Performance Analysis of a Cooperative Wireless Network with Unreliable Backhaul Links

Khan, T.; Kim, K.J.; Orlik, P.V.

TR2015-095  June 2015

Abstract

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2015 IEEE Communications Letters
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Talha Ahmed Khan, Student Member, IEEE, Philip Orlik, Senior Member, IEEE,
Kyeong Jin Kim, Senior Member, IEEE, and Robert W. Heath Jr, Fellow, IEEE

Abstract—A cooperative wireless network, where a cluster of $K$ single-antenna transmitters jointly serve a single-antenna receiver, is considered. Each transmitter is connected to the control unit (CU) via independent but unreliable backhaul links. The CU sends a common message to each transmitter over backhaul links, which upon successful reception, jointly transmit this message to the intended receiver. To facilitate analysis, a general expression is derived for the complementary cumulative distribution function of a sum of $K$ independent random variables, where each random variable is a product of an exponential and a bernoulli random variable. This result is applied to find a simple closed-form expression that characterizes the system outage performance as a function of network parameters and node geometry. The analytical model is validated using numerical simulations. As an application, the derived expression is also used for investigating the impact of backhaul assignment on the system performance.

Index Terms—Cooperative wireless networks, backhaul, bernoulli-weighted exponential.

I. INTRODUCTION

Wireless network infrastructure is being densely deployed to provide higher area spectral efficiency, in both cellular and wireless local area networks [1]. Providing wired backhaul to this infrastructure, however, remains a challenge. This can be attributed to the excessive capital required for wired backhaul deployment as well as to the cost of leasing existing backhaul. Wireless backhaul comes across as an alternative. Unfortunately, it is unlikely to be nearly as reliable as wired backhaul [2]. The problem is further compounded by topology and access-related issues, suggesting that many links will be non-line-of-sight (non-LOS), making them even more vulnerable to fading [3]. This marks a departure from conventional wireless networks which have traditionally been assumed to have highly reliable (fiber, ethernet or LOS) backhaul links.

Classical literature on base-station (BS) cooperation typically subsumes ideal error-free data-pipes for backhauling the BSs to the cloud or network backbone [4]–[6]. With the emergence of nontraditional backhaul links, these classical models need to be revisited. To this end, several papers have studied the impact of finite capacity backhaul on system performance and proposed optimal compression schemes (see [5] and references therein). In another line of work [7], game theory has been applied for studying the impact of heterogeneous backhaul on the downlink performance of a cooperative femtocell network. Similarly, networks with unreliable backhaul links have also been investigated [8]. In [8], the downlink of a coordinated multi-point system with unreliable backhaul links was considered. It was shown that unreliable backhaul could severely limit the performance gains promised by cooperation. Most previous work, however, relies on exhaustive simulations for performance analysis.

In this paper, we present an analytical framework for the performance analysis of a certain cooperative wireless network with unreliable backhaul links. To facilitate analysis, we derive the exact complementary cumulative distribution function (CCDF) of a sum of bernoulli-weighted exponential random variables. Applying this result, we find a simple closed-form expression to characterize the outage performance at the receiver under Rayleigh fading. Using this analytical expression, we investigate the impact of backhaul assignment on outage performance. Our framework is general as it can be leveraged for analyzing other cooperative setups where similar distributions may arise (e.g., in random access networks).

II. SYSTEM MODEL

We consider a portion of an orthogonal frequency division multiple access (OFDMA) based wireless network where a cluster of $K$ single-antenna transmitters (or nodes) attempt to send a common message to a stationary single-antenna user (receiver) over the same time-frequency resource block. Each node is connected to the cloud or control unit (CU) via dedicated but unreliable backhaul links [8]. In our transmission scheme, a node only transmits if it can successfully fetch the source message from the CU before the start of the next resource block. Let us define $\beta_i$, the backhaul reliability for node $i$, i.e., with probability $\beta_i$, node $i$ successfully decodes the source message sent over the backhaul link (before the start of the next resource block), whereas the message is erased with probability $1 - \beta_i$ due to unreliable backhaul. We assume the erasures to be independent across messages and model it using a bernoulli distribution $\text{Bern}(\beta_i)$. Here, the term backhaul is rather an abstraction and can also be used to model different operating conditions contributing to link failure. For instance, one could incorporate network congestion, hardware...
imperfections, etc. by defining $\beta$ to be a function of these parameters.

