

# Iterative Tuning Feedforward Speed Estimator for Sensorless Induction Motors

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## Abstract

For speed sensorless induction motors under field-oriented control (FOC), where the motor speed and angle are not measured, the speed control tracking bandwidth is mainly limited by the convergence rate of the state estimator. Prevailing speed-sensorless induction motors suffer significant performance degradation from removing the encoder, which limits their applications to fields requiring low or medium performance. This paper studies a new estimation approach for induction motors, aiming at improving the estimation bandwidth of sensorless induction motors, and thus enabling them for higher bandwidth drives. In the speed-sensorless estimation for induction motors, the classic model reference adaptive system (MRAS) approach [1, 2, 3, 4] was initially studied and remains appealing until today. Although the design was simple, this method often suffers from slow converging due to the adaptive estimation. Through the years, numerous estimation methods have been studied for induction motors, such as sliding mode observer [5, 6], extended Kalman filter (EKF) methods [7, 8], moving horizon estimation (MHE) methods [9], etc. In these designs the rotor's mechanical equation is often not included in the estimator model, based on the assumption that the mechanical dynamics is slow compared to the electrical dynamics. This assumption can significantly simplify the estimator design and allow the estimation to proceed without knowing the motor's mechanical parameters, but often results in slow transient. In this work, we propose a new induction motor state estimation method with the rotor's mechanical dynamics included, targeting at improving the speed estimation convergence rate and thus improve the speed tracking bandwidth of the sensorless induction motor. In the proposed estimator design, the estimations of rotor flux and the rotor speed are separated into two sequential steps, where the flux estimation is achieved by a linear filter, and the speed estimation uses a combination of feedforward and feedback. In order to address the difficulty of unknown rotor inertia and load torque, an iterative tuning method was used to automatically tune the feedforward gains. Experimental results show that the proposed estimation method has improved the estimation bandwidth over the baseline MRAS and EKF methods by 20 times, and a 0.01 s rise-time was demonstrated in the speed closed-loop step response.

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# ITERATIVE TUNING FEEDFORWARD SPEED ESTIMATOR FOR SENSORLESS INDUCTION MOTORS

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## INTRODUCTION

For speed sensorless induction motors under field-oriented control (FOC), where the motor speed and angle are not measured, the speed control tracking bandwidth is mainly limited by the convergence rate of the state estimator. Prevailing speed-sensorless induction motors suffer significant performance degradation from removing the encoder, which limits their applications to fields requiring low or medium performance. This paper studies a new estimation approach for induction motors, aiming at improving the estimation bandwidth of sensorless induction motors, and thus enabling them for higher bandwidth drives.

In the speed-sensorless estimation for induction motors, the classic model reference adaptive system (MRAS) approach [1, 2, 3, 4] was initially studied and remains appealing until today. Although the design was simple, this method often suffers from slow converging due to the adaptive estimation. Through the years, numerous estimation methods have been studied for induction motors, such as sliding mode observer [5, 6], extended Kalman filter (EKF) methods [7, 8], moving horizon estimation (MHE) methods [9], etc. In these designs the rotor's mechanical equation is often not included in the estimator model, based on the assumption that the mechanical dynamics is slow compared to the electrical dynamics. This assumption can significantly simplify the estimator design and allow the estimation to proceed without knowing the motor's mechanical parameters, but often results in slow transient.

In this work, we propose a new induction motor state estimation method with the rotor's mechanical dynamics included, targeting at improving the speed estimation convergence rate and thus improve the speed tracking bandwidth of the

sensorless induction motor. In the proposed estimator design, the estimations of rotor flux and the rotor speed are separated into two sequential steps, where the flux estimation is achieved by a linear filter, and the speed estimation uses a combination of feedforward and feedback. In order to address the difficulty of unknown rotor inertia and load torque, an iterative tuning method was used to automatically tune the feedforward gains. Experimental results show that the proposed estimation method has improved the estimation bandwidth over the baseline MRAS and EKF methods by 20 times, and a 0.01 s rise-time was demonstrated in the speed closed-loop step response.

