-Full Battery Is Partially Charged $(s_i \to s_k \to s_j; 0 < i < j < L + 1)$: In this case, the transition probabilities corresponding to an outage event and a non-outage event are respectively given by

$$\tilde{P}_{i, j} = \sum_{k=i+1}^{j} P_{i, k}^{(1)} \tilde{P}_{k, j}$$

and

$$\tilde{P}_{j, j} = \sum_{k=j+1}^{L+1} P_{j, k}^{(1)} \tilde{P}_{k, j},$$

where $P_{i, k}^{(1)}$ is given by (50), $\tilde{P}_{k, j}$ is given by (43), and $\tilde{P}_{k, j}$ is given by (44). Then, the transition probability of this case can be expressed as $P_{i, j} = \tilde{P}_{i, j} + \tilde{P}_{j, j}$.

8) The Non-Empty Battery Is Discharged $(s_i \to s_k \to s_j; 0 \leq j < i \leq L + 1)$: In this case, we have $P_{i, j} = 0$ since the battery always discharges with respect to $j < i$. Thus, the transition probability can be expressed as

$$P_{i, j} = \tilde{P}_{i, j} = \sum_{k=i+1}^{L+1} P_{i, k}^{(1)} \tilde{P}_{k, j},$$

where $P_{i, k}^{(1)}$ is given by (50) and $\tilde{P}_{k, j}$ is given by (44).

### IV. Numerical Results

This section presents some numerical results to validate the performance results of the developed schemes. In the simulations, the EH receiver sensitivity is set as $\varphi_{\text{min}} = -27$ dB.
Table I: Simulation Parameters

<table>
<thead>
<tr>
<th>No.</th>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Carrier frequency</td>
<td>868 MHz</td>
</tr>
<tr>
<td>2</td>
<td>Fixed transmission rate $R$</td>
<td>3 bps/Hz</td>
</tr>
<tr>
<td>3</td>
<td>Target information outage probability $\varepsilon$</td>
<td>1%</td>
</tr>
<tr>
<td>4</td>
<td>$L$ at $d_0 = 1$ m</td>
<td>$-50$ dB</td>
</tr>
<tr>
<td>5</td>
<td>Distances of dual-hop links: $d_1$ and $d_2$</td>
<td>8 and 18 m</td>
</tr>
<tr>
<td>6</td>
<td>Path loss exponent $\varphi$</td>
<td>2.5</td>
</tr>
<tr>
<td>7</td>
<td>Source/relay transmit antenna gain</td>
<td>18/8 dB</td>
</tr>
<tr>
<td>8</td>
<td>Additive noise power: $\sigma_s^2 = \sigma_d^2$</td>
<td>$-90$ dB</td>
</tr>
<tr>
<td>9</td>
<td>Energy coefficients: $\eta_R$ and $\eta_t$</td>
<td>0.4 and 0.75</td>
</tr>
</tbody>
</table>

dBm [37] and the size of the relay’s battery is set as $p_b = \rho p_s$, where $\rho = m_1 \theta_1$. Considering $\varphi_1 \geq \varphi_{\min}$ in practice, we set the actual number of energy levels of the relay’s battery to $L + 2$, where $L \triangleq \min(L, |p_b/\varphi_{\min}| - 1)$. For the sake of simplicity, we use the terms general HTS model and the GS policy interchangeably in the following. Furthermore, the optimal solution of (13) without any QoS constraint is denoted by $\alpha^\ast$. Unless otherwise stated, the remaining parameters used in the simulations are given in Table 1.

Fig. 3 investigates the e2e outage probability versus $\alpha$ of the considered schemes. In Fig. 3, we set $p_s = 26$ dBm, $g_r = -10$ dB, $m_1 = 4$, and $m_2 = 2$. As observed in Fig. 3, the e2e outage probability of the HT model first decreases with increasing $\alpha$. After reaching a minimum, the e2e outage probability of the HT model begins to increase with increasing $\alpha$. Furthermore, the e2e outage probability exhibits a piecewise behavior and approaches 1 in the high $\alpha$ region, as indicated by Proposition 1. Note that the HT model suffers serious outage performance degradation in the low and high $\alpha$ regions. As expected, when $\bar{\alpha}$ is applied, the achieved e2e outage probability is $P_{\text{out}} = 0.005$, which matches the minimum value of that of the HT model. When $\bar{\alpha}$ is applied, Fig. 3 shows that the target outage probability $\varepsilon = 0.01$ has been maintained. For the general HTS model, Fig. 3 shows that $\alpha = 0$ achieves the worst outage performance over all values of $\alpha$. Note that $\alpha = 0$ corresponds to the block-based HTS model. As $\alpha$ increases from 0 to the high $\alpha$ region, it can be shown the e2e outage probability first decreases dramatically and then approaches an outage floor of 0.003. Notably, the general HTS model achieves the best outage performance over $\alpha$ (note that $\bar{\alpha}$ and $\alpha$ are fixed).