Let us define $\{x_i\}_{i=1}^{K}$ to be the distances of the respective nodes from the user. We assume that the transmitter-receiver links undergo independent and identically distributed (IID) narrowband Rayleigh fading such that the corresponding channel power gains $\{H_i\}_{i=1}^{K}$ are exponential with unit means. Note that our approach can be generalized to the case where one or more channels have distinct means. In our cooperation model, the cooperating transmitters jointly transmit the same data to a single user using OFDM. Due to practical challenges, we do not assume any tight synchronization among the cooperating nodes. Unlike the transmitters, for which we do not assume any instantaneous channel knowledge, the receiver is required to know the composite downlink channel from the transmitters to perform coherent detection. With such a non-coherent joint transmission scheme (see [6, Appendix A] for details), the signal-to-noise ratio (SNR) at the receiver can be characterized as

$$\gamma \triangleq \frac{\sum_{i=1}^{K} \mathbb{E}[\eta_i H_i x_i^{-\eta}]}{\sigma^2} = \sum_{i=1}^{K} \mathbb{E}[\eta_i H_i x_i^{-\eta}]$$  \hspace{1cm} (1)

where $\mathbb{E}[\eta_i H_i x_i^{-\eta}]$ and $\mathbb{E}[\eta_i H_i]$ denote the transmit power at node $i$. The indicator function is used to model the unreliability of backhaul links such that $\Pr[\eta_i = 1] = \beta_i$ and $\Pr[\eta_i = 0] = 1 - \beta_i$. The receiver noise is assumed to be zero-mean complex Gaussian with variance $\sigma^2$. At the user, we define the probability of success $p_s(K, \{\beta_i\}_{i=1}^{K}, \theta) = \Pr[\gamma > \theta]$ as a function of the cluster size $K$, the transmit SNRs $\{\beta_i\}_{i=1}^{K}$, and the outage threshold $\theta$ as

$$p_s(K, \{\beta_i\}_{i=1}^{K}, \theta) = \Pr\left(\sum_{i=1}^{K} \mathbb{E}[\eta_i H_i x_i^{-\eta}] > \theta\right).$$ \hspace{1cm} (2)

With $\hat{H}_i \triangleq \mathbb{E}[\eta_i H_i x_i^{-\eta}]$, note that the term $\sum_{i=1}^{K} \hat{H}_i H_i x_i^{-\eta}$ in (2) consists of a sum of independent bernoulli-weighted exponential random variables, i.e., $H_i \sim \exp(\beta_i x_i^{-\eta})$. In the next section, we derive a generalized closed-form expression for the distribution of a sum of independent bernoulli-weighted exponential random variables. It is then used for characterizing the outage performance at the receiver.

### III. Sum of Bernoulli-weighted Exponentials

Consider a sum of $K$ independent bernoulli-weighted exponential random variables $\{\{e_{i}\}_{i=1}^{K}\}$ such that $S_K = \sum_{i=1}^{K} e_{i}$ with $e_{i} \overset{\Delta}{=} z_{i} G_{i}$. Here, $z_{i} \sim \text{Bern}(p_i)$, $p_i \overset{\Delta}{=} 1 - q_i$ and independent across $i$. Note that we do not require $\{z_{i}\}_{i=1}^{K}$ to have distinct means. Independently of $\{z_{i}\}_{i=1}^{K}$, we define independent random variables $\{G_{i}\}_{i=1}^{K}$ such that $G_{i} \sim \exp(\lambda_{i})$, $\lambda_{i} \overset{\Delta}{=} [\lambda_{1}, \cdots, \lambda_{K}]$, and $\Lambda$ has $\tau$ unique entries. Note that $1 \leq \tau \leq K$, where $\tau = 1$ when $\lambda_{i}$ are equal and $\tau = K$ when $\lambda_{i}$ are distinct. We further define $\{\delta_{i}\}_{i=1}^{K}$ to be the set of all unique elements of $\Lambda$, where $\delta_{i}$ has multiplicity $r_{i}$ in $\Lambda$. For ease of exposition, we hereby define $\Lambda \overset{\Delta}{=} [\lambda_{q_{1}}, \cdots, \lambda_{q_{K}}]$ and $Q \overset{\Delta}{=} \prod_{i=1}^{K} q_{i}$.