## PROBLEM FORMULATION

Assuming symmetric three phase excitation and linear magnetic circuit, the induction motor model in the stationary two-phase frame is given by

$$\begin{aligned} \dot{i}_{ds} &= -\gamma i_{ds} + \omega_1 i_{qs} + \alpha\beta\psi_{dr} + \beta\psi_{qr}\omega + u_{ds}/\sigma \\ \dot{i}_{qs} &= -\omega_1 i_{qs} - \gamma i_{ds} - \beta\psi_{dr}\omega + \alpha\beta\psi_{qr} + u_{qs}/\sigma \\ \dot{\psi}_{dr} &= \alpha L_m i_{ds} - \alpha\psi_{dr} + (\omega_1 - p\omega)\psi_{qr} \\ \dot{\psi}_{qr} &= \alpha L_m i_{qs} - (\omega_1 - p\omega)\psi_{dr} - \alpha\psi_{qr} \\ \dot{\omega} &= \frac{\mu}{J}(\psi_{dr}i_{qs} - i_{ds}\psi_{qr}) - \frac{T_L}{J} \\ y &= [i_{ds}, i_{qs}]^T, \end{aligned} \quad (1)$$

where the notation is defined in Table 1. Speed sensorless estimation problem for induction motor is roughly formulated as: design an estimator to reconstruct the full state of the induction motor system from measuring only the stator currents ( $i_{ds}, i_{qs}$ ) and voltages ( $u_{ds}, u_{qs}$ ).

## INDUCTION MOTOR STATE ESTIMATOR

Fig. 1 shows a block diagram of the proposed induction motor state estimator. Let us define state vector  $x = [i_{ds}, i_{qs}, \psi_{dr}, \psi_{qr}]^T$ . In the estimator design shown in Fig. 1, the current and flux estimation and the rotor speed estimation are decoupled. By separating out the mechanical dynamic equation and regarding the rotor speed  $\omega$

TABLE 1. Notations.

| Notation                           | Description                            |
|------------------------------------|--|
| $\hat{i}_{ds}, \hat{i}_{qs}$       | stator currents in $d$ - and $q$ -axes |
| $\hat{\psi}_{dr}, \hat{\psi}_{qr}$ | rotor fluxes in $d$ - and $q$ -axes    |
| $\omega$                           | rotor angular speed                    |
| $u_{ds}, u_{qs}$                   | stator voltages in $d$ - and $q$ -axes |
| $\omega_1$                         | angular speed of the rotating frame    |
| $T_L$                              | load torque                            |
| $J$                                | rotor inertia                          |
| $L_s, L_m, L_r$                    | stator, mutual, rotor inductances      |
| $R_s, R_r$                         | stator and rotor resistances           |
| $\sigma$                           | $(L_s L_r - L_m^2)/L_r$                |
| $\alpha$                           | $R_r/L_r$                              |
| $\beta$                            | $L_m/(\sigma L_r)$                     |
| $\gamma$                           | $R_s/\sigma + \alpha\beta L_m$         |
| $\mu$                              | $3L_m/(2L_r)$                          |

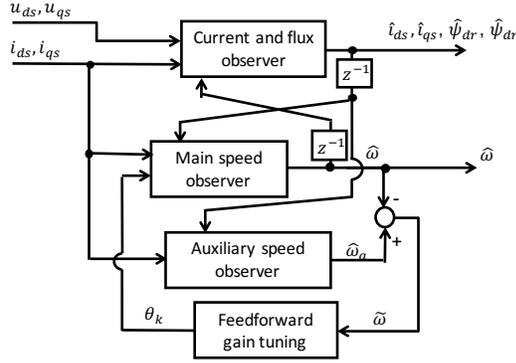


FIGURE 1. Block diagram of the induction motor state estimator.

as a time-varying parameter, the nonlinear induction motor model in (1) is reduced to

$$\begin{bmatrix} \dot{i}_{Ds} \\ \dot{i}_{Qs} \\ \dot{\Phi}_{Dr} \\ \dot{\Phi}_{Qr} \end{bmatrix} = \begin{bmatrix} -\gamma & 0 & \alpha\beta & \beta\omega \\ 0 & -\gamma & -\beta\omega & \alpha\beta \\ \alpha L_m & 0 & -\alpha & -\omega \\ 0 & \alpha L_m & \omega & -\alpha \end{bmatrix} \begin{bmatrix} i_{Ds} \\ i_{Qs} \\ \Phi_{Dr} \\ \Phi_{Qr} \end{bmatrix} + \begin{bmatrix} \frac{1}{\sigma} & 0 \\ 0 & \frac{1}{\sigma} \\ 0 & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} u_{Ds} \\ u_{Qs} \end{bmatrix}$$

$$y = [i_{Ds} \ i_{Qs}]^T.$$

The current and flux estimation for the linear induction motor model can be achieved by means of observers for linear systems, such as Luem-berger observer and Kalman Filter.