The average throughput versus $\alpha$ of the considered schemes is depicted in Fig. 4, where the simulation parameters are the same as those of Fig. 3. Fig. 4 clearly shows that the maximum average throughput of the HT model is achieved by $\alpha^\ast = 0.17$. Recalling the e2e outage performance in Fig. 3, it can be shown that the maximum average throughput of the HT model is achieved with $P_{\text{out}} \approx 0.1$, so that the outage performance seriously degrades compared to $\varepsilon = 0.01$. Although $\bar{\alpha}$ and $\alpha$ achieve a lower average throughput than the maximum point in the HT model, the corresponding outage performance is acceptable, as depicted in Fig. 3. Furthermore, $\alpha$ achieves a higher average throughput than that of $\bar{\alpha}$. For the general HTS model, Fig. 4 shows that the maximum average throughput is achieved by $\alpha = 0$. Unfortunately, we know that the corresponding outage performance is not acceptable by recalling the results of Fig. 3, so that the maximum average throughput achieved by $\alpha = 0$ is not useful. As $\alpha$ increases from 0, the achieved average throughput first decreases slowly and then maintains a fixed rate of decrease. Notably, the GS implementation of the general HTS model always achieves a higher average throughput than that of the HT model for all $\alpha$.

Fig. 5 shows the successful rate versus $\alpha$ of the considered schemes corresponding to Fig. 3 and Fig. 4. It is seen from Fig. 5 that the so-called optimal $\alpha^\ast = 0.17$ for the HT model achieves a zero successful rate due to the associated poor outage performance. Both $\bar{\alpha}$ and $\alpha$ achieve a non-zero successful rate. The GS policy achieves the highest successful rate for $\alpha$. Furthermore, the non-zero successful rate achieved by the GS policy corresponds to a wider $\alpha$ region than that of the HT model. When $\alpha = 0$, the GS policy achieves a zero successful rate. Therefore, it can be summarized that the general HTS model can maximize the successful rate among all the considered schemes by setting a non-zero TS factor.

The impact of $g_r$ on the e2e outage probability is depicted in Fig. 6, where we set $m_1 = 4$, $m_2 = 2$, and $p_s = 26$ dBm. As observed, the e2e outage probabilities of all the considered schemes increase with increasing $g_r$. For the HT model, the numerically optimized $\alpha^\ast$ achieves the worst outage performance in the low and middle $g_r$ regions. Fig. 6 also shows
that the e2e outage probability achieved by $\hat{\alpha}$ experiences a piecewise behavior, as suggested by Proposition 3. For the HT model, the best e2e outage performance is achieved by $\hat{\alpha}$. In the low $g_r$ region, $\bar{\alpha}$ results in an outage floor due to the fact that $\bar{\alpha} \to 1$ as $g_r \to 0$. For the HTS model, the GS policy with $\alpha = 0$ achieves a poor outage performance. Furthermore, in the high $p_r$ region, the GS policy with $\alpha = 0.15$ achieves the best outage performance among all the considered schemes.

The average throughput versus $g_r$ is depicted in Fig. 7, in which the simulation parameters are the same as those of Fig. 6. As observed in Fig. 7, except for $\hat{\alpha}$, the average throughputs achieved by all the schemes decrease with increasing of $g_r$. For $\bar{\alpha}$, the obtained average throughput increases with increasing $g_r$, which is consistent with the asymptotic analysis of the average throughput of Proposition 2. For the HT model, although $\alpha^*$ achieves the highest average throughput, we know that it has poor outage performance by recalling the results of Fig. 6. Fortunately, the GS policy achieves a higher average throughput than that of the HT model for all $g_r$. Note that the average throughput achieved by the GS policy with $\alpha = 0$ is higher than that of the GS policy with $\alpha = 0.15$, whereas the corresponding outage performance is worse than that of the latter.