**Theorem 1:** For $S_K$, a sum of $K$ independent bernoulli-weighted exponential random variables (as defined above), the CCDF $F_{S_K}(\theta) = \Pr\{S_K > \theta\} =$

\[
Q \sum_{u=1}^{r} \sum_{v=1}^{r} \left( \sum_{m=0}^{K-1} \left( \alpha_{m}(\Lambda) - \alpha_{m}(\Lambda) \right) \right) \sum_{n_u=1}^{m} \frac{Q(v, \delta_{u} \theta)}{\delta_{u} \theta} \prod_{j \neq u} \left( r_{j} + n_{j} - 1 \right) \delta_{j} \theta^{-(r_{j}+n_{j})}. \hspace{1cm} (3)
\]

For $\theta \geq 0$, we have

\[
\sum_{u=1}^{r} \sum_{v=1}^{r} \left( \sum_{m=0}^{K-1} \left( \alpha_{m}(\Lambda) - \alpha_{m}(\Lambda) \right) \right) \sum_{n_u=1}^{m} \frac{Q(v, \delta_{u} \theta)}{\delta_{u} \theta} \prod_{j \neq u} \left( r_{j} + n_{j} - 1 \right) \delta_{j} \theta^{-(r_{j}+n_{j})}. \hspace{1cm} (4)
\]

The summation in (4) is taken over all possible combinations of non-negative integers $n_{1}, \cdots, n_{K}$ that add up to $r_{u} + v$. Moreover, $Q(a, b) = \frac{1}{\Gamma(a)} \int_{b}^{\infty} e^{-t} \text{dt}$ denotes the regularized upper incomplete Gamma function. Furthermore,

$$c_{\Lambda}^{K-i} \overset{\Delta}{=} \prod_{i=1}^{K} \left( \frac{K}{i} \right)$$, \hspace{1cm} (5)

and $+ \sum_{i=1}^{\tau} \left( \frac{K}{i} \right)$ returns the sum of the elements of the set that it operates on. With a slight abuse of notation, $\left( \frac{K}{K-i} \right)_{\Lambda}$ is defined to be the set of all products of the elements of $\Lambda$ taken $K-i$ at a time.

The summation in (6) is taken over the elements of the set $\{ \frac{K}{K-i} \}_{\Lambda}$ returned by (5). Follow (5), (6) with the set $\Lambda$ now replaced by $\hat{\Lambda}$.

**Proof:** Please see Appendix A.

We now give an example to further clarify the notation. For $K = 3$ and $\Lambda = [\lambda_{1}, \lambda_{2}, \lambda_{3}]$, we have $\left( \frac{3}{1} \right)_{\Lambda} = [\lambda_{1}, \lambda_{2}, \lambda_{3}]$, $\left( \frac{3}{2} \right)_{\Lambda} = [\lambda_{1}, \lambda_{2}, \lambda_{3}, \lambda_{1}, \lambda_{3}]$ and $\left( \frac{3}{3} \right)_{\Lambda} = [\lambda_{1}, \lambda_{2}, \lambda_{3}, \lambda_{1}, \lambda_{3}, \lambda_{2}]$.

**Special Cases:** Some limiting cases of Theorem 1 are given below (for $\theta \geq 0$).

- $\{p_{i}\}_{i=1}^{K} \overset{\Delta}{=} p \rightarrow 1$. It is worth noting that as $p \rightarrow 1$, we can retrieve the well-known expression for the sum of $K$ independent exponential random variables as given in [9].
- $\theta \rightarrow 0$. Recall that $S_K$ is a mixed random variable with $\Pr\{S_K = 0\} = Q$ and $\Pr\{S_K > 0\} = 1 - Q$. As $\theta \rightarrow 0$, the expression in (3) simplifies to $F_{S_{k}}(0) = 1 - Q$.
- $K = 1$. The expression in (3) simplifies to $F_{S_{i}}(\theta) = (1 - q_{1}) e^{-\lambda_{1} \theta}$.
- $\theta \rightarrow 0$. Recall that $S_K$ is a mixed random variable with $\Pr\{S_K = 0\} = Q$ and $\Pr\{S_K > 0\} = 1 - Q$. As $\theta \rightarrow 0$, the expression in (3) simplifies to $F_{S_{k}}(0) = 1 - Q$.
• \( \tau = K \). When the exponential random variables have distinct means, i.e., \( \lambda_i \neq \lambda_j \) \( \forall \ i \neq j \), the expression in (3) simplifies to (for \( \theta \geq 0 \))
\[
F_{S_K}^c(\theta) = Q \sum_{j=1}^{K-1} \left( \sum_{i=0}^{j-1} \frac{\alpha_i(\hat{\Lambda}) - \alpha_i(\Lambda)}{\lambda_i \left( \prod_{l \neq j} \lambda_l - \lambda_j \right)} \right) e^{-\lambda_j \theta}.
\]

(7)

• \( \tau = 1 \). When the exponential random variables have identical means, i.e., \( \{\lambda_i\}_{i=1}^K \overset{\Delta}{=} \lambda \), the expression in (3) reduces to (for \( \theta \geq 0 \))
\[
F_{S_K}^c(\theta) = Q \sum_{i=0}^{K-1} \psi_i \sum_{j=0}^{i} (-1)^j \binom{i}{j} Q(K - j, \lambda \theta)
\]

(8)

where \( \psi_i = \frac{\lambda^i - \lambda^i}{K} \), and \( \alpha_i(\cdot) \) is as given in (5) and \( Q(\cdot, \cdot) \) denotes the regularized upper incomplete Gamma function.