The proposed speed estimator is consist of two parts: feedforward and feedback. The discrete-time speed estimation equation is given by

$$\hat{\omega}_k = \hat{\omega}_{k-1} + dt \left( \underbrace{\theta_k^1 (i_{qs} \hat{\psi}_{dr} - i_{ds} \hat{\psi}_{qr})}_{\text{feedforward term}} - \underbrace{K_{fb} (\tilde{i}_{qs} \hat{\psi}_{dr} - \tilde{i}_{ds} \hat{\psi}_{qr})}_{\text{feedback term}} \right), \quad (2)$$

where  $\hat{\omega}_k$  is the estimated rotor speed at the sam-ple instant  $k$ ;  $dt$  is the sample time;  $\theta_k^1$  and  $\theta_k^2$  are the feedforward gains;  $K_{fb}$  is the feedback gain; state variables with a hat denote their es-timated values, and the tilde notation indicates the estimation errors, such as  $\tilde{i}_{ds} = i_{ds} - \hat{i}_{ds}$  and  $\tilde{i}_{qs} = i_{qs} - \hat{i}_{qs}$ . In the following sections, the design details of the speed estimator are introduced.

### Feedforward speed estimation

The objective of the feedforward signal in the speed estimator is to ensure that the speed es-timation matches the rotor's dynamic equation. Compare the rotor dynamics in (1) and the speed estimator equation (2), one can deduce that with feedforward gains  $\theta_k^1 = \mu/J$  and  $\theta_k^2 = T_L/J$  will enable the speed estimate to track the rotor speed under the condition that the flux estimation has converged.

The inclusion of the feedforward term significantly improves the transient performance of the speed estimation. However, in practical applications, the motor mechanical system parameters, such as the rotor inertia and load torque, are often not ex-actly known. To address this difficulty, we use an iterative tuning speed estimator to tune the feed-forward controller gains automatically.

The iterative controller tuning is a method to fine-tune controllers in a repetitive process using only data collected in experiment runs. In this method, the controller parameters are chosen to minimize a certain cost function, and the values of the pa-rameters are updated iteratively with a gradient search. This algorithm has been widely used in the feedforward controller tuning for precision motion systems such as linear motors and wafer stages [10, 11, 12], and reference [13] provides a general overview of the algorithm and its applica-tions.

The tuning of the feedforward gain relies on the optimization of a certain objective function, which should represent the performance of the speed estimator. For controller tunings, the cost func-tion is often selected as the tracking error. For the induction motor speed estimation problem, let us define a vector of the feedforward gains  $\theta_k = [\theta_k^1, \theta_k^2]^T$ . If the motor speed could be di-rectly measured, the objective function could be selected as

$$V(\theta_k) = \tilde{\omega}_k^T \tilde{\omega}_k,$$

where  $\tilde{\omega}_k = \omega_k - \hat{\omega}_k$ , denoting the speed estima-tion error.

In the speed sensorless estimation circumstance, the motor speed is not available for direct measurement. In our design, the speed estimation from an auxiliary speed observer was selected as a ground truth signal, and the error between the two speed estimations is used in the cost function for tuning the feedforward gain in the speed estimator.

As is shown in Fig. 1, an auxiliary speed observer is running in parallel with the speed observer, and gives the estimated speed  $\hat{\omega}_a$ . One possible selection of the auxiliary speed observer is the adaptive estimation method introduced in [3]. In this method, the speed estimation was given by

$$\hat{\omega}_a = K_P(\tilde{i}_{ds}\hat{\psi}_{qr} - \tilde{i}_{qs}\hat{\psi}_{dr}) + K_I \int (\tilde{i}_{ds}\hat{\psi}_{qr} - \tilde{i}_{qs}\hat{\psi}_{dr}) dt,$$

where  $K_P$  and  $K_I$  are positive gains of the adaptive speed observe.

Using the auxiliary estimated speed  $\omega_a$  as an reference signal in the tuning of the feedforward term, the corresponding cost function can be written as

$$V(\theta_k) = (\hat{\omega}_a - \hat{\omega})^T (\hat{\omega}_a - \hat{\omega}). \quad (3)$$

Note that in this objective function the two speeds are all estimated.