Fig. 8 shows the successful rate versus $g_r$ of the considered schemes corresponding to Fig. 6 and Fig. 7. Fig. 8 shows that the so-called optimal $\alpha^*$ for the HT model achieves a zero successful rate. In the low and middle $g_r$ regions, the successful rate achieved by $\hat{\alpha}$ is higher than that of $\bar{\alpha}$. In the high $g_r$ region, both $\hat{\alpha}$ and $\bar{\alpha}$ achieve a zero successful rate. Notably, the GS policy with $\alpha = 0.15$ achieves the highest successful rate among all the considered schemes. Only when $g_r$ becomes larger than -2 dB, does the GS policy with $\alpha = 0.15$ achieve a zero successful rate.

Fig. 9 shows the e2e outage probability versus $p_s$. In Fig. 9, we set $m_1 = 4$, $m_2 = 2$, and $g_r = -10$ dB. As observed, the piecewise behavior occurs for the e2e outage probability achieved by $\hat{\alpha}$. For the HT model, the best outage performance is achieved by $\hat{\alpha}$ for the considered values of $p_s$. For the HTS model, the GS policy with $\alpha = 0$ approaches an outage floor with increasing $p_s$. Moreover, the best outage performance is achieved by the GS policy with $\alpha = 0.15$ in the middle and high $p_s$ regions among all the considered schemes.

The average throughput versus $p_s$ is shown in Fig. 10, where the simulation parameters are the same as those of Fig. 9. As observed in Fig. 10, $\alpha^*$ achieves the highest average throughput among all the schemes in the HT model. However, the highest average throughput achieved by $\alpha^*$ also results in a poor outage performance as depicted in Fig. 9. As expected from Corollary 1, Fig. 10 shows that $\hat{\alpha}$ achieves a higher average throughput than that of $\bar{\alpha}$ in the high $p_s$ region. For the general HTS model, Fig. 10 also shows that although the GS policy with $\alpha = 0$ achieves the highest average throughput,
the corresponding outage performance degrades seriously, as depicted in Fig. 9. Notably, in the low and middle $p_s$ regions, the GS policy with $\alpha = 0.15$ achieves a higher average throughput than all the schemes of the HT model.

Fig. 11 shows the successful rate versus $p_s$ of the considered schemes corresponding to Fig. 9 and Fig. 10. Fig. 11 shows that the so-called optimal $\alpha^*$ for the HT model achieves a zero successful rate in most of the considered $p_s$ region. For the HT model, $\hat{\alpha}$ always achieves a higher non-zero successful rate than that of $\bar{\alpha}$ for the considered range of $p_s$. Furthermore, the GS policy achieves a higher non-zero successful rate than that of $\hat{\alpha}$ in the middle $p_s$ region. However, in the high $p_s$ region, $\alpha$ achieves a higher non-zero successful rate than that of the GS policy. The reason for this phenomenon is that we set the fixed $\alpha = 0.15$ for the GS policy intuitively, whereas $\hat{\alpha}$ becomes smaller with increasing $p_s$.

V. CONCLUSION

This paper has studied QoS-constrained relay control for an FDR-assisted SWIPT network in terms of the e2e outage probability, average throughput, and successful rate. Conditioned on the RSI channel power, the e2e outage probability has been derived for the HT model. Subject to the QoS-constraints of minimizing outage probability and maintaining a target outage probability, two optimal TS factors that maximize the average throughput have been respectively presented for the HT model. To enable energy accumulation and scheduling across channel realizations, the general HTS model has been proposed by employing TS within each transmission block. To analyze the e2e outage probability of the general HTS model, the residual energy levels of the relay’s battery have been modeled as an MC with a two-stage state transition. Based on this uniform framework, the block-based HTS model can be analyzed as a special case of the general HTS model. The practical significance of the proposed QoS-constrained control schemes over the HT model without the QoS-constraints and the block-based HTS model has been verified by numerical results.