IV. SIMULATIONS AND APPLICATIONS

A. Validation

In this subsection, we use simulations to validate the closed-form expression given in Theorem 1. Fig. 1 plots the CCDF of \( S_K \) for various values of \( K \). It can be seen that the simulation results are in complete agreement with the analytical results.

![Fig. 1. Analytical (anlt) and simulation (sim) results for the CCDF of \( S_K \) for various values of \( K \) with \( \Lambda = [10, 10, 0.1, 0.01] \), and \( \{p_1 = 0.8, p_2 = 0.7, p_3 = 0.6, p_4 = 0.5\} \).](image)

B. Applications

In this subsection, we apply the derived result to model the network described in Section II. Let us define \( B = \{\rho_1^{-1}x_1^\eta, \cdots, \rho_{K-1}^{-1}x_{K-1}^\eta\} \), \( \hat{B} = \{\rho_1^{-1}x_1^\eta + \rho_2^{-1}x_2^\eta, \cdots, \rho_{K-1}^{-1}x_{K-1}^\eta + \rho_K^{-1}x_K^\eta\} \) and \( \hat{B} = \prod_{i=1}^{K-1} (1 - \beta_i) \). Using (7), the success probability can be compactly expressed as \( p_s(K, \{\rho_i\}_{i=1}^K, \theta) = \)
\[
\hat{B} \sum_{j=1}^{K} \left( \frac{\sum_{i=0}^{j-1} \alpha_i(\hat{\Lambda}) - \alpha_i(\Lambda)}{\rho_j^{-1}x_j^\eta \left( \prod_{l \neq j} \rho_l^{-1}x_l^\eta - \rho_j^{-1}x_j^\eta \right)} \right) e^{-\rho_j^{-1}x_j^\theta}. \]

(9)

While (9) is useful for analyzing clusters that have asymmetric geometries, we also consider the case when the cluster geometry is symmetric, i.e., \( \{x_i\}_{i=1}^K \overset{\Delta}{=} d \). Using (8), the success probability for the symmetric case can be expressed as
\[
p_s(K, \rho, \theta) = \hat{B} \sum_{i=0}^{K-1} \psi_i \sum_{j=0}^{i} (-1)^j \binom{i}{j} Q(K - j, \rho^{-1}d^\eta \theta)
\]

(10)

where \( \psi_i = (\rho^{-1}d^\eta)^{i-K} \alpha_i(\hat{B}) - (-1)^i \binom{i}{K} \), and the nodes are assumed to transmit with the same power, i.e., \( \{\rho_i\}_{i=1}^K = \rho \) (from here on). In Fig. 2, we plot the CCDF of \( \gamma \) (same as \( p_s(K, \rho, \theta) \)) for the symmetric case for various values of the cluster size \( K \). There is a complete match between the analytical and simulation results. Moreover, this figure shows that the performance is limited by the product of backhaul unreliabilities (\( \hat{B} \)) in the outage regime since \( p_s \) converges to \( 1 - \hat{B} \) as \( \theta \) vanishes. For example, for \( K = 3 \), \( p_s \) converges to 0.958 which is the same as \( 1 - \hat{B} \). Furthermore, it can be seen that the success probability increases with \( K \) due to an additional diversity gain.

![Fig. 2. The success probability for various values of \( K \) with a symmetric node geometry. Analytical (anlt) results match the simulation (sim) results. Simulation parameters are \( \rho = 10, \eta = 3.6, \{x_i\}_{i=1}^K = 3 \), and \( \beta_i \in \{0.7, 0.65, 0.6, 0.55\} \).](image)

We now consider a backhaul assignment problem at the CU for the asymmetric case and apply the result in (9) for analysis. Assume that the CU has information about node geometry \( \{x_i\}_{i=1}^K \) and backhaul reliability \( \{\beta_i\}_{i=1}^K \). It, however, has no knowledge about the instantaneous fading realization \( \{h_i\}_{i=1}^K \). How should the backhaul resources \( \{\beta_i\}_{i=1}^K \) be assigned to different nodes in order to maximize the success probability at the user? For example, backhaul links may correspond to non-overlapping frequency bands, while \( \{\beta_i\}_{i=1}^K \) may depend on the average interference seen by each band.