It is assumed that no constraints are present on the selection of the feedforward estimation gain, *i.e.*, the optimization problem for searching the feedforward gains is unconstrained. Since the difference between the two estimated speed  $\hat{\omega}_a - \hat{\omega}$  is a linear function of the feedforward estimation gain  $\theta_k$ , the objective function in (3) is convex, which implies that the global optimal is achievable. The value of the feedforward gain can be updated iteratively using the gradient based search method, as

$$\theta_{k+1} = \theta_k - \alpha_k R_k^{-1} \frac{\partial V(\theta_k)}{\partial \theta_k},$$

where  $\theta_k$  is the feedforward gain at the  $k$ -th iteration,  $\alpha_k$  is the step size,  $R_k$  is a matrix to modify the search direction, and  $\frac{\partial V}{\partial \theta_k}$  is the gradient of the objective function with respect to the feedforward gain evaluated at the present value of  $\theta_k$ .

### Feedback speed estimation

In the design of the feedback term in the speed estimator, a nonlinear feedback gain  $K_{fb}$  was designed to address the trade-off between (a) the

requirement of using high feedback gain during transient for bandwidth improvement, and (b) the requirement of using small feedback gain during steady state operation for mitigating measurement noise and model mismatches.

Let us define the feedback term  $e_T = \tilde{i}_{ds}\hat{\Phi}_{qr} - \tilde{i}_{qs}\hat{\Phi}_{dr}$ . The signal  $e_T$  contains the information of the motor operation mode.

When the motor is in steady state running, the feedback signal  $e_T$  demonstrates a periodic oscillation slightly above and below its mean value, and the fundamental frequency of this periodic oscillation is the rotational frequency of the induction motor. This speed ripple is because of the parametric model error, un-modeled motor dynamics, and inherent induction motor torque ripple. During the steady operation, the feedback gain in the speed estimator should be small to avoid magnifying this undesired AC component in the feedback signal, since it will result in large harmonic oscillation in the estimated speed.

On the other hand, during motor speed transient, *e.g.*, when the motor is accelerating or decelerating, the signal  $e_T$  presents relatively large magnitude, and will converge to its steady state value after the transient. During the motor speed transient, a large feedback gain is desired to increase the speed estimation bandwidth.

Based on the aforesaid observations, the feedback gain in the speed estimation  $K_{fb}$  is designed as a function of the feedback term  $e_T$ , which has a small amplitude when  $e_T$  is smaller than its typical magnitude, and has a large amplitude when  $e_T$  is around its typical value of during transient. In our implementation, the feedback gain  $K_{fb}$  is selected to be the form of

$$K_{fb} = \min(a_1 + a_2 e_T^2 + a_3 e_T^4, M_{fb}/e_T),$$

where  $a_1, a_2, a_3, M_{fb}$  are positive constants, and  $M_{fb}$  is the maximum magnitude of the feedback term  $K_{fb}e_T$ .

### EXPERIMENTAL VALIDATIONS

The proposed estimation method was experimentally tested with a 250 W three-phase induction motor. Control algorithm is compiled by Matlab/Simulink, and downloaded to dSPACE DS1104 for realtime operation. PWM inverter was used to energize the motor. The control loop runs at a sample frequency of 4 kHz, and the PWM frequency is also 4 kHz. The induction motor being

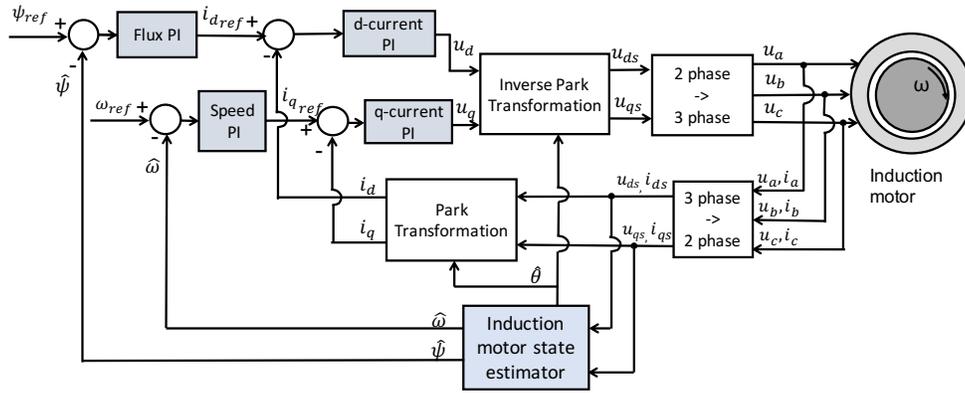


FIGURE 2. Block diagram of the induction motor field-oriented control (FOC).