APPENDIX A: A PROOF OF PROPOSITION 1

The e2e outage probability can be rewritten as

$$P_{\text{out}}^{(\text{HT})} = \Pr \left\{ g_1 < \frac{S_{\text{min}}}{p_s} \right\} + \Pr \left\{ (g_1 \geq \frac{S_{\text{min}}}{p_s}) \cap (\gamma_{c2s} < \gamma_{th}) \right\}. \tag{A.1}$$

Since $g_1$ follows the gamma distribution, we have

$$\Pr \left\{ g_1 < \frac{S_{\text{min}}}{p_s} \right\} = F_{g_1} \left( \frac{S_{\text{min}}}{p_s} \right) = 1 - \frac{\Gamma_u \left( m_1, \frac{m_{\text{min}}}{p_s} \right)}{\Gamma(m_1)}. \tag{A.2}$$

and

$$\Pr \left\{ g_1 \geq \frac{S_{\text{min}}}{p_s} \right\} = 1 - F_{g_1} \left( \frac{S_{\text{min}}}{p_s} \right) = \frac{\Gamma_u \left( m_1, \frac{m_{\text{min}}}{p_s} \right)}{\Gamma(m_1)}. \tag{A.3}$$

Then, the task is to evaluate $P_{\text{out}}^{\text{th}} = \Pr \{ \gamma_{c2s} < \gamma_{th} \}$. By substituting (9) into $P_{\text{out}}^{\text{th}} = \Pr \{ \gamma_{c2s} < \gamma_{th} \}$, conditioned on the RSI channel power, $P_{\text{out}}^{\text{th}}$ can be expressed as

$$P_{\text{out}}^{\text{th}} = \frac{\left\{ \Pr \left\{ g_1 g_2 < \frac{\gamma_{th} \sigma^2 g_1}{\kappa \rho_1 \left( 1 - \kappa \gamma_{th} g_1 \right)} \right\}, \text{ for } g_r < \frac{1}{\kappa \gamma_{th}} \right\}}{1 - \frac{1}{\kappa \gamma_{th}}}. \tag{A.4}$$

Define $x_i = g_i / \theta_i$ for $i = 1, 2$ and $y = x_1 / x_2$. Notably, $x_i$ ($i = 1$ and 2) has the standard gamma distribution and $y$ is the product of two independent gamma variables. By utilizing the result of [43], the CDF of $y$ can be shown as

$$F_y(y) = \left( \frac{2^{m_2 - m_3} \Gamma(m_2)}{\Gamma(m_1) \Gamma(m_2)} \right) D_{\mu, \nu}(2\sqrt{y}), \tag{A.5}$$
where \( \mu = m_1 + m_2 - 1 \), \( \nu = m_1 - m_2 \), and \( D_{\mu, \nu}(y) = \int_{y}^{\infty} x^\mu K_{\nu}(x)dx \). Then, conditioned on \( g_r \), \( P^1_{\text{out}} \) can be further expressed as

\[
P^1_{\text{out}} = \begin{cases} \Pr \left( y < \frac{\gamma_{\text{th}} \sigma^2_r (1 + \kappa g_r)}{\kappa \nu \theta_1 \theta_2 (1 - \kappa g_r) + 1} \right), & g_r < \frac{1}{\kappa \gamma_{\text{th}}} \\
= \frac{F_y \left( \frac{\gamma_{\text{th}} \sigma^2_r (1 + \kappa g_r)}{\kappa \nu \theta_1 \theta_2 (1 - \kappa g_r) + 1}, 0 \right)}{1 + \eta g_r \gamma_{\text{th}}} \leq \alpha < 1 \\
= \frac{\eta_{\text{th}} \sigma^2_r (2 \sqrt{2} \xi)}{2 \theta_1 \theta_2 (2 \sqrt{\xi})}, & 0 \leq \sqrt{\xi} \leq 1 \\
= \frac{\eta_{\text{th}} \sigma^2_r (1 + \kappa g_r)}{2 \theta_1 \theta_2 (1 - \kappa g_r) + 1}, & \alpha < 1, \end{cases} \tag{A.6}
\]

where \( \xi \equiv \frac{\gamma_{\text{th}} \sigma^2_r (1 + \kappa g_r)}{\kappa \nu \theta_1 \theta_2 (1 - \kappa g_r) + 1} \). By substituting (A.2), (A.3), and (A.6) into \( P_{\text{out}} = \Pr \left( g_1 < \frac{\xi_{\text{th}}}{\rho_{\text{th}}} \right) + \Pr \left( g_1 \geq \xi_{\text{th}} \right) \), we arrive at (15).