Fig. 3 plots the success probability for all possible \( (K!) \) backhaul assignments given \( K = 3 \) and \( \beta_i \in \{0.2, 0.4, 0.6\} \). As expected, this figure shows that the optimal strategy is to assign the backhaul links in descending order of reliability starting with the closest node, i.e., \( \beta_1 = 0.6, \beta_2 = 0.4 \), and \( \beta_3 = 0.2 \). We also study the case where the CU is faced by a given set of choices. This can, for example, model
that Φ for the considered system. used to determine the optimal backhaul assignment strategy backhaul link failure. The developed framework can also be used to model different network dynamics which contribute to liable backhaul links. Our framework is general as it can be the outage threshold is smaller than -6 dB. outperforms curve 6 when the outage threshold is around -6 of the receiver, albeit at the expense of further sacrificing the however, it is better to have a reliable backhaul in the vicinity of the receiver, albeit at the expense of further sacrificing the backhaul reliability at the closest node. For example, curve 4 outperforms curve 6 when the outage threshold is around -6 dB or higher while the latter gives a better performance when the outage threshold is smaller than -6 dB.

V. CONCLUSION

We have derived a closed-form expression to characterize the performance of a cooperative wireless network with unreliable backhaul links. Our framework is general as it can be used to model different network dynamics which contribute to backhaul link failure. The developed framework can also be used to determine the optimal backhaul assignment strategy for the considered system.

APPENDIX A: A PROOF OF THEOREM 1

We begin the proof of Theorem 1 by stating the following lemma. 

Lemma 1: For a set Ω = (ω1, · · · , ωk) consisting of elements which are not all zero, and a variable x, it follows that

$$\prod_{i=1}^{K} (\omega_i - x) = \sum_{i=0}^{K} (-1)^i \frac{\Omega^{(K-i)}}{x^i}$$

(A.1)

where CΩK\underline{=}1.

Proof: The above expression can be verified by expanding both sides in variable x. The proof is omitted for brevity. ■

We now find the characteristic function Φs(ˆΛ) of Λ = ζ|G|.

$$\Phi_{s}(jt) = \int_{-\infty}^{\infty} e^{jt\gamma} f_s(\epsilon) d\epsilon = q_i + p_i \frac{\lambda_i}{\lambda_i - jt} \Phi_{\epsilon_i}(jt)$$

(A.2)

where (A.2) follows by modeling the mixed distribution as $f_s(\epsilon) = q_i \mathbb{I}_{(\epsilon=0)} + p_i \lambda_i e^{-\lambda_i \epsilon} \mathbb{I}_{(\epsilon>0)}$, where $\mathbb{I}_{(\cdot)}$ is 1 when the condition in the subscript is true and is zero otherwise. We next find the characteristic function of $S_K$.

$$\Phi_{S_K}(jt) = \prod_{i=1}^{K} \frac{\lambda_i - q_i jt}{\lambda_i - jt} = \prod_{i=1}^{K} q_i \frac{\lambda_i q_i^{-1} - jt}{\lambda_i - jt}$$

(b) $Q \sum_{m=0}^{K} \alpha_m(\hat{\Lambda}) y^m$

(c) $Q \left( 1 + \sum_{m=0}^{K-1} (\alpha_m(\hat{\Lambda}) - \alpha_m(\Lambda)) y^m \right)$

(d) $Q \left( 1 + \frac{\sum_{m=0}^{K-1} (\alpha_m(\hat{\Lambda}) - \alpha_m(\Lambda)) y^m}{\sum_{i=1}^{K} (\lambda_i - y)} \right)$

(A.3)

where (a) follows from the property that the characteristic function of a sum of mutually independent random variables equals the product of individual characteristic functions. (b) results by substituting $y = jt$ and applying Lemma 1 to the numerator and the denominator in (a). Adding and subtracting $Q \sum_{i=0}^{K} \alpha_i(\Lambda)y^i$ from the numerator in (b), and using $\alpha_K(\hat{\Lambda}) = \alpha_K(\Lambda)$, we obtain an expression with a proper fraction in (c). Finally, the partial fraction expansion method is used to obtain the result in (d), where

$$\Upsilon_m(r_u, v) = \left. \left[ \frac{(-1)^a}{a!} \frac{\partial^a}{\partial y^a} \left( y^m \sum_{i=0}^{\tau} (\delta_i - y)^{-r_i} \right) \right] \right|_{y=r_u}$$

(A.4)

with $a = r_u - v$. Evaluating (A.4) results in (4). Applying inverse transform formula on (A.3) gives the probability density function, which upon integration, yields the CCDF of $S_K$ as given in (3).

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