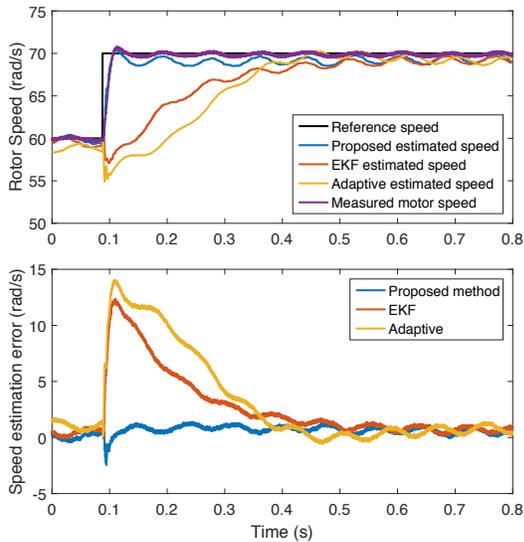


FIGURE 3. Upper plot: reference, measured, and the estimated speed of the induction motor. Lower plot: speed estimation error of the proposed method and the baseline methods.

tested has parameter values:  $R_s = 11.05 \Omega$ ;  $R_r = 6.11 \Omega$ ;  $L_s = L_r = 0.3165 \text{ H}$ ;  $L_m = 0.2939 \text{ H}$ ;  $J = 5 \times 10^{-4} \text{ kgm}^2$ . No load was added during the test.

The tracking controller implements a standard indirect field oriented control (IFOC) as is shown in Fig. 2. Four proportional and integral (PI) controllers are used to regulate speed, rotor flux amplitude, and stator currents in d- and q-axis, respectively. The rotor flux magnitude estimate  $\hat{\psi}$  is achieved by  $\hat{\psi}_r = (\hat{\psi}_{dr}^2 + \hat{\psi}_{qr}^2)^{1/2}$ , and in the speed and flux control the estimated signals are used for feedback.

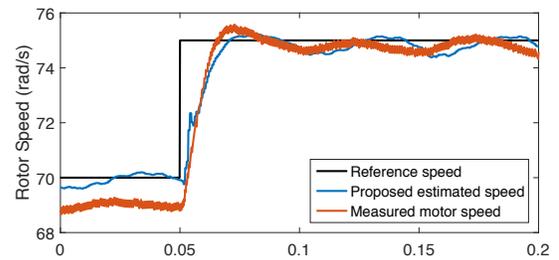


FIGURE 4. Sensorless induction motor step response with the estimated speed used for feedback.

Fig. 3 shows the speed step response of the induction motor and the estimated speed using different methods. In this experiment, the motor speed measured with an optical encoder was used for feedback for a fair comparison between the estimation schemes. These data were taken after the feedforward gain auto-tuning has converged. The proposed method was compared with the adaptive estimator (MRAS) and extended Kalman filter (EKF). It can be seen that in the speed estimation error plot, the estimation error of the proposed method was shooting downwards during transient, which implies that the speed estimation is leading the true motor speed. This is a direct effect of the feedforward terms. Compared with both baseline estimation methods, the proposed speed estimation converges much faster: the transient converges within 0.015 s, while the baseline EKF and adaptive methods use about 0.3 s to converge.

Fig. 4 shows the sensorless induction motor's speed step response when the motor's speed control loop uses the estimated speed for feed-

back. In this experiment, with the proper controller and estimator tuning, the induction motor speed step response demonstrated a rise time of 0.01 s, which enables the motor to be used for mechatronic systems with relatively fast dynamics requirements.

### CONCLUSION AND FUTURE WORK

In this paper, a new state estimation method for sensorless induction motors was proposed and tested. The proposed speed estimation method uses a combination of feedforward and feedback, where the feedforward gains are automatically tuned using an iterative tuning algorithm, and the feedback gains are designed to be nonlinear to better balance the trade-off between the high bandwidth and low noise requirements. Experimental results demonstrate that the proposed method can improve the speed estimation convergence rate significantly, and the speed control of the sensorless induction motor with the proposed estimation method has demonstrated a rise time of 0.01 s.

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