**APPENDIX B: A PROOF OF PROPOSITION 2**

From (A.6), we know that \( P^1_{\text{out}} = 1 \) when \( \alpha \in \left[ 0, \frac{1}{1 + \eta g_r \gamma_{\text{th}}} \right] \). When \( \alpha \in \left( 0, \frac{1}{1 + \eta g_r \gamma_{\text{th}}} \right) \), \( P^1_{\text{out}} = F_y (2 \sqrt{\xi}) \), which is a non-decreasing CDF of \( y \) with respect to \( 2 \sqrt{\xi} \), where \( \xi \) is given by (17). Thus, minimizing \( P^1_{\text{out}} \) is equivalent to minimizing \( \xi \) with respect to \( \alpha \in \left( 0, \frac{1}{1 + \eta g_r \gamma_{\text{th}}} \right) \). Since there exists a one-to-one mapping from \( \alpha \in (0, 1) \) to \( \alpha \in (0, \infty) \), minimizing \( \xi \) with respect to \( \alpha \in \left( 0, \frac{1}{1 + \eta g_r \gamma_{\text{th}}} \right) \) is equivalent to minimizing \( \xi \) with respect to \( \kappa \in \left( 0, \frac{1}{1 + \eta g_r \gamma_{\text{th}}} \right) \).

The second order derivative of \( \xi \) with respect to \( \kappa \) can be expressed as

\[
\frac{\partial^2 \xi}{\partial \kappa^2} = -\frac{2 \eta_{\text{th}} \sigma^2_r (1 - 3 g_r \kappa \gamma_{\text{th}} + 3 g_r^2 \kappa^2 \gamma_{\text{th}}^2 + g_r^3 \kappa^3 \gamma_{\text{th}}^2)}{\kappa^3 p_s \theta_1 \theta_2 (-1 + g_r \kappa \gamma_{\text{th}})^3}. \tag{B.1}
\]

Based on the fact that \( \kappa \in \left( 0, \frac{1}{g_r \gamma_{\text{th}}} \right) \), we have \( -1 + g_r \kappa \gamma_{\text{th}} < 0 \) and \( 1 - 3 g_r \kappa \gamma_{\text{th}} + 3 g_r^2 \kappa^2 \gamma_{\text{th}}^2 + g_r^3 \kappa^3 \gamma_{\text{th}}^2 > 0 \). Thus, it can be shown that \( \frac{\partial^2 \xi}{\partial \kappa^2} > 0 \) and \( \xi \) is a convex function of \( \kappa \in \left( 0, \frac{1}{g_r \gamma_{\text{th}}} \right) \). By solving \( \frac{\partial \xi}{\partial \kappa} = 0 \) with respect to \( \kappa \), the achievable minimum \( \xi \) is obtained by

\[
\bar{\kappa} = \frac{1}{g_r \gamma_{\text{th}}} \left( \sqrt{\gamma_{\text{th}} (\gamma_{\text{th}} + 1)} - \gamma_{\text{th}} \right). \tag{B.2}
\]

Then, by substituting (B.2) into \( \alpha = \frac{\bar{\kappa}}{\gamma_{\text{th}}} \), we have

\[
\bar{\alpha} = \frac{1}{1 + \eta g_r (\sqrt{\gamma_{\text{th}} (\gamma_{\text{th}} + 1)} + \gamma_{\text{th}})}. \tag{B.3}
\]

which is the TS factor that achieves the minimum information outage probability. Due to the uniqueness of \( \bar{\alpha} \), it is also the TS factor that achieves the allowable maximum average throughput. By substituting (B.3) into (16), the achieved minimum information outage probability can be expressed as

\[
P^1_{\text{out}} = \frac{\eta_{\text{th}} \sigma^2_r (2 \sqrt{\xi})}{\bar{\alpha}}, \tag{B.4}
\]

where

\[
\xi = \frac{g_r \gamma_{\text{th}} \sigma^2_r}{\bar{\alpha}} \left( 2 \left( \sqrt{\gamma_{\text{th}} (\gamma_{\text{th}} + 1)} + 1 \right) \right). \tag{B.5}
\]


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